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‘Me and maths’: towards a definition of attitude grounded on students’ narratives

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Abstract The attitude construct is widely used by teachers and researchers in mathematics education. Often, however, teachers’ diagnosis of ‘negative attitude’ is a causal attribution of students’ failure, perceived as global and uncontrollable, rather than an accurate interpretation of students’ behaviour, capable of steering future action. In order to make this diagnosis useful for dealing with students’ difficulties in mathematics, it is necessary to clarify the construct *attitude* from a theoretical viewpoint, while keeping in touch with the practice that motivates its use. With this aim, we investigated how students tell their own relationship with mathematics, proposing the essay “Me and maths” to more than 1,600 students (1st to 13th grade). A multidimensional characterisation of a student’s attitude towards mathematics emerges from this study. This characterisation and the study of the evolution of attitude have many important consequences for teachers’ practice and education. For example, the study shows how the relationship with mathematics is rarely told as stable, even by older students: this result suggests that it is never too late to change students’ attitude towards mathematics.

Keywords Attitude towards mathematics · Teachers’ education · Students’ failure in mathematics · Narrative research

Theoretical background

The construct *attitude* finds its origin in social psychology (Allport 1935), in connection with the problem of predicting individuals’ behaviour in contexts that involve choices based on simple preferences like buying goods or voting. In these studies attitude is

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generally described as a predisposition to respond to a certain object either in a positive or in a negative way. Early studies about attitude in mathematics education are placed in this framework, and focus on the relationship between attitude towards mathematics and school mathematics achievement, trying to highlight a *causal* relationship. As Neale (1969, p. 631) underlines:

> Implicit (…) is a belief that something called 'attitude' plays a crucial role in learning mathematics. (…) positive attitude toward mathematics is thought to play an important role in causing students to learn mathematics.

In actual fact, a meta-analysis on existing literature carried out by Ma and Kishor (1997) shows that the correlation between attitude and achievement is statistically not significant, and the results emerging from different studies are often non-comparable and even contradictory.

A critical feature of early studies in mathematics education is the scarce attention paid to the interaction between emotional and cognitive aspects in the context of specific mathematical activities. In the late 1980s, something changed after the book Affect and Mathematical Problem Solving (edited by McLeod and Adams 1989) was published. Following the theory of emotions developed by the psychologist George Mandler, there is a great emphasis on the cognitive origin of emotional factors, and vice versa emotional factors are invoked to interpret the behaviour of students involved in mathematical problem solving: for the first time, affective factors are taken into account to explain students' behaviour *internal* to mathematical activities. In one of his two contributions in this book, Mandler stresses some typical issues of previous research on attitude, posing some questions about the *aversion to mathematics*—'When in a child's school life do the first signs of aversion to mathematics appear? How are these signs first expressed in the learning situation?'—and underlining the importance of investigating 'how the «curiosity machine» [the student] turns into a «mathematical idiot»' (Mandler 1989, p. 240). The whole book is full of new and interesting research questions, but no complete answers are provided, as Mayer (1990, p. 36) claims: 'Affect and Mathematical Problem Solving raises interesting questions but provides few answers'. Anyhow, interest in the issues introduced by this book has been one of the main engines for the development of a specific research field devoted to study affect in mathematics education, in which more recent research about attitude is located (Zan et al. 2006).

In the early 1990s, McLeod (1992) stressed the need to develop a theoretical framework for affect in mathematics education, and identified three distinct constructs within the affective domain: emotions, beliefs and attitudes. DeBellis and Goldin (1999) proposed values as a fourth construct.

The need for a theory in the field of affect—useful to inform teacher education—involved in particular the construct of *attitude towards mathematics* and led to identifying some critical issues in existing research.

The lack of theoretical clarity

In mathematics education a large portion of studies about attitude do not provide a clear definition of the construct itself: often attitude is defined implicitly and a posteriori through the instruments used to measure it (Leder 1985; McLeod 1992; Ruffell et al. 1998; Daskalogianni and Simpson 2000; Di Martino and Zan 2001, 2002, 2003). Moreover, studies that explicitly give a definition of attitude do not share a single definition. In the

variety of meanings attributed to the construct, three main different types may be identified:

(a) A *simple* definition that describes attitude as the positive or negative degree of affect associated with mathematics (Haladyna et al. 1983).
(b) A *tripartite* definition that recognises three components in attitude: emotional response towards mathematics, beliefs regarding mathematics and behaviour related to mathematics (Hart 1989).
(c) A *bi-dimensional* definition in which, with respect to the previous one, behaviours do not appear explicitly (Daskalogianni and Simpson 2000).

It is natural to wonder: which is the *right* definition of attitude? The question does not allow a simple answer. As Kulm (1980, p. 358) suggests, analysing the use of the construct from several points of views: 'It is probably not possible to offer a definition of attitude toward mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful'. According to this point of view, the variety of definitions of attitude is not limiting but rather enriching for researchers, since different research problems can require different definitions. Hence, the previous question naturally changes from 'which is the *right* one?' to 'which is the *suitable* one for a certain research problem?' This is exactly how the definition of attitude takes up the role of a *working definition* (Daskalogianni and Simpson 2000). An important consequence of this position is that, in the context of mathematics education, the construct of 'attitude' is useful if it is characterised as an instrument capable of taking into account problems typical of mathematics education. As a matter of fact, as a working definition, the simple definition of attitude seems capable of predicting some students' choices (for example, the choice of their future educational career), but not adequate to face more complicated aspects, such as the way to possibly modify those choices, and to interpret students' behaviour in the mathematical activity (for example, problem solving), in which decisions that are more complex than choices are involved.

Another critical issue is related to the characterisation of *positive/negative* attitude: only in the case of the simple definition does this characterisation seem *natural*, since positive (negative) attitude is identified with positive (negative) emotional disposition towards mathematics. However, in the case of multidimensional definitions of attitude, what do 'positive' or 'negative' refer to? To each dimension individually? To one of them? Is it rather to be meant as a way to consider the *positive/negative* degree of the different dimensions? If that is the case, what sort of balance may be made?

