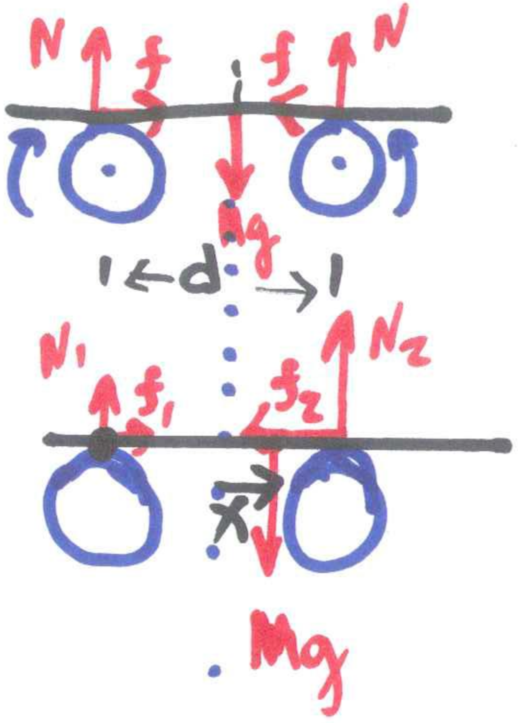
Σαβ. 15/1/22

1



M, g, μ

$f = \mu N$

$N_1 + N_2 = Mg$

$Mg\left(\frac{d}{2} + x\right) = N_2 d$

$$f_1 - f_2 = \mu N_1 - \mu N_2$$
$$= \mu(Mg - N_2) - \mu N_2$$
$$= \mu Mg - 2\mu N_2$$
$$= \mu Mg - 2\mu \frac{Mg}{d}\left(\frac{d}{2} + x\right)$$
$$= \cancel{\mu Mg} - \cancel{\mu Mg} - \frac{2\mu Mg}{d} x$$

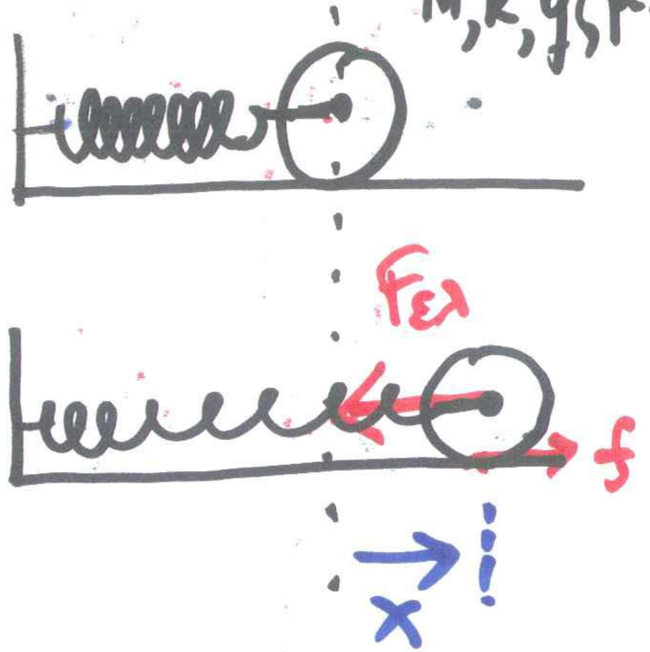
$$\Sigma F = -\left(\frac{2\mu Mg}{d}\right) \cdot x$$

