$$\Rightarrow r\frac{dT}{dr} = C_1 \Rightarrow \frac{dT}{dr} = \frac{C_1}{r} \Rightarrow dT = \frac{C_1}{r}\cdot dr \Rightarrow$$

$$\Rightarrow T(r) = \int \frac{C_1}{r}\,dr + C_2 = C_1\cdot \ln r + C_2 \Rightarrow \frac{d^2T}{dx^2} = 0$$

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\cancel{k}\, r\frac{\partial T}{\partial r}\right) + \cancel{g}^{\,0} = \rho\cdot c\,\cancel{\frac{\partial T}{\partial t}}^{\,0} \Rightarrow \frac{1}{r}\cdot\frac{d}{dr}\cdot\left(r\frac{dT}{dr}\right) = 0$$

$$\frac{d^2T}{dx^2} = 0 \Rightarrow T(x) = C_1\cdot x + C_2 \qquad f(r)$$

$$\frac{1}{r}\cdot\frac{d}{dr}\cdot\left(r\frac{dT}{dr}\right) = 0 \Rightarrow \frac{d}{dr}\boxed{\left(r\frac{dT}{dr}\right)} = 0 \Rightarrow \frac{d}{dr}f(r) = 0$$

$$\left\{\begin{array}{l} f(r) = C_1 \Rightarrow \\ r\cdot\frac{dT}{dr} = C_1 \end{array}\right.$$

$\underline{\underline{n=2}}$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\cdot\left(\cancel{k}\, r^2\frac{\partial T}{\partial r}\right) + \cancel{\dot{g}}^{\,0} = \rho\cdot c\,\cancel{\frac{\partial T}{\partial t}}^{\,0}$$

$$\frac{1}{r^2}\cdot\frac{d}{dr}\cdot\underbrace{\left(r^2\frac{dT}{dr}\right)}_{f(r)} = 0 \Rightarrow \frac{1}{r^2}\cdot\frac{d}{dr}\cdot f(r) = 0 \Rightarrow$$

$$\Rightarrow \frac{d}{dr} f(r) = 0 \Rightarrow f(r) = C_1 \Rightarrow$$

$$\Rightarrow r^2\frac{dT}{dr} = C_1 \Rightarrow dT = \frac{C_1}{r^2}dr \Rightarrow T(r) = \int \frac{C_1}{r^2}dr + C_2$$

$$\Rightarrow T(r) = -\frac{C_1}{r} + C_2$$

$$dT = \left[-\frac{\dot g}{2k} r + \frac{1}{r}\right] dr \Rightarrow T(r) = -\frac{\dot g}{4k} r^2 + C_1 \cdot \ln r + C_2$$

$$\frac{1}{r} \cdot \frac{d}{dr}\left(k \cdot r \frac{dT}{dr}\right) + \dot g = 0 \qquad k = 62000.$$

$$\frac{1}{r} \cdot \frac{d}{dr} \cdot \left(r \cdot \frac{dT}{dr}\right) + \dot g / k = 0 \Rightarrow \frac{1}{r} \frac{d}{dr} \cdot \left(r \frac{dT}{dr}\right) = -\dot g / k$$

$$\frac{d}{dr} \cdot \left(\overbrace{r \frac{dT}{dr}}^{f(r)}\right) = -\frac{\dot g}{k} \cdot r \Rightarrow \frac{d}{dr} f(r) = -\frac{\dot g}{k} r \Rightarrow$$

$$\Rightarrow df(r) = -\frac{\dot g}{k} r\, dr \Rightarrow f(r) = -\frac{\dot g}{2k} r^2 + C_1 \Rightarrow$$

$$\Rightarrow r \frac{dT}{dr} = -\dot g / 2k \, r^2 + C_1 \Rightarrow \frac{dT}{dr} = -\frac{\dot g}{2k} r + \frac{C_1}{r} \Rightarrow$$