Summarising, on the one hand, the simple definition appears not adequate to be a key to interpret students' behaviour in mathematical activities, on the other hand, the multidimensional definitions do not allow an immediate characterisation of positive/negative attitude.

The 'measurement' of attitude

Another critical point in research on attitude towards mathematics, related to the choice of a definition, is its *measurement* (Di Martino and Zan 2001, 2003). In most studies, questionnaires and Likert scales are the instruments typically used in research to assess attitude. These instruments generally propose items like: 'Mathematics is useful', 'I like problem solving', 'I think about arithmetic problems outside school'. Since these items are related to the three different dimensions—emotions, beliefs, behaviours—questionnaires make

implicit reference to the tripartite model. Assessment is generally made through measurement, typically according to the following methodology:

(a) for every item a different score is assigned to every possible answer,
(b) a total score is obtained adding the scores corresponding to the single items,
(c) then *positive* is associated with a high score and *negative* with a low one.

This methodology opens up a number of questions:

- How to choose the items? Are we sure that the object of the items is relevant for the respondent? For instance, in the case of beliefs, the respondents have to give an opinion about some beliefs chosen by the researcher: these beliefs might not be psychologically central in the respondent's belief system (using the terminology of Green 1971).
- How to choose the scores for the items? In other words: what is a positive emotion/belief/behaviour?
- If we do not take into account the interaction of the different dimensions for the final score, would this be consistent with the implicit use of a multidimensional definition of attitude? Is it consistent to *measure* a multidimensional construct with a single score?

Therefore, on the one side, criticism about *measurement* of attitude relates to the kind of instruments used (Ma and Kishor 1997), on the other side, it relates to the claim of the need and even the possibility of measuring attitude (Ruffell et al. 1998) rather than describing it. The need to measure attitude appears a necessary step in the traditional approach (essentially normative) which aims to point out a cause/effect relationship between attitude and achievement in mathematics, or more generally between attitude and behaviour. This aim leads to several theoretical issues. Many studies on attitude that make use of questionnaires assume an implicit cause and effect relation between beliefs, emotions and behaviour. The cause/effect relationship between beliefs and behaviour is often inferred from the transitivity of the chain:

$$\text{beliefs} \rightarrow \text{emotions} \rightarrow \text{behaviour}$$

In other words, a causal relationship between *negative* emotions and failing behaviour, as well as a causal relationship between certain beliefs and negative emotions, are implicitly assumed. From these assumptions, a causal relationship between beliefs and behaviour is inferred, but these implicit assumptions are questionable.

As we showed in a previous study (Di Martino and Zan 2002), the same belief can elicit different emotions in different individuals. We used a purposefully designed questionnaire in which students were asked to give not only their opinion regarding a belief, but also to choose which kind of emotion (like/dislike) they attached to this belief. For example:

☐ In mathematics there is always a reason for everything
☐ It is not true that in mathematics there is always a reason for everything
And...
☐ I like ☐ I don't like ☐ I am indifferent to
…this characteristic of mathematics

The results of the study were that all the six possible combinations in the following table were significantly covered:

	I like	I don't like	I am indifferent
In mathematics there is always a reason for everything			
It is not true that in mathematics there is always a reason for everything			

Then the study highlights that the interaction affect/cognition depends on the individual, thus questioning the possibility of finding the general cause/effect laws, and even questioning the purpose of searching for them. This suggests the need, in research on attitude, for a shift from a normative approach to an interpretative one: if in a normative approach 'attitude' is a construct finalised to *explain causes* of behaviour, thus enabling researchers to predict behaviour, in an interpretative one, it becomes a construct useful to describe the interaction between affect and cognition, enabling the observer to *understand motives* of intentional actions.

Results of research on the attitude construct, starting from the critical issues illustrated so far, were the basis of a theoretical framework for a narrative study we carried out with the aim of constructing a characterisation of attitude that strongly links to the problems emerging from practice, and, at the same time, being able to shape it. This study also inspired an Italian National project on the issue of negative attitude towards mathematics, founded by the Ministry of Instruction, Research and University (MIUR) in the years 2001–2004.[1]

In the next section, we will briefly report on some findings of the Project that highlight the significance of the construct 'negative attitude' for teachers' practice, and also suggest the need for a theoretical clarification, so that the construct might be used more effectively in teaching.

Teachers' use of the construct attitude: findings of an Italian National Project

The Project, named 'Negative attitude towards mathematics: analysis of an alarming phenomenon for culture in the new millennium', aimed to tackle the problems posed by an increasingly worrying educational emergency in mathematics, linked to a dramatic decrease in the number of students accessing scientific undergraduate courses in Italy. The Project was structured in two phases: the first phase aimed to test the educational significance of the construct 'attitude'; the second one was a study of the attitude towards mathematics of a number of different categories of people (teachers, mathematicians, adults and students). An in-depth theoretical study of the attitude construct cut across all the activities. We only report here a description of the second activity of the first phase (Polo and Zan 2006), aimed to recognise if, and eventually how, teachers actually use the construct 'negative attitude' in their practice. More precisely, the goals of the study were to

[1] Besides the authors, several Italian researchers participated in the Project: P. Ferrari (Alessandria); M. Polo (Cagliari); F. Furinghetti, F. Morselli (Genova); N. Malara (Modena); R. Tortora, D. Iannece, V. Vaccaro, M. Mellone (Napoli).

see whether, in their practice, teachers use the construct of negative attitude when they diagnose difficulty, and, if this is the case, to see how they use it, investigating:

(a) what type of definition they make reference to;
(b) if and how the diagnosis of negative attitude constitutes an instrument for intervening in a more targeted way on recognised difficulties.

A questionnaire was administered to 146 teachers from various school levels (primary, middle and high school).

As regards the use of the construct negative attitude in diagnosing students' difficulty, the results were that the diagnosis 'This student has a negative attitude toward mathematics' is used by most teachers (only 34% of the sample claim they have used it practically never or rarely).

