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Doing Mathematics in the Workplace
A Brief Review of Selected Literature[1]

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Abstract

The aim of this review of selected literature on research into mathematics in the workplace is to offer researchers from the field of adult mathematics education an opportunity to become more familiar with this specialised area. In recent years much progress has been made, building on earlier research, but with increasingly nuanced understandings. In particular, sociocultural activity theory has played an important role, and several articles reviewed here have drawn on this theory. Their focus has been on gaining a comprehensive understanding of what workers actually do involving aspects of mathematics, set in the rich context of a functioning workplace. This encompasses taking account of workplace artefacts (technological & otherwise), various forms of communication (verbal & non-verbal), different forms of skills, and even the concept of boundary crossing; most importantly, how workers learn

Key words: workplace mathematics, technological competence, sociocultural activity theory

Introduction

The world of work has, throughout history and across cultures, incorporated mathematical thinking and communication into its tools, symbols, and organisational practices, as part of the production of goods and services. However, for the most part, the main objective is to get the job done as efficiently as possible, to satisfy a range of stakeholders, be they customers (external, internal, upstream, downstream), employers, shareholders, patients, audiences, and so forth.

There is no definitive workplace. Workplaces can range from a single trader (or tradesperson) right through to branches of multinational companies; workplace activities may be located at fixed sites, sites varying from job to job, or conducted virtually; work may be officially registered and recognised, or part of the informal economy. The work undertaken can vary in intensity, complexity, and responsibility; weekly hours can vary, and employment can be full-time, part-time, or casualised. Work regimes themselves are not necessarily desirable for

[1] This article is an extended version of a paper presented at ALM 19, Auckland, New Zealand, in June 2012.

individuals: too little or too much work, too many hours or too few, infrequent or unpredictable calls on their labours. Similarly, conditions of work can vary within and between workplaces.

Mathematics-containing workplace technologies in the form of tools or equipment, from everyday implements to highly technical robotic or other instrumentated substitutes for human labour, often vastly exceed human capacities in their speed, accuracy, memory capacity, tolerance of dangerous or extreme physical conditions, and so forth. Besides being embedded in technological artefacts, mathematics is also rendered invisible in technologies of management, such as Quality Control [QC] regimes, standard operating procedures [SOPs], or other prescribed routines (FitzSimons, 2002). Formal mathematics —the academic mathematics familiar to most researchers— often only becomes visible in situations of breakdown.

Early research generally tried to identify and map workplace mathematics onto existing school mathematics curricula — itself an arbitrary selection from the discipline (Ernest, 1991). School mathematics, regarded as a proxy for academic mathematics, was taken as the norm, and workplace mathematical activity was accorded lower status. The earliest researchers tended to focus only on that mathematics which was visible (Harris & Evans, 1991), ignoring not only the crystallised mathematics embedded in workplace technological artefacts and routines, but also the entire sociocultural, political, and historical contextual settings of their observations.

In formal education, both general and vocational, the development of mathematical skills and techniques is the focus of attention and the object of the activity in the mathematics classroom; whereas, in the workplace, mathematics — if it is noticed at all — is almost always regarded as but one tool in the process of achieving a desired outcome. Workers, past and present, generally do not regard what they do as being mathematics nor themselves as being mathematical (e.g., Wedege, 1999).

Over a decade ago, I published[2] a comprehensive review of research into workplace mathematics (FitzSimons, 2002). In recent years much progress has been made, building on earlier research, but with increasingly nuanced understandings. In particular, sociocultural activity theory — among others — has played an important role, and several articles reviewed here have drawn on this theory. Their focus has been on gaining a comprehensive understanding of what workers actually do involving aspects of mathematics, set in the rich context of a functioning workplace.

This article aims to provide a synthesis of selected workplace research over the last decade or so, with a view to answering the question of what knowledge has been gained since then. The focus is on the ‘world of work’ in relation to the mathematics that workers use mostly unconsciously, transform, or even create locally, and communicate through language, gestures, signs, and other non-verbal communication, in order to address the ever-evolving problems of their particular workplace activity.