α.α.τ. με $\omega = \sqrt{\frac{2\mu g}{d}}$

ΑΣΚΗΣΕΙΣ
ΣΤΟ ΑΚΑΜΠΤΟ
ΣΩΜΑ

2

$M, R, g(\mu_s), \kappa$



$$a = \alpha R$$

$$\underline{F_{\varepsilon\lambda} - f = Ma}$$

$$cm: \tau = I\alpha$$

$$f\cancel{R} = \kappa M R^{\cancel{2}} \cdot \frac{a}{\cancel{R}}$$

$$\underline{f = \kappa M a}$$

$$F_{\varepsilon\lambda} = (1+\kappa) M a$$

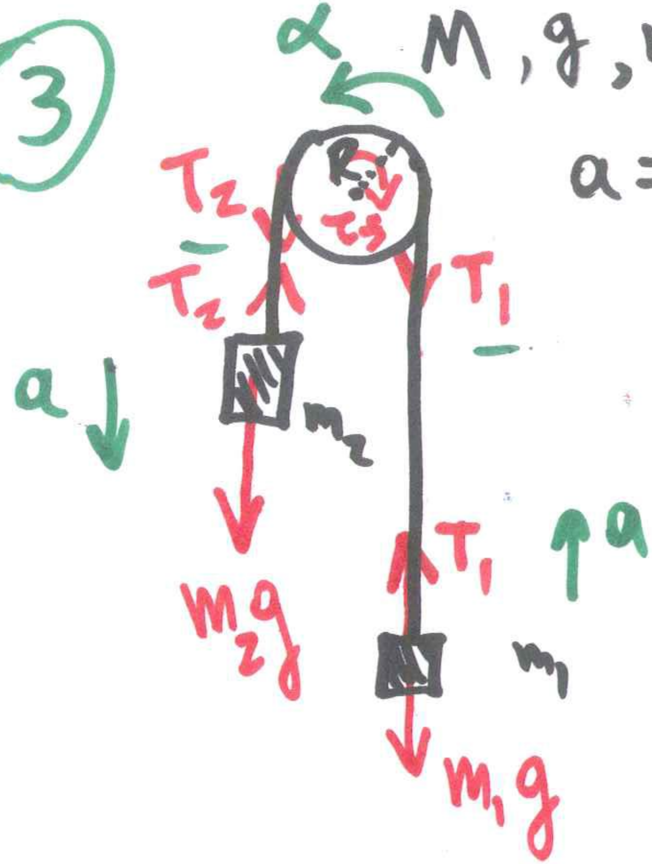
$$-kx = (1+\kappa) M a$$

$$a = -\left(\frac{k}{M} \frac{1}{1+\kappa}\right) x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m(1+\kappa)}}$$

3



M, g, k

$a = \alpha R$

$$T_1 - m_1 g = m_1 a$$

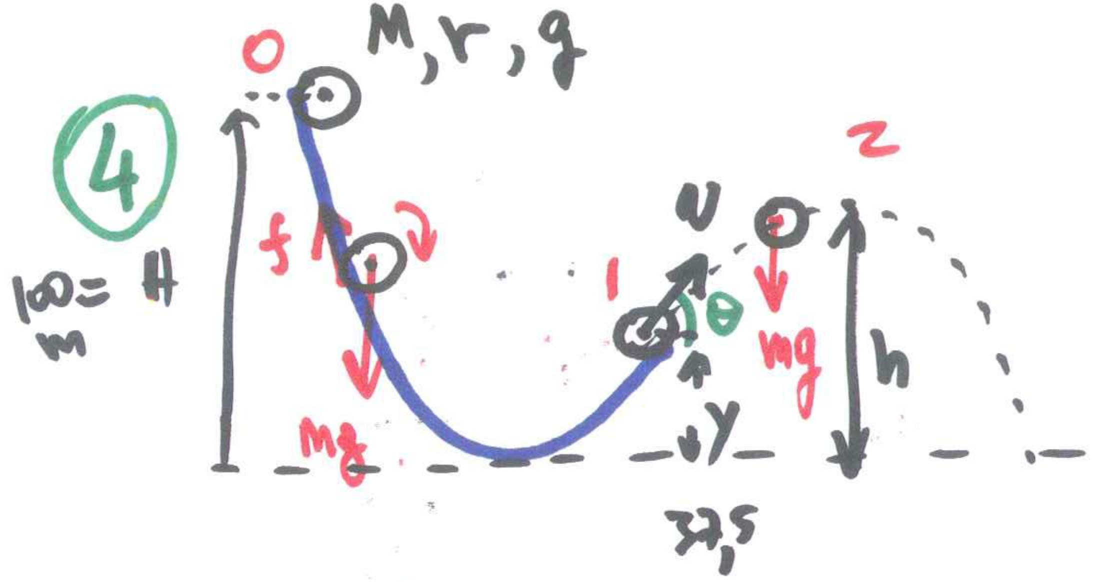
$$m_2 g - T_2 = m_2 a$$

$$-\tau_f + (T_2 - T_1)\cancel{R} = (kM R^{\cancel{2}}) \cdot \frac{a}{\cancel{R}}$$