Regarding the type of definition teachers make reference to the study highlighted that they never refer to the simple definition of attitude. Some teachers refer to students' beliefs about mathematics ('I diagnose negative attitude when the student says: mathematics is useless, difficult, made of mechanical rules,…'), others refer to the student's low mathematics self-concept ('He/she believes he/she is unsuited, not able to understand,…'), others to students' emotions (boredom, anxiety, fear, hatred, dislike…), and finally others to students' behaviour (little work at home, mechanical application of rules,…). Actually teachers interpret and use in different ways the same diagnosis 'negative attitude toward mathematics', thus producing a babel of meanings, that make communication difficult.

The most interesting results though, emerge from our investigation concerning if and how the diagnosis of negative attitude constitutes an instrument for intervening in a more targeted way on recognised difficulties. The study suggests that the diagnosis: 'this student has a negative attitude toward mathematics' is the teacher's causal attribution of the student's failure that the teacher perceives as uncontrollable. It seems to be often the final step of a series of unsuccessful didactical actions: a claim of surrender rather than a diagnosis capable of steering future action, a sort of *black box* rather than an accurate interpretation of a student's behaviour. Opening this black box, in order to turn the negative attitude diagnosis into a useful instrument for both practitioners and researchers, makes it necessary to clarify the construct from a theoretical viewpoint, while keeping in touch with the practice that motivates its use. This was exactly the aim of the narrative study that we describe in more details in the next paragraphs.

Towards a definition of attitude grounded in practice

In order to construct a characterisation of attitude, in particular of negative attitude, we investigated which dimensions students use to describe their relationship with mathematics. We needed an instrument consistent with an interpretative approach, capable of capturing students' relationship with mathematics, giving voice to the students through the possibility of talking about the aspects *they* considered relevant for their own experience with mathematics. To reach this goal, we chose to use students' narratives about their own story with mathematics.

In educational research the main claim for the use of narrative is, according to Connelly and Clandinin (1990, p. 2) 'that humans are storytelling organisms who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world'. The scholars identify different modalities for collecting

narrative data: field notes of shared experiences, journal records, interviews, storytelling, letter writing, autobiographical and biographical writing, and other narrative data sources.

In the field of mathematics education, narratives and other non-traditional methods are more and more often used, especially in research about teachers' beliefs and teachers' practice (Brown and Cooney 1991; Chapman 1997, 2002; Da Ponte 2001), where many scholars underline the need for social and anthropological approaches (Eisenhart 1998; Arsac et al. 1992; Bishop 1998). Indeed, narratives include the *tacit knowledge* underneath practice which is difficult to express in explicit form (Polanyi 1958): this tacit knowledge embeds teachers' deep beliefs that influence practice. Outside the field of teacher education, less numerous studies in mathematics education make use of narratives: some have students as their object (Ruffell et al. 1998), others adults (Karsenty and Vinner 2000), and some researchers also used narrative to report their own research (Hannula 2004).

In our study to stimulate students' narration we proposed the essay 'Me and maths: my relationship with mathematics up to now'. The only indication given by the researchers to the students is the title: differently to what happens with questionnaires, the narrator is not required to express his/her position about items chosen by others (probably on aspects not relevant for him) and he/she can talk about the aspects he/she considers relevant for his/her own experience with mathematics.

Our hypotheses in choosing autobiographical essays is that pupils will tend to explicitly evoke those events and remarks about their past that they deem important *here and now*, and they will also tend to paste fragments, introducing some causal links, not in a logical perspective but rather in a social, ethical and psychological one (Bruner 1990). Furthermore, we assume that in order to describe the kind of relationship an individual has with mathematics, this pasting process—typical of autobiographical narratives—is more important than an *objective* report of one's experience with the discipline at school. These remarks lead us to a delicate issue, related to the use of autobiographical stories in research: their *reliability*. If it is true that any instrument poses this problem—for instance Schoenfeld (1989) underlines the issue of the mismatch between beliefs exposed and beliefs in practice—it is equally true that, in the case of narrative, the issue of truth is approached with methods that differ from positivist ones. The interesting thing is the construction of what Spence (1982) calls *narrative truth* which may be closely linked, loosely similar, or far removed from *historical truth*. As Bruner (1990, pp. 119–120) claims:

> It does not matter whether the account conforms to what others might say who were witnesses, nor are we in pursuit of such ontologically obscure issues as whether the account is 'self-deceptive' or 'true'. Our interest, rather, is only in what the person thought he did, what he thought he was in, and so on.

The hypothesis underlying our research is that the narrative and autobiographic data collected would have allowed us to identify the dimensions students use to describe their relationship to mathematics, thus suggesting a characterisation of attitude towards mathematics that strictly links to practice.

The study

In this section, our study is described in details: data collection, type of approach and type of analysis.

The data

We proposed the essay to a large sample of students—precisely 1,496—ranging from grade 2 to grade 13: 707 from primary school (grade 2–5), 369 from middle school (grade 6–8), 420 from high school (grade 9–13).

One of the peculiarities of our research is exactly the large amount of data that we collected: usually studies using narratives involve small groups of individuals due to the large amount of time needed to analyse narrative materials (the collected essays constitute a convenient sample, i.e. not fixed on a statistical basis but rather obtained through a collaboration with teachers and heads of schools who accepted our requests).

For the administration of the essays we gave some guidelines in order to leave the students free to describe even criticism and strong negative emotions towards either mathematics or teachers: essays were anonymous, assigned and collected in the class not by the class mathematics teacher.