Research Methodologies and Important Theoretical Concepts Used in the Workplace Literature Reviewed

The studies discussed in this literature review have adopted predominantly qualitative methods, with most adopting ethnographic approaches and many utilising socio-cultural activity theory, based on the work of Vygotsky and Leont’ev (see, e.g., Roth & Lee, 2007, for a comprehensive review). In these approaches, the *unit of analysis* is the *activity* itself which is generally undertaken by a group of people, (such as a work unit) in order to satisfy a *motive* (such as the production of a good or a service, in their broadest interpretation). Various *actions* are undertaken to achieve a range of *goals* supporting the activity, and, in turn, these depend on *unconscious operations* or skills. (For a further description of activity theory in relation to mathematics education, see FitzSimons, 2008.)

[2] Bear in mind that there is a considerable lag between the production of a manuscript and its eventual publication in hard copy, although this is now being reduced with online technologies.

Any *breakdown* in this system, including communication, is an opportunity for learning to take place — for workers and for researchers. If the worker's necessary skills are not in place (e.g., needing to learn a new IT technique), this becomes an immediate goal in the ongoing activity. As the context changes, different goals and different actions come into play for the worker. A feature of workplace activity is that problems arise on a daily basis that cannot always be predicted in advance, and locally new solutions must be generated. For researchers, a breakdown in a routine activity offers the possibility for gaining deeper insights as participants are probed for more detailed explanations of things that were previously taken-for-granted.

Communication — in its many forms — is, of course, a central issue in activities associated with work; and the concepts of *boundary crossing* and *boundary objects* are raised by many researchers (see Akkerman & Bakker, 2011, for a comprehensive review). Workers at all levels interact with others within and beyond their designated practice: between the work unit and customers or suppliers, internal or external; or between various organisational layers. Communication can encompass dialogue, written texts, diagrams, signs, symbols, and gestures. People often need to communicate across the boundaries of their particular work setting and, since mathematics is often an integral part of the work process, mathematical thinking and reasoning is likely to be embedded in this communication. *Workplace artefacts* such as graphs, tables, spread sheets, historic records of production, data collection and output (including quality control protocols), are likely to be in use as well. As a means of communication between different groups, these boundary objects — as they have come to be known in the literature, building on the original work by Star in 1988 and 1989 (Star, 2010) — are often wrongly assumed by their producers and users to be transparent, leading to communication breakdowns.

In the studies reviewed here, data collection has generally included participant observation, interviewing, and collection of artefacts. Not surprisingly, many articles have focused on the use and the influence of technology as an integral component of contemporary workplaces.

Any formulation of skills is time- and place-specific, and is often embedded in labour relationships and broader social structures encompassing social actors, institutions, as well as social values and norms. Wedege's (2000) understanding of *workplace technology* views mathematics as being integrated in four dynamically inter-related dimensions: (a) technique/machinery; (b) work organisation; (c) vocational qualifications; and (d) workers' competences. In addition, as part of this multidimensional view of workplace mathematics, Wedege (2011, p. 3) provides a definition of *technological competence, which she defines* as "workers' capacities (cognitive, affective, & social) for acting effectively, critically, and constructively" to challenges in the technological workplace.

Research Findings

The literature reviewed was drawn from over 50 refereed journal articles published over the last decade or so. Articles were selected on the basis of their capacity to illustrate: (a) differences in approaches to mathematics-related tasks between school students and workers, (b) graphic details of some mathematics-related activities actually carried out by particular workers, (c) differing structural resources available in the workplace, (d) workers' attitudes to school mathematics or the vocational mathematics intended to be relevant to their future occupations, and (e) how technology-enhanced artefacts can help workers to uncover and find meaning in mathematics rendered invisible by automation.

Problem Solving: Plumbers vs. Students

One objective of many studies is to make comparisons between the workplace and the school setting. Jurdak and Shahin (2001) documented, compared, and analysed the nature of spatial reasoning by plumbers in the workplace and by school students while constructing the 'same' solids from plane surfaces. The authors drew on activity theory for its potential to explain the differences between the two settings in terms of motives, tools available and accessible, and constraints. Data were collected from a plumbing workshop with five experienced adult plumbers having little or no school experience, and five tenth-grade students (two girls & three boys), while constructing a cylindrical container of capacity one-litre and height of 20 cm.