$$-\frac{\tau_f}{R} + (m_2 - m_1) g = (m_1 + m_2 + kM) a$$

$$a = \frac{(m_2 - m_1) g - \tau_f / R}{m_1 + m_2 + kM}$$

4



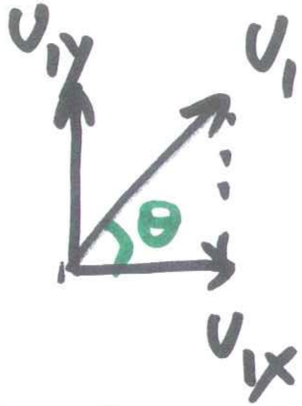
$\sin\theta = 0{,}6$

$\cos\theta = 0{,}8$

$v_{cm} = \omega R$

$U_g = 0$

$k = 1$



$$K = K_T + K_R = \frac{1}{2} m U_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 = \frac{1}{2} m U_{cm}^2 + \frac{1}{2} k M \cancel{R^2} \cdot \frac{U_{cm}^2}{\cancel{R^2}}$$

$$K = \frac{1+k}{2} m v_{cm}^2 \xrightarrow{k=1} m v_{cm}^2$$

$$E_0 = E_1 \Rightarrow \cancel{m} g H = \cancel{m} g y + \cancel{m} v_1^2 \Rightarrow g(H-y) = v_1^2 = 10 \cdot 62{,}5 = 625$$

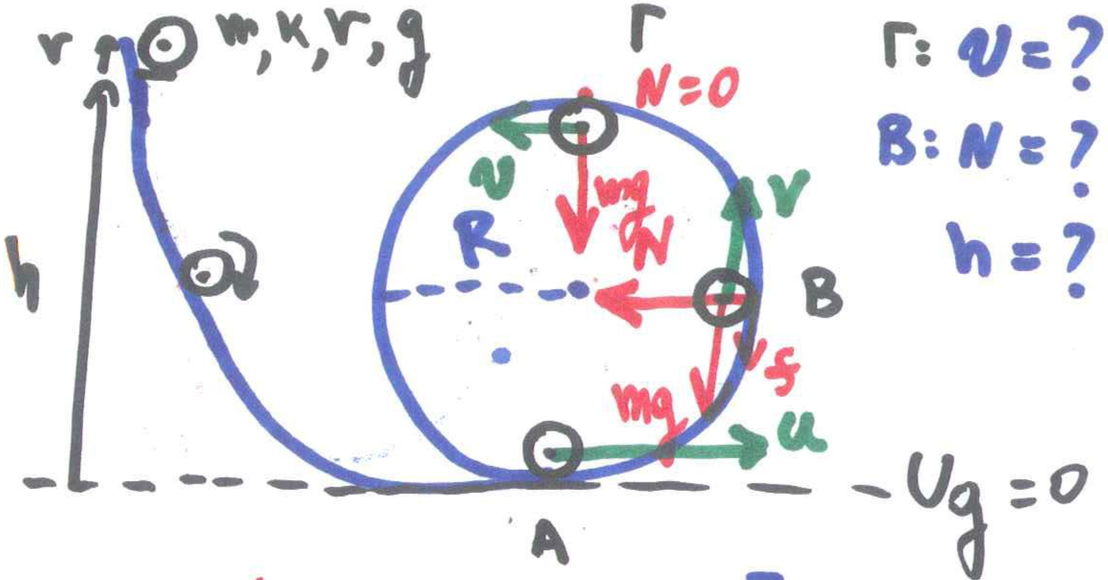
$$\cancel{m} g y + \frac{1}{2} m \cancel{v_{1x}^2} + \frac{1}{2} \cancel{m} v_{1y}^2 + \frac{1}{2} \cancel{I \omega_1^2} = \cancel{m} g h + \frac{1}{2} \cancel{m v_{2x}^2} + \frac{1}{2} m v_{2y}^2 + \frac{1}{2} \cancel{I \omega_2^2}$$

$v_1 = 25$ m/s

$v_{2y} \to 0$

$$h = y + \frac{v_{1y}^2}{2g} = 37{,}5 + \frac{(25 \cdot 0{,}6)^2}{2 \cdot 10} = \ldots \text{ m}$$