The approach to the data

Demazière and Dubar, in their study on the analysis of autobiographic interviews in the field of sociology, warn researchers about two types of temptations: the *illustrative* approach and the *restitutory* approach. In the former case, the collected material is used to illustrate the researcher's theoretical standpoints; in the latter case, material is returned in its original form, with no comments or interventions. These apparently antithetical choices both provide *key in hand solutions* to the 'problem of structuring collected data and theoretical issues, people's words and scientific concepts' (Demazière and Dubar 1997, p. 44). Moreover, Demazière and Dubar tackle the problem of what is the (subjective) *sense* of a story emerging from an interview, as well as the problem of identifying this sense through the analysis of the written text, with a step by step illustration of their own analysis of some interviews. In the approach they propose—called *analytical* approach—the text is analysed in order to systematically produce sense starting from people's words. Final outcome of this analytical process is the construction of a set of categories, properties, relationships, aimed at *understanding* behaviour: what Glaser and Strauss (1967) call a *grounded theory*, i.e. a theory discovered from the collected data, the construction of which requires a continuous back and forth between the different research phases. According to Glaser and Strauss, grounded theory is capable of realising one of the interrelated roles of theory, that is 'to be usable in practical applications—prediction and explanation should be able to give the practitioner understanding and some control of situations' (Glaser and Strauss 1967, p. 3). In actual fact, through

> an initial, systematic discovery of the theory from the data (…) one can be relatively sure that the theory will fit and work. And since the categories are discovered by examination of the data, laymen involved in the area to which the theory applies will usually be able to understand it, while sociologists who work in other areas will recognise an understandable theory linked with the data of a given area. (Glaser and Strauss 1967, pp. 3–4)

The analysis

An analysis fitting with the approach we adopted leads us to wonder, using Demazière and Dubar's words (1997, p. 37): 'How to analyze in order to understand?' In our case, we need

to *interpret* texts and identify a set of categories, properties, relationships, aimed at *understanding* the narrator's mathematical experience.

Lieblich et al. (1998), focusing on the process of reading and analysing a narrative (in particular life stories), identify two main independent dimensions:

(a) Holistic versus categorical.
(b) Content versus form.

Using their words:

> The first dimension refers to the unit of analysis, whether an utterance or section abstracted from a complete text or the narrative as a whole. […] The second dimension, that is, the distinction between the content and form of a story, refers to the traditional dichotomy made in literary reading of texts. (Lieblich et al. 1998, p. 12)

Combining these dimensions results in four modes of reading a narrative:

1. Holistic—content mode of analysis: the complete life story of an individual is used and focus is on the content presented by it.
2. Holistic—form—based mode: the plots or structure of complete life stories are examined.
3. Categorical—content mode (*content analysis*): categories of the studied topic are defined, and separate utterances of the text are extracted, classified and gathered into these categories/groups.
4. Categorical—form mode: focus is on discrete stylistic or linguistic characteristics of defined units of the narrative (for example, which kind of metaphors the narrator is using,…).

Since the four modes of analysis provide different kinds of information, we used all of them to better grasp the *meaning* of the text:

1. holistic—content to identify the core theme of an essay;
2. holistic—form to study its plot;
3. categorical—content to investigate categories;
4. categorical—form to identify recurrent metaphors used and compare different types of 'incipit', i.e. the beginning of the text.

For part of the analysis, we made use of T-Lab,[2] software consisting of linguistic and statistical tools to analyse texts. In order to use the software all the essays were transcribed, and some variables (for instance the school level) were chosen, as they were needed to carry out a comparative analysis (identification of specificities).

The richness of both results and remarks emerging from the analysis of the essays cannot be described globally.[3] Therefore, we present findings that we view as particularly significant for teachers' practice and teachers' education:

– the dimensions that students used to describe their own relationship to mathematics;
– the stories characterised by one or more changes in the quality of this relationship and the factors that seem to play a role in these changes.

[2] The bibliography related to T-lab is available on-line: http://www.tlab.it/en/presentazione.asp.

[3] The essays were translated by a bilingual expert, who tried to keep the sense of jargon terms and expressions as closer as possible to the original one.

The dimensions that students used to describe their own relationship to mathematics

According to a *grounded theory* approach, we used the collected data to *discover* a set of categories aimed at understanding how students describe their own relationship to mathematics. The construction of these categories required a repeated reading of the texts and a continuous back and forth between the different research phases: each new reading suggested new points for reflection and categories, which in turn led us to modify and refine the next phases of reading. In the following we briefly describe this process and its outcomes.

As a first step, we started from the investigation of how relevant the emotional disposition is, and in particular the use of the sentence 'I like/dislike mathematics' in students' description of their own relationship to mathematics. As we discussed earlier, in actual fact, the simple definition of attitude identifies this construct with the emotional disposition towards mathematics, concisely expressed by 'I like/dislike mathematics'. This reading led us to identify 986 essays (65.9% of the sample). In some essays *like/dislike* is the almost exclusive prevailing aspect.

> I do like mathematics because I like the teacher and the topics we deal with. Example: I like oral calculations. I enjoy it because I like worksheets, problems (…) and times tables. I also like Italian, but I like mathematics more than that. [2P.12][4]

Many essays of this type can be found, but only in the first years of primary school: this might point to the fact that this expression is mainly relevant at primary school level. This hypothesis is confirmed by the statistical analysis carried out by T-Lab which, while calculating specificities among categories of the various school levels (χ^2 calculation), highlights that the word 'piacere' ('to like') is the most characteristic of primary school essays, compared to the total number of essays, with a very high χ^2 value (613.39). This is probably a proof of the fact that reducing one's relationship with mathematics to the expression 'I like/dislike mathematics' means over-simplifying it. This simplification is useful to young students (like those in the first years of primary school) who avail of a limited lexicon and care for declaring the *direction* (positive/negative) of their relationship with mathematics.

In actual fact, when the expression 'I like/don't like mathematics' is used in older students' essays, it is found at the very beginning of the essay:

> To be honest, I like mathematics! (…) [2H.56]

In cases like this, the expression seems to be used in order to introduce the reader to the text, providing him/her with a warning about the *direction* of the relationship so that he/she might be somehow *guided* in reading the rest of the essay.

Even more often the expression 'I like/dislike mathematics' can be found at the very end:

> (…) in the end, I like mathematics very much. [1M.11]

In this case, the student seems to feel the need to summarise, both for himself and for the reader, all he has previously described, by defining the *direction* of his own relationship with mathematics.

[4] Here, as well as in the next excerpts, the first number refers to the class level, the letter refers to the school level (Primary/Middle/High), the last number indicates the progressive numbering of the essay within the category.