Based on the method of structural analysis, Jurdak and Shahin (2001) identified differences in the types and sequence of each group's actions as well as in the degree of complexity. Jurdak and Shahin contrasted the two groups:

> In the course of constructing the task container, both the plumber and the environment changed. The plumber started with perceptual action, responded to what has been executed by continuously reviewing the model container, executed more actions, and so on. The goals for container construction changed as the container evolved and its actuality became possible. This interaction elicited critical skills such as recognizing opportunities or problem finding, knowing when and how to apply skills that have been learned in other contexts, and exploiting properties of the present situation. (p. 312)

On the other hand,

> The students' interaction with their physical environment was minimal and they approached the problem of constructing the task container by implementing the procedure that was derived directly from the classroom practice. ... they relied [almost] exclusively on mnemonic and cognitive tools by reading the problem, selecting the formula, calculating the unknown, and writing the answer. … The students showed little control over the problem solving process, they strongly believed in the power of formulas and algorithms and hence did not feel the need for self-monitoring other than checking the correctness of their calculations. (p. 312)

Jurdak and Shahin (2001) observed that, whereas in school mathematics is a conceptual tool, detached from the situations which give it meaning, in the workplace mathematics is a concrete tool which takes its meaning from the situation at hand to solve problems that may arise within that context. They concluded that although using mathematics in the workplace is more meaningful, school (i.e., formal) mathematics has more power and is more generalisable beyond the specific context of application.

Glass Factory Computer-Aided Design and Manufacture [CAD/CAM] Technicians

Magajna and Monaghan (2003) conducted a case study of six CAD/CAM technicians who design and produce moulds for glass factories; their research focused mostly on volume calculations. The technicians' work was in many ways related to mathematics (e.g., constructing shapes, calculating or managing the cutting operations on machine tools). To help describe the structure of emergent goals, Magajna and Monaghan followed Saxe's four parameter model:

1. *Activity structures*: In order to understand a particular aspect of observed practice it is necessary to "consider the whole production cycle from the client's initial order to the manufacturing of the mould and the use of the mould in the production of bottles in the (distant) glass factories" (p. 105).
2. *Social relations/interactions* influence the emergent goals.
3. *Conventions and artefacts*: Although some methods used to determine the volume may be traditional, the introduction of computers and CNC machines has also led to new methods.
4. *Prior understandings* brought by individuals, both constrain and enable the goals that individuals construct in practices.

Magajna and Monahan (2003) found that "there was an evident discontinuity between the school mathematics used and the observed mathematical practices. This discontinuity was evident at both a subjective and an objective level" (p. 117). At the objective level, school-like concepts and practices, such as linear equations, trigonometry and Pythagoras's theorem, were used in the context of practice, even though the technicians did not necessarily understand all the mathematical background. School mathematics knowledge was used 'as is', without being questioned discussed or modified. However, discontinuity was also evident at the subjective level when the technicians claimed that there was no school mathematics in their jobs.

A second finding of Magajna and Monahan (2003) was that technology in this CAD/CAM workplace was so all-pervasive that it virtually structured the technicians' activity and played a crucial role mathematically. Not only were their mathematical procedures shaped by the technology they used, but their mathematical understandings were used as a means to achieve technological results. Mathematical correctness was negotiated amongst the technicians and also with customers and contacts in the glass factory, in contrast to the school situation.

Third, reflecting the findings of Jurdak and Shahin (2001), Lave (1988), and others, Magajna and Monaghan (2003) found that in mathematical breakdown situations the practitioners either simply chose another mathematical or construction method or, more commonly, overcame the problem by technological means rather than mathematising the cause of the breakdown in the sense of reasoning about mathematical procedures. They "did not solve problems involving mathematical abstractions in this workplace" (p. 120). In both cases, the workers' actions should be interpreted within the goal-oriented behaviour of their occupation: Technicians' work is embedded within the imperatives and constraints of the factory's production activity cycle which included: (a) time pressures demanding almost immediate solutions; (b) many mathematical procedures being frozen in the technology, often invisible, and poorly understood by practitioners who had limited control over them; (c) the final product and not the mathematics was what counted (i.e., mathematical correctness does not necessarily mean technological correctness); and (d) a lack of practitioner confidence in understanding the complex procedures involved.