5



Γ: $\upsilon = ?$

B: $N = ?$

$h = ?$

$$E_\Gamma = E_B$$

$$\frac{1+k}{2} m\upsilon^2 + mg(2R - r) = \frac{1+k}{2} m v^2 + mgR$$

$$\Gamma: \quad \cancel{N} + mg = m \cdot \frac{\upsilon^2}{R - r} \Rightarrow \upsilon^2 = g(R - r)$$

οριακή ανακύκλωση

$$\upsilon = \sqrt{gR} \quad \text{υλικ. σημ.}$$

$$B: \quad N = m\frac{v^2}{R - r} = \frac{m}{\cancel{R - r}} \cdot \frac{3+k}{1+k} g\cancel{(R - r)} = mg \cdot \frac{3+k}{1+k}$$

$$\frac{(1+k)}{2} g(R-r) + g(\cancel{2}R-r) = \frac{1+k}{2} v^2 + \cancel{gR}$$

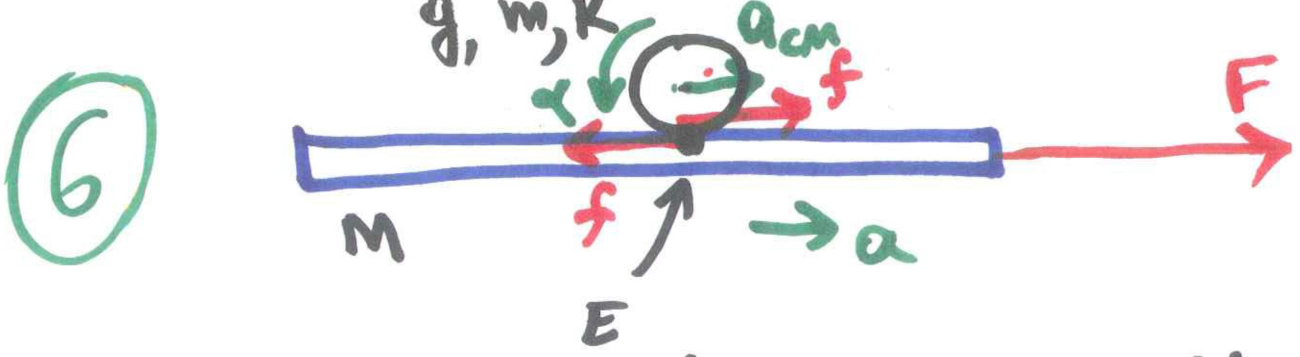
$$g(R-r)\left(\frac{1+k}{\cancel{2}} + \cancel{1}^{\,2}\right) = \frac{1+k}{\cancel{2}} v^2$$

$$v^2 = \left(\frac{3+k}{1+k}\right) g(R-r) \xrightarrow{\text{vl.6yp}} v = \sqrt{3gR} \checkmark$$

$$E_0 = E_r \Rightarrow mg(h+r) = \frac{1+k}{2} g(R-r)$$

$$\Rightarrow h = \dots$$

⑥



επαφής $\quad v_E(\text{σφαίρ}) = v_E(\text{χαλί})$

$$v_{cm} + \omega R = v \Rightarrow a_{cm} + \alpha R = a$$

χαλί: $\quad F - f = Ma \Rightarrow F - ma_{cm} = M(a_{cm} + \alpha R)$

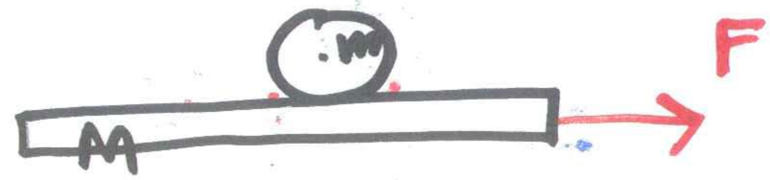
σφαίρα: μεταφ. $\quad f = ma_{cm} \qquad F = ma_{cm} + Ma_{cm} + M\dfrac{a_{cm}}{k}$

$$\boxed{a_{cm} = \frac{F}{m + M(1 + 1/k)}}$$

περιστρ.: $\quad fR = kmR^2 \cdot \alpha$

$$f = kmR\alpha \Rightarrow \alpha R = \frac{f}{km} = \frac{\cancel{m}a_{cm}}{k\cancel{m}} = \frac{a_{cm}}{k}$$