In any case, it seems that the expression 'I like/dislike' is generally used not to provide a detailed description of one's relationship with mathematics, which is usually done in the core part of the essay, through a narration of one's experiences and emotions; it is rather used because of a sort of need to decide a direction (positive or negative) for one's relationship with mathematics.

One interesting aspect, though, is that the use of the expression 'I like/dislike' is not necessary to describe either positive or negative features of one's relationship with mathematics from the emotional point of view, even in younger students' essays. This conclusion strongly emerges from the reading of essays in which the expression is not used at all, and the emotional dimension of the relationship is anyway transmitted in a (linguistically) complete and detailed way:

> To me mathematics is only a waste of time because once you have learned numbers, you can even stop, but no, we continue and lessons start to torture you slowly and it is an awful feeling when I write and don't understand, and it seems to me I'm going down to hell: I start sweating from head to feet, I go completely red and I feel like I'm exploding. [3P.28]

However, the emotional dimension, viewed in its general aspects and not reduced to 'like/dislike mathematics', has been studied by most of the researchers who investigate on attitude (regardless of the type of definition of attitude they refer to). Therefore, we also considered those essays which explicitly refer to (strong) emotions, such as *hate*, *love*, *fear*, *anger*,… At the end of this process, we obtained a total of 1,085 essays (72.5% of the sample) that explicitly refer to the *emotional dimension*.

The second step was to detect within this group the essays, in which emotions associated with mathematics are somehow justified (through utterances such as 'I like/hate/fear,… mathematics, *because*…'). The analysis of these motives aimed to identify a posteriori other significant dimensions used by the students to describe their relationship with mathematics, thus *grounding* on the collected data the next research phases.

In some essays the conjunction 'because' does not introduce a motive underlying the declared emotions, but only lists the *activities* that characterise the writer's experience with mathematics:

> I like mathematics very much because I like to calculate numbers, solve expressions and problems. [2M.30]

In all other cases we may identify three main types of *causes*. One frequent causal attribution of the writer's emotional disposition towards mathematics refers to the *teacher*, who acts as a crucial mediator in his/her relationship with mathematics:

> I liked the mathematics of the last five years because I had a teacher who used to explain the things I did not understand even for ten times. [1M.135]

It is therefore the role of the teacher which emerges as a mediating factor for the writer's relationship with mathematics. The other two types of causes we identified directly refer to that relationship, or anyway to at least one of the two subjects involved in this relationship, i.e. *mathematics* and the *writer*.

As regard 'mathematics', we most frequently found motives underlying a declared emotional disposition referring to the writer's *vision of mathematics*:

> I don't like it because there are many rules to make a tiny little operation you must divide one number by the other one, take away the number you had before and so on. Moreover, if you forget a rule you run into troubles! [1M.16]

> I never liked to learn things by heart (except for some formulae) and this subject, together with Physics, gives me a chance to think and discuss. I like it, because it is a subject which needs reasoning. [3H.16]

Reading these two excerpts, a question comes to our minds: is it the same mathematics? On the one hand, rules without reasons, leading to the need of memorising; on the other hand, knowing both what to do and why, thus stressing the role of reasoning. A widely spread distinction between these two visions of mathematics is that drawn by Skemp (1976) between *instrumental* mathematics and *relational* mathematics. This classification, similarly to others, represents a simplification and it is not an easy task to find neat positions or even recognise them by reading an essay. However, some indications emerge, often through the writers' *theories of success* (Nicholls et al. 1990), that is their beliefs about what needs to be done to be successful in mathematics. In particular, an instrumental view can be spotted in theories of success which emphasise the role of memory and recall a vision of mathematics as a set of rules to be memorised.

The link between vision of mathematics and emotional disposition is anyway a subjective one: shared aspects of mathematics can elicit different emotional dispositions in different people (Di Martino and Zan 2002). An example drawn from the essays concerns the widespread belief 'Mathematics is not an opinion':

> It is fascinating because it is not an opinion, it is a rational subject (like my own character), which needs no interpretation; … it is so. [5H.4]

> This does not mean that I like mathematics, actually I completely hate it, simply because it is a subject I feel really far from me. When you have to solve an equation you don't need to be creative, to interpret or say what you feel; mathematics is empty of feelings, just think of the well-known saying «mathematics is not an opinion». [5H.1]

Also the other most frequent cause that students report to justify a declared emotional disposition refers to the writer's relationship to mathematics, and involves both poles of it: 'I' and 'mathematics'. It is marked by utterances like 'I succeed/fail in mathematics', 'I understand/don't understand mathematics', 'I get good/bad marks in mathematics',… We classify this type of motive as referring to the writer's *perceived competence in mathematics.*[5]

> I like mathematics very much because I find it easy [4P.80]

> I don't like mathematics because my head can't take it. When the teacher explains something and I understand it, I like mathematics, but when I don't understand it I feel like crying. [2M.28]

As a third step, we analysed the essays which did not refer to the emotional dimension, in the light of those categories emerged in the previous step which *directly* refer to the writer's relationship to mathematics, i.e. vision of mathematics and perceived competence. This analysis points out that vision of mathematics and perceived competence turn out to

[5] We follow Pajares and Miller (1994), who see *perceived competence in mathematics* as part of *mathematical self-concept*, which includes beliefs of self-worth.

be meaningful themes also in the essays that do not refer to the emotional disposition: among the 411 essays which do not explicitly refer to the emotional dimension, 379 (92.2%) refer to vision of mathematics or perceived competence (in particular 21.7% *only* to vision, 27% *only* to perceived competence). Furthermore, in those essays we can almost always notice that the description of one's relationship to mathematics develops around one of these themes or rather both:

> To me mathematics is a subject immersed in a calculator which calculates and calculates, but never expresses anything, it's an iron heartless machine, only able to send out a couple of numbers, always the same ones. But mathematics is the history of numbers and if we want to talk about feelings, well, we got wrong because our heart is the house of feelings and it's better keep them in. [4P.85]

> I haven't got a good relationship with mathematics, I would rather say I am poor, because since primary school I have never been good, because I did not engage enough, I did little homework, especially in mathematics. In the class I was rarely diligent, and when the teacher saw me chatting with my classmates, started to shout and often she punished me, she was the most strict among the teachers. Of course I was not happy to get bad marks, I would have liked to improve, but I really could not get interested, because explanations were difficult and I could not understand. [3M.7]

At the end of this analysis, we obtained only 32 essays (2.1% of the entire sample) that did not refer to at least one of the three dimensions:

- Emotional disposition.
- Vision of mathematics.
- Perceived competence.