Telecommunication Technicians: Tool Mediation

Triantafillou and Potari (2010) studied a group of technicians in a telecommunication organisation. Guided by an activity theory framework, they identified, classified, and correlated the tools that mediated the technicians' activity, and studied the mathematical meanings that emerged.

> The technicians' typical daily activity was to fix a number of reported faults in the local underground copper-wiring network. Their major working tools were wire pairs that were bundled together into cables consisting of between 100 and 600 pairs. This wiring network started from the main organization building in the center of the city, went through specific boards, the telecommunication closets, and then distributed to a number of boxes around the area of the closet, and finally was directed to the local subscribers. The largest part of this network was underground. The technicians, in order to accomplish their daily main activity, had to perform a series of actions that were usually sequential. These actions were to: (a) trace the reported pair of wires on a set of physical tools (the telecommunication closet and the boxes around the area) by using the information given on an instruction sheet, (b) use a technical map to trace the underground wiring network from the closet to the subscribers' boxes, and (c) use two measuring instruments ... to locate the exact point of fault. (p. 281)

In relation to these three actions, Triantafillou and Potari categorised the tools as (a) mathematical (communicative, processes, & concepts), and (b) non-mathematical (physical & written texts), which they presented in a comprehensive systemic network (see p. 290, Figure 5). They found that the technicians' emerging mathematical meanings in relation to place value, spatial, and algebraic relations were expressed through personal algorithms and *metaphorical* and *metonymic* reasoning (e.g., using analogies or alternative representations such as graphs), indicating the situated character of their mathematical knowledge. Triantafillou and Potari identified differences from typical mathematical representations in the technicians' explanations or their physical tools and written texts, in keeping with the findings of Magajna and Monaghan (2003). However, they also noted that a number of processes and strategies shared the same structure and characteristics with those developed in a school context.

From these relatively small-scale studies, I turn to a major UK research project where the concept of boundary crossing has been an important component, in addition to activity theory. Kent, Noss, Guile, Hoyles, and Bakker (2007) construed the term *techno-mathematical literacies* (TmL) to emphasise both the mediation of mathematical knowledge by technology and the breadth of knowledge required in the context of contemporary technology-rich workplaces that are both highly automated and increasingly focused on flexible responses to customer needs. They were also interested in how boundary objects "may facilitate effective communication between and within work teams and between work teams and customers" (p. 67). In the first phase of their research they carried out ethnographic case studies in 10 companies; in the second they carried out design experiments: I discuss findings from each, in the manufacturing and financial services sectors, respectively.

Process Improvement in Manufacturing

Kent, Bakker, Hoyles, and Noss (2011) conducted several case studies of process improvement in manufacturing companies, focusing on measurement aspects. Here, I discuss two: a production line bakery and a pharmaceutical packaging company.

A process improvement (PI) project in a large cake production line bakery. Kent, Noss, Guile, Hoyles, and Bakker (2011) observed the work of a PI team in a factory which involved cakes continuously moving on conveyor belts within linear ovens many metres long. The entire production line was several hundred metres long, with every stage of the baking process monitored using a combination of automated and manual measurements. In the PI process, the workers used measurements throughout the whole baking process to develop a capacity profile: a summary chart of speeds of the different machines and processes, intended to make visible bottlenecks in the process which could be targeted for improvement.

However, management had not been aware of the importance of creating a culture in which workers appreciated the need to measure and to take results seriously. Because the workers had not received any training in these requirements for PI, Kent et al. (2011) found that, initially, team members with less formal education were under-skilled in this kind of work. Measurements were taken inconsistently, from variable location points in relation to the cake and the machinery, and were not systematically recorded, hindering the optimisation techniques used in PI. After a 2-week project to improve performance on this production line, the workers' views and actions were transformed through their participation in acts of measurement, reading the graphs, noticing things that were previously invisible to them, and thinking about solutions. Nevertheless, Kent et al (2001) noted that even when PI recommendations are made on logical (mathematical) grounds, there are often good practical reasons (e.g., cost) for management not accepting them.