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(k r^n \frac{\partial T}{\partial r}\right)+\dot{g}=\rho c \cancel{\frac{\partial T}{\partial t}} \to 0 \qquad n=2$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\cancel{k}\, r^2 \frac{dT}{dr}\right)+\dot{g}=0 \Rightarrow \qquad f(r)=r^2\frac{dT}{dr}$$

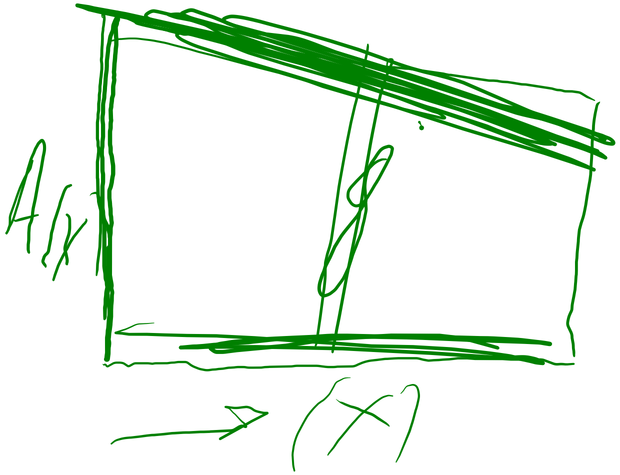
$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}\cdot r^2 \Rightarrow \frac{d}{dr}f(r)=-\frac{\dot{g}}{k}r^2$$

$$f(r)=-\frac{\dot{g}}{3k}\cdot r^3+C_1 \Rightarrow r^2\cdot\frac{dT}{dr}=-\frac{\dot{g}}{3k}r^3+C_1$$

$$\Rightarrow \frac{dT}{dr}=-\frac{\dot{g}}{3k}\cdot r+\frac{C_1}{r^2} \Rightarrow T(r)=-\frac{\dot{g}}{6k}r^2-\frac{C_1}{r}+C_2$$

$$\frac{1}{A} \cdot \frac{\partial}{\partial x}\left(k A \cdot \frac{\partial T}{\partial x}\right) + \dot{q} = \rho \cdot c \, \frac{\partial T}{\partial x} \to 0$$

$$\frac{d}{dx} \cdot \left(A(x) \frac{dT}{dx} \right) = - \frac{\dot{q}}{k} A(x)$$



① κυλινδρική συμμετρία

② σημείο παραγωγής θερμότητας ($\dot{g}$)

③ k δεν είναι σταθερό $\rightarrow$ $k(T)$ $\rightarrow$ $k(T(r))$, $k(r)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k \cdot r \frac{\partial T}{\partial r}\right) + \dot{g} = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(k \cdot r \frac{dT}{dr}\right) + \dot{g} = 0 \Rightarrow \frac{d}{dr}\left(k r \frac{dT}{dr}\right) = -\dot{g} \cdot r$$

$$k \cdot r \cdot \frac{dT}{dr} = -\frac{\dot{g}}{2} r^2 + C_1 \Rightarrow k \frac{dT}{dr} = -\frac{\dot{g}}{2} r + \frac{C_1}{r}$$

$$k \equiv k(r) \quad k(r) = k_0 + \lambda \cdot r \quad (k_0 + \lambda r)\frac{dT}{dr} = -\frac{\dot{g}}{2} r + \frac{C_1}{r} \Longrightarrow$$

$$(k_0+\lambda\cdot r)\cdot\frac{dT}{dr}=-\frac{\dot{g}}{2}r+\frac{C_1}{r}\Rightarrow$$

$$\Rightarrow dT=\left[-\frac{\dot{g}}{2}\cdot\frac{r}{k_0+\lambda r}+\frac{C_1}{r\cdot(k_0+\lambda r)}\right]dr\Rightarrow$$