Summarising, we may claim that, when they describe their own relationship to mathematics, nearly all students refer to one or more of these dimensions: emotions, vision of mathematics and perceived competence. There are some essays that develop around only one of these three dimensions, more frequently essays make reference to all three (although they might be possibly centred on one of them):

> Mathematics and I mainly got on well, except for lower secondary school times when I went through a critical moment because I did not get on with the teacher and her teaching methods.
> I like mathematics, compared to other subjects, because you don't need to study for hours, if you understand how it works you just need to engage yourself and practice. It is not a boring subject, such as history for example, where you need to study a lot, remember names, dates and events and then after one year you have forgotten 90% of the things you've done.
> In mathematics you need to learn by heart at most some formulas or theorems. Now I haven't got big problems with this subject, doing rather well, the lessons are reasonably fun and this is a stimulus to pay attention in the classroom, at home I go back to the book to read about the explained topics and I do my homework, which is fortunately given in the right dose, some teachers actually overdo, and others never give homework.
> Anyway, at the moment mathematics and I have a good relationship. [3H. 98]

Usually when essays make reference to all three dimensions, these are deeply interconnected. As regards the connection between emotional dimension and perceived competence, it is so strong in some essays, that the terms that recall the two dimensions are used as synonyms:

> At the beginning of the year in grade 1, I didn't like mathematics much but as I went on I saw it was easy and I could understand it. [5P.348]

Here, there is a transition from emotions ('I like') to perceived competence ('I could understand it') by means of the word 'but'. The link between emotional disposition and perceived competence in mathematics is very complex, also because the perceived competence is linked to one's idea of success in mathematics. One of the most interesting outcomes of the reading of the essays, from both an educational and a theoretical perspective, is that *success in mathematics* has not a shared meaning among students. In some essays *succeeding* is identified with school success, i.e. with getting good marks, and thus it is up to the teacher to acknowledge one's success. In some other cases, it is identified with *understanding*, and therefore the student might be the one who acknowledges his/her own success. These positions are not easily distinguishable, because very often understanding is not well defined in essays, and it seems to be certified through achievement only. Even in those cases when success is clearly associated with understanding, things are complicated, since different meanings of understanding emerge: sometimes, it is identified with knowing the rules and being able to apply them correctly (an *instrumental* understanding); in other cases, a *relational* understanding appears, referring to one's awareness of why the rules work and how they are linked to one another.

> Now I'm in the second year and I attended the supplementary course and participated in the lessons with my teacher and I understood a bit but then I forget the mechanism. [2H. 53]

> Up to middle school I have always succeeded in mathematics, because I have always understood the reasoning paths, because at middle school we used to study more theory and we had more time to understand a topic than we have had during this school year. [1H.15]

The vision of mathematics is thus relevant regarding both the emotional disposition and the perceived competence in mathematics. The connection between these two dimensions—vision of mathematics and perceived competence—is often expressed by the student's causal attributions of success/failure (Weiner 1974):

> Sometimes I can't do it because there are divisions, multiplications, subtractions, additions that take too long. [4P.145]

> I am not done for mathematics because there are too many things to remember. [2M.10]

Theories of success and attributions of success/failure are thus indicators of both one's vision of mathematics and perceived competence. Moreover, attributions of success/failure often provide information about the writers' beliefs about him/herself, thus contributing to outline his/her mathematical self-concept:

> I can't do mathematics because sometimes I don't understand it, either because I'm 'thick', or because I don't get focused enough. [3H.27]

The reading of the essays, pointing out the subjectivity of the connection among the three dimensions, again underlines that a normative approach to these issues is not appropriate. Nevertheless it is still possible to identify recurrent patterns linked to a story of difficulty or unease: the most typical of these patterns is characterised by a negative emotional disposition ('I dislike mathematics') together with emotions like boredom, anger, frustration, fear and with an instrumental vision of mathematics joint to a low perceived competence.

An interesting finding is that, from the essays' analysis, the pattern characterised by a negative emotional disposition, a relational vision of mathematics and a high perceived competence *never* emerges. In other words, where a negative emotional disposition is explicitly stated, this disposition is always associated with either an instrumental vision of mathematics or to a low perceived competence.

At this point, it becomes interesting from both a theoretical and an educational viewpoint to analyse *if* and *because of what* the relationship with mathematics changes over time. In order to do so, we analyse those essays that tell stories characterised by changes in the quality of the student's relationship with mathematics: we call these essays *stories of change*.

Stories of change

In the literature different meanings of the term *story* can be found. In particular, there is a variety of positions about the difference between *stories* and *narratives*: some scholars use 'story' and 'narrative' as synonyms, others use 'narrative' in a broader sense that 'story', assuming that a story has a protagonist, a plot, and a turning point leading to a resolution; others, on the contrary, use 'story' in a broader sense than 'narrative', seeing narrative as a story told by a narrator. We consider all essays as narratives, and stories only those essays where a sequence of events in time can be recognised—what is called *a plot*—characterised by a beginning state, a middle action, a final state. Typically, the essays of younger students are not stories according to this meaning.

The development of the plot over time has been analysed by Lieblich et al. (1998). They underline that the first phase of the analysis is to identify the thematic focus for the development of the plot: in our case, we here refer to the plot of *the relationship with mathematics.* Three basic formats can be identified, depending on the progression of the narrative: a *progressive narrative*, in which the story advances steadily; a *regressive narrative*, characterised by a course of deterioration or decline; a *stable narrative*, in which the plot is steady. These three basic formats can be combined to construct more complex plots.