Overall equipment effectiveness (OEE) in a pharmaceutical packaging company. In this company, tablets were brought to be packed into foil and plastic blister packs, with a number of these packaged together into a cardboard box. Just prior to the research taking place, manually operated packing lines, each run by six operators, had been replaced by fully automated packing lines, now operated by one technician. The production manager described the activity as:

> Literally you pour tablets in one end and feed it cartons, film, foil, leaflets—and out of the other end comes a sealed pack, and groups of packs boxed and labelled—all automatically; all the operator does with that is to stack it on the pallet … the main thing is the ability to be able to tweak the machine to keep it going at the optimum speed we require. (Kent et al., 2011, p. 757)

Instead of using hand tools and having physical access to many parts of the old production process, the technician now had to operate the line through a computerised control panel—one level of abstraction away from the physical process, also from the measurement process. Kent et al. (2011) noted that there was more to tweaking and running machines well than recognised by management; especially managing product changeovers which varied significantly between operators. This required the technician: (a) to understand the physical mechanisms of the machine and to be able to identify and remediate problems, or (b) to be able to communicate what was wrong to the skilled maintenance engineers in the factory. It also required operators to be able to make sense of the measurements and (idiosyncratic) graphical data generated by the packing machine's internal computers.

Overall equipment effectiveness (OEE) combines three generic variables for production, Performance, Quality and Availability, to construct a more abstract measure concerned with maintaining a balance between the three variables. Although OEE measures created a visibility about the system for senior managers, they were not discussed with the operational employees. Kent et al. (2011) concluded that technology brings an extra layer of complexity to measurement. Although it is commonly assumed that automation leads to a reduction in human error, many managers expressed the concern that it also leads to less engagement with the production process, and several preferred to have employees measure and report manually. As described above, the complexities faced by technicians in running the machinery highlight the importance of how their tacit knowledge interacts with their codified knowledge to get the job done effectively.

Financial Services Workers

One case study (Kent et al., 2007), design experiments was set in the financial services sector, and focused on the annual pension statement, a boundary object designed to facilitate boundary crossing between the company and customers. However, this pension statement routinely failed in its communicative role, largely due to the invisible factors of the underlying mathematical-financial models not being made available either to customers or to the Enquiry Team. Straightforward customer enquiries were to be resolved immediately by telephone, if possible. However, automation of the IT system, intended to ensure the accuracy of information sent to customers, had actually disempowered employees who lacked any real understanding of the models and calculations, inhibiting their communications with colleagues and customers.

Although the mathematics involved in finance seems superficially similar to secondary school mathematics, Kent et al. (2007) observed that in the workplace context every mathematical procedure, no matter how simple, is part of a whole range of decisions and judgments about complex processes or products. Employees need to be able to mathematically appreciate computer outputs, interpreting them in their context, and recognizing which components are hidden by the IT system. They also need to be able to reason about the mathematical models embedded in the system in terms of the key relationships between product *variables* (e.g., percentage rates, management fees, sales commissions) and their effect on *outputs* presented in the form of graphs or tables.

Bakker, Kent, Hoyles, and Noss (2011) detailed an intervention in the same industry where they designed *technology-enhanced boundary object*s (TEBOs) in order to improve employees' understandings of the mathematics behind the mortgages they sold (i.e., their techno-mathematical literacies). Their goal was *not* to teach employees the mathematics behind boundary objects such as the mortgage package, but to engage with the different aspects of their underlying mathematical models. They developed software to model or reconstruct actual practice, using data drawn from a current account mortgage (CAM) which integrated a regular current account with the property mortgage. The company had previously published a booklet containing a standard repayment graph which was actually mathematically unrealistic. Bakker et al. (2011) described it as *pseudo-mathematical*: mathematical in form but not in function. Interviews revealed that