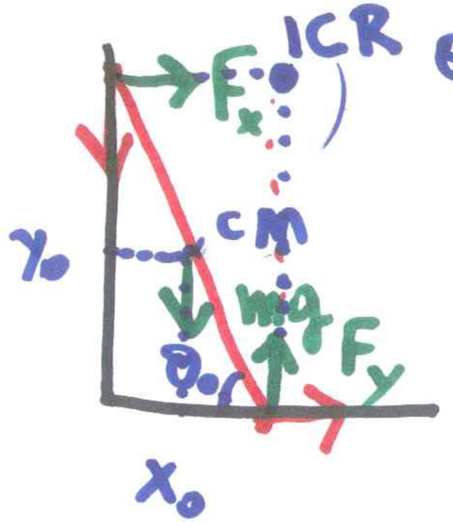
$$a = a_{cm} + \alpha R = a_{cm} + \frac{a_{cm}}{k} = \frac{(1 + 1/k)F}{m + M(1 + 1/k)}$$



$$A = \frac{F}{m+M}$$

$$A = \frac{m a_{cm} + M a}{m+M}$$

ελέγξτε
ότι είναι ίσα



$\theta_0 = 85^0$ m, L, g

(7)

$\theta = ?$ χάσιμο επαφής

$F_x = 0 \Rightarrow a_x = 0 \Rightarrow v_x = \max$

$x_0 = L/2 \cos\theta$
$y_0 = \frac{L}{2}\sin\theta$ $\Rightarrow$ $x = \frac{L}{2}\cos\theta$
$y = \frac{L}{2}\sin\theta$ $\Rightarrow$ $\dot{x} = -\frac{L}{2}\sin\theta \cdot \dot{\theta}$
$\dot{y} = \frac{L}{2}\cos\theta \cdot \dot{\theta}$ $\Rightarrow$ $\ddot{x} = -\frac{L}{2}\cos\theta\,\dot{\theta}^2 - \frac{L}{2}\sin\theta\,\ddot{\theta} = 0$

$mgy_0 = mgy + \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}w^2 \Rightarrow$

$v^2 = \dot{x}^2 + \dot{y}^2 = \frac{L^2}{4}\sin^2\theta\,\dot{\theta}^2 + \frac{L^2}{4}\cos^2\theta\,\dot{\theta}^2$

$v^2 = \frac{L^2}{4}\dot{\theta}^2$

$$\cancel{m}g(y_0-y) = \frac{1}{2}\cancel{m}\frac{L^2}{4}\dot{\theta}^2 + \frac{1}{2}\cdot\frac{1}{12}\cancel{m}L^2\cdot\dot{\theta}^2$$

$$g\frac{\cancel{L}}{\cancel{2}}(\sin\theta_0 - \sin\theta) = \frac{1}{\cancel{2}}\left(\frac{1}{4}+\frac{1}{12}\right)L^{\cancel{2}}\dot{\theta}^2$$

$$\boxed{\dot{\theta}^2 = \frac{3g}{L}(\sin\theta_0 - \sin\theta)}$$

$$2\cancel{\dot{\theta}}\cdot\ddot{\theta} = -\frac{3g}{L}\cos\theta\cdot\cancel{\dot{\theta}}$$

$$\boxed{\ddot{\theta} = -\frac{3g}{2L}\cos\theta}$$

$$\ddot{x} = 0 \Rightarrow \cos\theta \cdot \dot{\theta}^2 + \sin\theta\, \ddot{\theta} = 0$$

$$\cancel{\cos\theta} \cdot \frac{\cancel{3g}}{\cancel{L}}(\sin\theta_0 - \sin\theta) - \sin\theta \frac{\cancel{3g}}{2\cancel{L}} \cancel{\cos\theta} = 0$$

$$2\sin\theta_0 - 2\sin\theta = \sin\theta$$

$$\boxed{\sin\theta = \frac{2}{3}\sin\theta_0} \qquad y = \frac{2}{3}y_0$$

$$\theta = \sin^{-1}\left(\frac{2}{3}\sin 85^\circ\right) = \sin^{-1}\left(\frac{2}{3}\cdot 0{,}996195\right)$$