The analysis of plots related to the relationship with mathematics in our essays is of high theoretical and educational interest. First, it enables us to identify the middle level as the critical school level for pupils' relationship with mathematics;[6] furthermore, it shows how the relationship with mathematics is rarely told as stable, even by older students. This variability in particular, leads to an extremely important conclusion from the educational viewpoint: it is never too late to change one's own relationship with mathematics.

When an important change occurs, the writer describes with most details the moments in which this happens, moments that Bruner (1990) calls *turning points* (p. 121):

[6] This can be partly explained by the higher relevance of assessment starting from middle school level.

(…) at critical junctures, 'turning points' emerged, again culturally recognizable, produced almost invariably by an access of new consciousness aroused by victory or defeat, by betrayal of trust, and so on.

In this context, we call *stories of changes* those in which the plot describes an *inversion* in the quality of the relationship to mathematics.

Reading the essays we can notice that the relationship to mathematics is often characterised by *ruptures*, i.e. by *discontinuity points*. In these cases, change is described with more details, thus giving more information about the possible causes. The fact that these moments are narrated as jumps is probably a sign that the narrator views them as unexpected and traumatic, and tries to find a possible explanation for that discontinuity.

The relationship with mathematics can move from negative to positive:

(…) when I was at primary, it was the subject I most hated (…)
As I grew up I got more and more fond of mathematics, because I started to solve mathematical expressions (which are the things I prefer about this subject); so, after realising that I was starting to succeed with them, then mathematics or, to be more precise, algebra, became my passion. [2H.74]

Or from positive to negative:

I remember that I had a good relationship with mathematics: if I'm not wrong up to the second year of secondary school I enjoyed doing mathematics.
My relationship with this subject turned upside down when I changed my mathematics teacher in grade 10, since then I neither listened to nor studied mathematics.(…)
The third year of secondary school marked the final break with mathematics and the definite rejection towards studying mathematics, not only because I failed that year but also because the teacher changed again and this one was worse than the previous one. [5H. 7]

However, most frequently *ups and downs* can be found:

The first time I met mathematics was in the first year of primary school, and then my hatred for mathematics started, because of the times tables. (…)
And then I went to lower secondary school and there I most hated mathematics, as a matter of fact I didn't understand anything, mathematics was Arabic to me.
We were not made to be together, but then, who knows why, in grade 8 there was a Boom, I was like a sort of mathematician, I was so good that equations and problems and theorems seemed to be brothers of mine, I almost appeared as a genius in mathematics.
But, as it happens in dreams, good things never last for long and actually my achievement dropped in grade 9, but nothing serious: my relationship to mathematics depends on the moment. [1H.42]

The stories of change provide the highest amount of information on the factors that influence one's relationship with mathematics because the narrator often dwells upon details concerning turning points. We sometimes find turning points identified with some specific episodes:

> One day the teacher dictated the so called problem, I never understood how you were supposed to do it, and when I had that bad mark because of that problem, I stopped loving mathematics. [1H.39]

More often change is due to specific topics/activities:

> At primary and also at middle school (up to the beginning of literal expressions) I used to like mathematics. But when we started with the literal expressions at middle school some problems came up between mathematics and me. [1H.55]

Or even to the transition from one school to another one:

> Mathematics and I have not always had a bad relationship but, I must say that since I got to high school we become like + and −, i.e. two opposite things that are not mutually attracted. [4H.10]

This underlines how delicate the transition between school levels is a transition which brings about a change in the teacher's explanation and assessment methods, but also in the environment, regarding classmates and teachers. As far as the teacher is concerned, surely it is the most recurrent factor linked to change in the relationship with mathematics:

> My relationship with mathematics did not start well, because my primary school teacher only looked after the best pupils and this was not fair to me (…) My relationship with mathematics at lower secondary school got better because I had a teacher who looked after me; whereas my relationship at higher secondary school is rather good, maybe because the teacher is looking after me enough. [1H.27]

However, the role of the teacher—as we already pointed out in the analysis of causal links—is not limited to turning points. The teacher emerges as a crucial mediating factor with respect to the main three themes used to describe the relationship with mathematics (the emotional disposition, the vision of mathematics and the student's perceived competence), as significantly expressed in the following excerpt:

> To me mathematics would not be the same if we had another teacher. [4P.19]

Discussion

The results presented suggest a characterisation of attitude and in particular of *negative/positive* attitude towards mathematics. A multidimensional model emerges from students' description of their relationship with mathematics (Fig. 1). This model is characterised by three strictly interconnected dimensions:

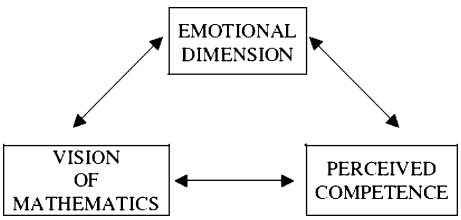

Fig. 1 The three-dimensional model for attitude

– Emotional disposition towards mathematics.
– Vision of mathematics.
– Perceived competence in mathematics.

The study also highlights the subjectivity of the interaction among the emotional disposition and the other dimensions, thus confirming the need for instruments capable of capturing this subjectivity.

The multidimensionality of the model underlines the inadequacy of the positive/negative dichotomy for attitude referred only to the emotional dimension (like/dislike), and rather suggests to consider an attitude as *negative*, when at least one the dimensions is *negative*. In this way, we can outline *profiles* of negative attitude, depending on the dimension that appears to be *negative*.

If the profile of negative attitude needs to be an effective instrument for both diagnosis and intervention, it would be appropriate to diminish its complexity. This is possible in particular, if we reduce each dimension to a dichotomy:

– Emotional disposition: positive/negative.
– Vision of mathematics: relational/instrumental.
– Perceived competence: high/low.

The simplification is twofold: on the one hand, it is a simplification of each of the three dimensions, reduced to one single aspect (for example, the vision of mathematics involves other than simply the instrumental/relational aspect); on the other hand, it comes from a radicalisation of the modalities of each of these aspects. In this way, we obtain eight different profiles, seven of which have at least one *negative* component.