> … the sales agents were largely unaware of any but the simplest relationships between interest rates and repayment schemes, even though they were able to explain mortgages in general financial terms (e.g., capital and interest).… sales agents saw the different interest rates as labels for instruments: annual rates as labels attached to a mortgage arrangement or a savings account, monthly rates as labels attached to credit card or loan debts. (p. 29)

In other words, sales agents had not understood the mathematical meanings or relationships. One TEBO reconstructed the mortgage graph in a spreadsheet with all the input variables and calculations made explicit in order that the graphs could be produced according to the input variables that matched different customer scenarios. Another TEBO modelling credit-card debts revealed to participants the problems with paying back only the obligatory monthly charge. Anecdotally, this intervention seemed to have improved employees' understanding and confidence, but they were forbidden by management for legal reasons to use the TEBOs in actual practice.

Black Boxes in Industry

Williams and Wake (2007a), drawing on their earlier work from a major study (Wake & Williams, 2001), sought to expose contradictions between College and work in order to explain the apparent disappearance of mathematics in the workplace. They were attempting to answer two questions:

- How is mathematics shaped, indeed often hidden, by workplaces?
- What processes serve to shape mathematics in workplaces differently from that in Colleges, and hence cause a 'gap' for students when presented with mathematics from workplaces? (p. 318)

Instead of focusing specifically on the differences between the two systems of workplace and College, they sought to engage with these differences as contradictions in practice. Grounding their research in cultural-historical activity theory, and conducting multiple case study visits to workplaces by college students and teacher-researchers, they observed that mathematical processes have been crystallised in *black boxes* shaped by workplace cultures. They identified two key processes through which this happens: automation and the historical development of instrumentation.

According to Williams and Wake (2007a), automation occurs when the work of mathematics is crystallised in instruments, tools, and routines that control operations, and these operations are then automated — for example, in the design of a machine, or in the writing of a program to perform precision cutting operations in three dimensions. These artefacts serve as boundary objects between the mathematics performed historically, in their original design, and their current use by operatives who use them on a daily basis, but without needing to know or understand how they work. Williams and Wake described this as a distribution of mathematics in *time*. As a consequence, they concluded, this workplace mathematical genre is mediated by idiosyncratic conventional forms, the mathematical origin and significance of which may have vanished over time. As such it is also opaque to outsiders, even to the mathematically trained researchers themselves.

Williams and Wake (2007a) also described how the historical development of instrumentation could result in sub-units of the workplace community being protected from mathematics by a division of labour, supported by communal rules, norms, and expectations. These are often regulated by boundary objects (e.g., data collection forms) which seem to distribute and crystallise work *horizontally*, in a division of labour across social *space*. Workers collecting data, for example, may have no understanding of the importance of the data collection, the uses to which it is put, or even the need for accuracy — in the manner comparable to research by Kent et al. (2011) on process improvement on the cake production line.

As identified in the work of Triantafillou and Potari (2010), the use of metaphor and metonymy is a topic of growing interest for researchers involved with school mathematics teaching and learning. Williams and Wake (2007b, p. 346) asked: "How can models and metaphors mediate communication between students, workers, and teacher-researchers?" Explanations by workers that were productive sometimes drew upon cultural models, including metaphors and mathematical models, that made connections with relatively more concrete, well understood resources such as commonly used metaphors of communication, time and space, supported by gestures and/or words making reference to time, place, and so on. They gave an example of such a conversation with an engineer, which showed how they, as outsiders, could come to make sense of what at first appeared to be a very mysterious and opaque spreadsheet formula and to understand the mathematics inside. Williams and Wake (2007b) used the expression *workplace genres of mathematics* because workplaces, technologically mediated or otherwise, tend to develop their own conventions and terms. However, more technological workplaces may also draw on elements of other widely recognised social and cultural forms of mathematical language, such as those known as *engineering mathematics*, *spreadsheet mathematics*, and so on.