$$= \sin^{-1}(0{,}83549)$$

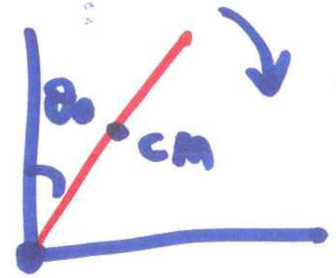
$$\approx 41{,}6^\circ$$

$$\tau_{ICR} = I_{ICR} \cdot \alpha \qquad I_{ICR} = I_{cm} + md^2$$

$$\cancel{m}g \cdot \frac{L}{\cancel{2}} \cos\theta = \left(\frac{1}{12} \cancel{m} L^{\cancel{2}} + \cancel{m} \left(\frac{L}{2}\right)^{\cancel{2}} \right) \alpha$$

$$\alpha = \frac{3g}{2L} \cos\theta = \ddot{\varphi} \Rightarrow \ddot{\theta} = -\frac{3g}{2L}\cos\theta$$

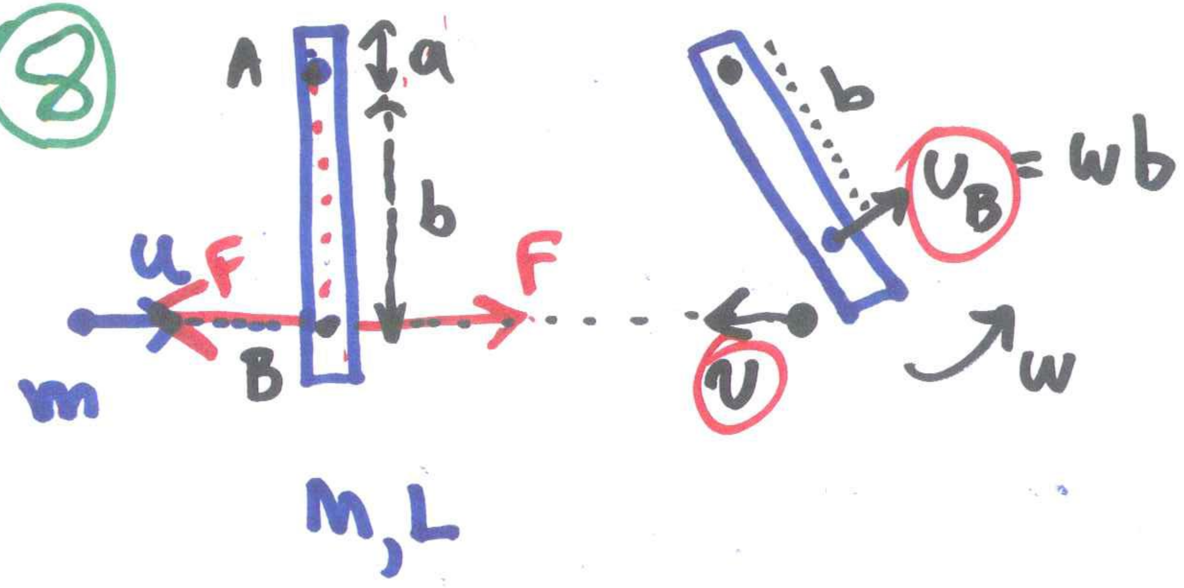
$$\varphi + \theta = \frac{\pi}{2} \Rightarrow \dot{\varphi} = -\dot{\theta} \Rightarrow \ddot{\varphi} = -\ddot{\theta}$$



ICR

→ παλι, $y = \frac{2}{3} y_0$

(8)



$$I_a = \frac{1}{3} M \left(L^2 - 3La + 3a^2\right)$$

$$\frac{I_a}{b^2} = m_{ab}$$

$$L'_A = L_A \Rightarrow mvb + I_a\omega = mub \Rightarrow m(u-v) = \frac{I_a}{b^2}\omega b = m_{ab} v_B$$

$$K' = K \Rightarrow \cancel{\tfrac{1}{2}} m v^2 + \cancel{\tfrac{1}{2}} I_a \omega^2 = \cancel{\tfrac{1}{2}} m u^2 \Rightarrow m(u-v)(u+v) = \frac{I_a}{b^2}\omega^2 b^2$$

$$\boxed{m(u-v) = m_{ab} v_B}$$

$$m(u-v)(u+v) = m_{ab} v_B^2$$

$$\boxed{u + v = v_B}$$

$$\boxed{v = \frac{m - m_{ab}}{m + m_{ab}} u \qquad v_B = \frac{2m}{m + m_{ab}} u}$$