$$\Rightarrow T(r)=-\frac{\dot{g}}{2}\cdot\int\frac{r\,dr}{k_0+\lambda r}+C_1\cdot\int\frac{1}{r\cdot(k_0+\lambda r)}dr+C_2$$

$$\int\frac{r}{k_0+\lambda r}dr=$$

$$u=k_0+\lambda\cdot r\Rightarrow du=\lambda\cdot dr\Rightarrow dr=\frac{du}{\lambda}$$

$$\lambda r=u-k_0\Rightarrow r=\frac{u-k_0}{\lambda}$$

$$\int\frac{\frac{u-k_0}{\lambda}}{u}\cdot\frac{du}{\lambda}=\frac{1}{\lambda^2}\int\frac{u-k_0}{u}du=\frac{1}{\lambda^2}\cdot\int\left[1-\frac{k_0}{u}\right]\cdot du$$

$$\frac{1}{\lambda^2}\int\left[1-\frac{k_0}{u}\right]du = \frac{1}{\lambda^2}\left[u - k_0 \ln u\right]$$

$$\left(\ln(k_0+\lambda r)\right)' = \frac{1}{k_0+\lambda r}\cdot\lambda$$

$$u = k_0+\lambda r$$

$$\frac{1}{\lambda^2}\cdot\left\{k_0+\lambda r - k_0\cdot\ln\left[k_0+\lambda r\right]\right\}$$

$$\int\frac{1}{r\cdot(k_0+\lambda r)}dr = \int\frac{(\ln r)'}{k_0+\lambda r}dr$$

$$g(x) = \ln r$$

$$f(x) = \frac{1}{k_0+\lambda r}$$

$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int g(x)\cdot f'(x)$$

$$\frac{1}{\lambda}\cdot\int\frac{\left(\ln(k_0+\lambda r)\right)'}{r}dr$$

$$\frac{1}{k_0+\lambda r}\cdot\ln r + \lambda\int \ln r\cdot\frac{1}{(k_0+\lambda r)^2}dr$$

$$\int \frac{1}{r(t_0+\lambda r)}\,dr = \frac{1}{\lambda}\int \frac{[\ln(t_0+\lambda r)]'}{r}\,dr$$

$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int g(x)\cdot f'(x)\,dx$$

$$\frac{\ln(t_0+\lambda r)}{r} - \int \ln(t_0+\lambda r)\cdot\frac{1}{r^2}\,dr$$

$$\int \ln(t_0+\lambda r)\cdot\left(-\frac{1}{r}\right)'\,dr$$

$$= \frac{\ln(t_0+\lambda r)}{r} + \frac{1}{\lambda}\int \frac{1}{r}\cdot\frac{1}{(t_0+\lambda r)}$$

$k(T)$

$$k(T)\cdot\frac{dT}{dr} = -\frac{g}{2}r + \frac{C_1}{r}$$

$$k(T) = k_0 + \lambda\cdot T^2$$

$$(k_0 + \lambda T)\,dT = \left[-\frac{g}{2}r + \frac{C_1}{r}\right]dr$$

$$k_0\cdot T + \frac{\lambda}{2}T^2 + C_2 = -\frac{g}{4}r^2 + C_1\ln r + C_3$$

$$\frac{\lambda}{2}T^2 + k_0 T = -\frac{g}{4}r^2 + C_1\cdot\ln r + C_3 - C_2 \quad C$$

$$T^2 + \frac{2k_0}{\lambda}T = -\frac{g}{2\lambda}r^2 + \frac{2C_1}{\lambda}\ln r + C'$$

$$T^2 + \frac{2k_0}{\lambda}T + \frac{g}{2\lambda}r^2 - \frac{2C_1}{\lambda}\ln r - C' = 0$$

$$Ax^2 + Bx + \Gamma = 0$$

$$\Delta = B^2 - 4A\Gamma$$

$$T_{1,2}(r) = \frac{-B \pm \sqrt{\Delta}}{2A}$$