Apprentice Electricians Learning to Bend Pipes

In an empirical study framed by cultural-historical activity theory, specific differences between college and workplace were identified and theorised, not by boundary crossing but by the concept of *personality* (discussed below). Roth (2012) investigated (a) the geometrical practices of electrician apprentices learning to bend electrical conduits in college and on the job, and (b) how they handled the relation between differing practices. The requirements for doing well in the two activity systems were very different: exhibiting knowledge of trigonometry in one, and doing a good job that makes bending and subsequent pulling of wires practical in the other. Formal trigonometry was the reference in the classroom, whereas the codified rules of practice were the main reference on the job.

In college, students intending to become electricians are taught conduit bending theory, and are required to study basic trigonometry. The textbook provides "magic circles" to help calculate such functions as the sine, cosine, and tangent. Apprentices carry out extensive calculations and measurements to determine angles, their positions on the tubing, and the distances between the angles. Once a student has calculated the distances, s/he uses measuring tape and bender to produce the tubing such that it properly bypasses the obstacle provided. However, in their practical conduit bending class, apprentices encounter a specialised conduit bender on which much of the required information is inscribed, rendering the trigonometric calculations superfluous (see Fig. 4, Roth, 2012, p. 7 [online]).

Roth's (2012) study illustrates the radical differences between the (mathematical) practices of bending electrical metal tubing, in college and in the workplace, calling into question the usefulness of vocational courses which emphasise formal mathematics that is treated as irrelevant in the workplace. Nevertheless, the electrical apprentices managed to move between the different activities with a sense of coherence, integrating these differing experiences as part of the electrician personality they develop. Having gone to college allows the electricians to work according to the formal and legal requirements of the electrical code, while meeting the practical demands and operating within the workplace constraints.

Roth (2012) noted that in addition to the experienced differences between college and workplace, the electricians' discourse about those differences was both topic and resource in their workplace conversations. However, in the case of mathematics, there was little cross-reference between college and workplace, in story telling or in practice. According to Roth, stories encode not only practical knowledge but also the very process of *subjectification*, which describe the changes *within* an activity system (school, work); and *personality*, the changes the individual undergoes as s/he moves repeatedly *between* systems of activity. Being able to talk about the contradictions between the two systems is as much part of being a recognized practitioner as is competent practice.

Chemical Spraying and Handling

The processes of preparation, application, handling, storage and transport of chemicals are key elements of a range of economically significant industries, and place high demands on workers' literacy, and especially numeracy skills. Many of these skills are acquired during employment on-the-job or in associated off-the-job training. However, a substantial body of research evidence demonstrates that such skill transfer is achieved only with difficulty, and that numeracy skills are highly context-dependent. FitzSimons, Mlcek, Hull, and Wright (2005) undertook 13 case studies of enterprises which used chemicals extensively in industries including rural production, amenity horticulture, local government, outdoor recreation, and warehousing.

The mathematical processes and strategies used by workers to undertake calculations included: estimation, written methods or basic calculator use; oral or written communication of mathematics to other workers, and the interpretation of their mathematics; consultation with prescriptive calculations sheets and with historical records or online data; and completion of up-to-date records of chemicals used and the corresponding amounts. Complex contextual factors included date/time of spraying; block area; specific crop to be sprayed; crop stage; weed/pest/disease targeted; chemical group, rate per hectare, litres of spray applied, method of application; temperature, wind speed, wind direction, rainfall, humidity; and variations in equipment used, from small-scale backpacks to broad-acre mechanised spraying. Considerations such as these demand that the workers have a broad understanding of the entire work process of the area in which they are involved.

The accuracy of calculations was highly dependent on these factors as well as the degree of accuracy available on the equipment used. Importantly, economic and legal contingencies are strongly implicated within these contextual factors. Safety standards are critical, both in terms of the risk of spillage or misapplication of chemicals which may be harmful to people (workers & consumers) and/or the environment. Errors or carelessness may even cause spoilage of the end-product itself (such as an entire crop or vintage or sports ground) or the imposition of large fines for environmental pollution, thereby risking the company's financial viability.