Although the main purpose of our study was not to highlight relations of cause/effect between attitude and behaviour, our reading of the essays highlighted recurrent patterns in the stories characterised by failures and unease: a constant element is a low perceived competence, often joint to an instrumental vision of mathematics. A low perceived competence is reinforced by repeated experiences perceived as failures: in these cases, the student might become convinced that he/she has not suitable resources to control his/her own success in mathematics and consequently come to feel as useless to invest his/her own resources. This attitude, that might be defined as *fatalist*, may result in giving up thinking, and therefore in a failing behaviour, such as avoiding giving an answer, or answering randomly.

Within one's vision of mathematics, the idea of success and of the possibility of keeping under control associated factors thus appears as one of the most crucial for one's perceived competence and in particular for the interpretation of a possible failure.

Implications for teachers' practice and teachers' education

The results discussed above suggest implications for teachers' practice and for teachers' education. From a previous study, we observed that the teachers' diagnosis 'this student has a negative attitude toward mathematics' was a sort of black box, a claim of surrender by the teacher, rather than an accurate interpretation of the student's behaviour capable of steering future didactical action. We have tried to open this black box, to make the negative attitude construct a useful instrument for both practitioners and researchers. To render the construct 'usable in practical applications', giving the teacher 'understanding and some

control of situations' (Glaser and Strauss 1967), we have *grounded* a definition of attitude on the data collected from students' narratives.

The characterisation of attitude we proposed—in particular of negative attitude—is a tool for teachers to construct an accurate diagnosis, structured in the observation of the three identified dimensions (often underestimated in teaching), and aimed to identify the student's profile of attitude. In this way, the teacher may intervene towards a change of attitude focusing on the possible *negative* components of the profile.

The critical dimension will hardly be only the emotional one, since—as we saw earlier—the profile 'negative emotional disposition/relational vision/high perceived competence' did not emerge from any of the 1,496 essays. This suggests that the negative emotions towards mathematics a student expresses, might be important signs for the teacher, that may direct his/her attention to the vision of mathematics that student is constructing and to his/her perceived competence.

The identification of typical/recurrent patterns in stories of difficulty underlines the need for intervening on a negative attitude, as well as the stories of change highlight the possibility of actually doing it (also with secondary school students).

Assuming both the possibility and the need of changing a student's attitude towards mathematics, the question arises: how can the teacher act towards a change? The central role played by a low perceived competence, as emerging from the essays, underlines how important is for students to experience success in mathematics, and prior to this, how important it is to identify success so that its achievement becomes possible.

A shift in the idea itself of success in mathematics, from the production of correct and quick answers to the activation of meaningful thought processes, can result in a significant change in both perceived competence and emotional disposition. An activity centred on mathematical processes rather than on products—such as problem solving—can therefore become a valuable strategy to either prevent or overcome certain profiles of negative attitude. This is an example of how an intervention aimed at changing one dimension might impact the other dimensions as well, due to the deep interconnections of the three dimensions.

Particularly relevant for the dynamic relations between these interconnections appear to be the student's theories of success and causal attributions of failure/success, mutually deeply interconnected as well: they are precious indicators for the teacher to grasp students' vision of mathematics and perceived competence, which need to be continuously monitored.

The study also suggests some possible uses of the instrument 'essay' in teachers' practice (see for example McGivney, quoted in Fiore 1999).

First of all, the essay enables us to have a deeper individual knowledge of pupils and therefore to set up more targeted teaching strategies. Moreover, the vision of mathematics that emerges from the essays of the whole class may become a sort of monitoring of one's own practice: the teacher might for example be convinced he/she is conveying a relational image of mathematics, whereas his/her pupils' essays show an instrumental vision of mathematics.

More generally, the fact that the teacher proposes an essay to students conveys the message that he/she is interested in what they think and feel, and not only in their achievement in mathematics. Of course using essays in teaching practice has very different aims from using them in research: the choice of making pupils write their essays anonymously and that of involving other teachers may be both revised depending on the particular situation.

The use of the essay at school level, i.e. in the community of teachers, has further potential and may permit study of the evolution of pupils' relationship with mathematics

through the years: this might provide an opportunity to monitor the effects of the school and leave a trace of the stories of its pupils.

The importance of the dimensions that emerge from students' narratives suggests that teachers need to learn how to deal with students' emotions, vision of mathematics and perceived competence. From this need follows the importance of introducing these aspects in teacher education, as part of teachers' pedagogical content knowledge (according to Shulman's categorisation 1986).

We believe that the instrument 'essay' has potential in this direction as well. In actual fact, teaching someone to deal with students' affect is not an easy task, and it is important to create new instruments to allow working on this aspect with prospective teachers. Some of the students' essays, carefully selected, can be a useful material: for instance, teachers might be asked to recognise students' beliefs, emotions, theories of success, causal attributions of success/failure and a collective discussion about the possible answers might follow.

Mastering both theoretical knowledge and a specific language regarding the construct of attitude and more in general regarding affect, may help the single teacher in the processes of observation, interpretation and intervention and also make communication among teachers on their own practice easier.

Reflection on the role of the teacher in the construction of students' attitude towards mathematics naturally leads to the need to reflect with prospective teachers on *their own* vision of mathematics and on *their own* theories of success. Therefore, this suggests the need for a reflection with prospective teachers on *their own* attitude towards both mathematics and its teaching. The instrument 'essay' might be useful in this context as well, to open up and favour this type of reflection.

We would like to conclude by underlying how this study has enriched us both as researchers and as teachers and educators. Reading the essays, we got the opportunity to listen to meaningful stories, as well as to understand how important is for students to get the chance to talk about them:

> It's the first time that I write things like these on a sheet (…)
> This has been a great experience. Thanks for all. [5P.122]

Dealing with the use of narratives in research about teachers, Cortazzi (1993) claims that educational investigations, in general, have paid too little attention to teachers' voices. We think that *students'* voices were even more neglected. Our study made us convinced of how important is to let these voices talk.

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