For these workers involved in chemical handling and spraying, it was a requirement that they already held a basic vocational certificate in the area. Learning on the job was largely experiential, with opportunities for them to become enculturated into communities of practice through interrelationships with other employees. Supervisors were often involved in initial training and check regularly on work practices; in some cases novices were given trial areas to spray and then asked to account for any discrepancies in expected area coverage and/or spray consumption. Most workplaces placed a strong emphasis on ensuring that workers are prepared to check before acting, of being unafraid to ask a seemingly *dumb* question.

Learning in these kinds of workplace differs significantly from formal institutionalised education in that it is rich in context, supported by historical records, and mediating artefacts such as tools, equipment, manuals, charts, and so forth, as well as communication of a qualitatively different kind from the classroom. In this workplace environment the object is satisfactory task completion, with much more at stake than appropriately accurate calculations. (See FitzSimons & Wedege, 2007, for a more comprehensive review of related literature on workplace numeracy and workplace learning.)

Conclusion and Implications

It is pleasing to see a burgeoning of research in this subfield of mathematics and work, especially in journals such as this, ALM's own journal, and in many international conference proceedings. Several conclusions may be drawn, each with implications for further research. It is abundantly clear that school and workplaces are, in general, completely different activity systems, even though learning permeates both. In the language of activity theory, their motives and goals rarely intersect, even though their operations may in fact appear to do so. School and workplace have two different logics, as shown by the data with respect to the purposes and uses of technology, behaviours expected of workers and students, and relative values placed on cognitive and practical skills. As pointed out by Evans (1999), among others, the issue of transfer remains problematic.

Automation has had a major impact, especially on primary and secondary industries, with a consequent reduction in the visibility of the mathematical processes it envelops. The term *black box* originated in the 20th century with the development of cybernetics, but Maaß and Schlöglmann (1988) argue that as a process it dates back to the Stone Age: Its success relies on its relatively context-free transfer of results, where knowledge without understanding can be passed from developers to end-users relatively easily. However, it is this hidden complexity—not only in technology but permeating many workers' daily activities—which presents a great challenge to workplace mathematics researchers and vocational mathematics educators. It is worth reiterating the importance of the interaction between the tacit knowledge of workers and the codified knowledge of the workplace — see Eraut (2004) for a comprehensive analysis.

A recurrent question concerns the relationship between formal vocational mathematics education and actual workplace practice. On the one hand, in traditional trade-related vocational mathematics programs there are contradictions between school and workplace practice—as shown by Roth (2012); see also FitzSimons (2002). On the other hand, competency based training programs, which require only the atomised mathematical skills immediately visible, can disempower workers if they have no real mathematical understanding to build upon. Yet, the very nature of contemporary work and society requires people being able to cope with, and even contribute towards addressing the inevitable unforeseen changes and ever increasing degrees of complexity—and this needs real meaning making in mathematical aspects and appropriate skills in communication.

As outsiders, researchers interested in workplace mathematics have to learn something of the work process (norms, rules, & division of labour), and the workplace technology and jargon, if they are to make sense of the practices they observe there. There is also a need for them to respect the mathematics of workers that may not conform to the traditional school model. Williams and Wake (2007a) recognised that this demands inquiry skills and appropriate predispositions, as well as social confidence, which many of their College students—as prospective workers—seemed to lack. They made a series of recommendations for college students and trainee workers:

1. to understand the mathematics of others, including comprehending a diversity of unfamiliar technical media;
2. to become proficient at using spreadsheets, including decoding particular spreadsheets, taking into account contextualisation within the given work process;
3. to experience mathematics as a communal activity as is common in workplace situations;
4. to encounter a diversity of mathematical conventions and methods; and
5. to develop mathematical thinking within realistic, complex workplace contexts.

In order to make sense of workplace mathematics, outsiders need to develop flexible attitudes to the way mathematics looks, to the way it is rendered invisible by tools and artefacts and by the division of labour. Sociocultural activity theory offers one methodological framework for researchers to investigate the social, cultural, and historical factors that contribute to the complexity of work environments where adults do mathematics: where they use the mathematics they already know, transform it, possibly create (locally) new mathematics; always with the intention of solving problems that are meaningful and important to them personally or professionally, embedded in practical constraints, and with serious, sometimes life and death, consequences.

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