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USING TWO LANGUAGES WHEN LEARNING MATHEMATICS

ABSTRACT. This article reviews two sets of research studies from outside of mathematics education to consider how they may be relevant to the study of bilingual mathematics learners using two languages. The first set of studies is psycholinguistics experiments comparing monolinguals and bilinguals using two languages during arithmetic computation (language switching). The second set of studies is sociolinguistic research on young bilinguals using two languages during conversations (code switching). I use an example of a mathematical discussion between bilingual students to illustrate how sociolinguistics can inform analyses of bilingual mathematical conversations.

During courses and conference presentations I often share data from classrooms where bilingual students participate in mathematical discussions, sometimes using two languages. Students, researchers and practitioners are interested and sometimes puzzled by students using two languages. They often wonder when, how, or why students switch from one language to another. These questions motivated me to consider when bilingual learners might use two languages as they learn mathematics and to explore what research in psycholinguistics and sociolinguistics might offer for understanding these practices.

Two situations when bilinguals either report or have been observed using two languages are while carrying out arithmetic computations and during mathematical conversations (with a teacher, a peer, or during class discussions). In this article I review research from two fields outside mathematics education, psycholinguistics and sociolinguistics, related to language practices in these two situations. I consider whether and how this research is relevant to the study of bilingual students learning mathematics in classrooms. I use a mathematical discussion between two bilingual students to illustrate how work in sociolinguistics can inform analyses of bilingual mathematical conversations.

Theoretical perspective

This article reviews studies from two different theoretical perspectives, psycholinguistics and sociolinguistics, that differ both in how they explain and how they explore language practices. Psycholinguistics views language as an individual cognitive phenomenon. Sociolinguistics stresses the social nature of language, starting from the assumption that language is not only

cognitive but also cultural, social, and situated. Studies from a sociolinguistic perspective examine language use in naturally occurring settings. In contrast, studies from a psycholinguistic perspective have been limited to experimental settings.

From a sociolinguistic perspective, psycholinguistic experiments provide only limited knowledge about speakers' competence or how people use language.

> The speaker's competence is multifaceted: How a person uses language will depend on what is understood to be appropriate in a given social setting, and as such, linguistic knowledge is situated not in the individual psyche but in a group's collective linguistic norms. (Hakuta and McLaughlin, 1996).

Sociolinguistics provides theoretical frameworks and methodologies for studying discourse (for example, Gee, 1996, 1999), concepts such as register (Halliday, 1978), and analyses of classroom discourse (for example, Cazden, 1986,1993; Mehan, 1979). Research in second language acquisition, bilingualism, and biliteracy (Bialystok, 2001; Hakuta and Cancino, 1977; Valdés-Fallis, 1978, 1979; Zentella, 1997) also provides theories, empirical results, concepts, and definitions necessary for studying mathematics learning in bilingual (and multilingual) classrooms.

I use a sociolinguistics perspective for reviewing research and critiquing assumptions about language practices. I draw primarily on research with two U.S. Latino populations, Mexican-American and Puerto Rican. I use the definition of bilinguals provided by Valdés-Fallis (1978) as "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (p. 4). This definition describes bilingualism not only as individual but also as a social and cultural phenomenon involving participation in language practices and communities. I first provide brief definitions and clarify distinctions among concepts, leaving more detailed examples for later sections.

Bilingualism

Bilingualism is simultaneously an individual, social, cultural, historical and political phenomenon. While being bilingual in some languages and settings is a sign of education, "in other languages and other places (it) may be synonymous with poverty and supposed cultural deprivation" (De Avila and Duncan, 1981). For example, Latino bilinguals in the U.S. have a particular history as part of a language minority. Some bilingual Latinos came to the U.S. as immigrants, others are the descendants of immigrants, and others never emigrated or immigrated anywhere but live in territories that were originally Mexico and later became the U.S. In the U.S., bilingualism

is not always considered an asset. In particular, Spanish is not a high-status second language. As a heritage language, Spanish shows a pattern of loss from one generation to another (Tse, 2001). Bilingual Latino learners in the U.S., rather than being viewed as having additional language skills, have often been described by deficiency models (Garcia and Gonzalez, 1995).

Definitions of bilingualism range from "native-like fluency in two languages," to "alternating use of two languages" (De Avila and Duncan, 1981), to "belonging to a bilingual community" (Valdés-Fallis, 1978). Contemporary scholars studying bilingualism view "native-like control of two or more languages" as an unrealistic definition. This definition does not reflect evidence that the majority of bilinguals are rarely equally fluent in both languages:

Bilinguals acquire and use their languages for different purposes, in different domains of life, with different people. It is precisely because the needs and uses of the languages are usually quite different that bilinguals rarely develop equal fluency in their languages (Grosjean, 1999, p. 285). In contrast, Grosjean proposes we shift from using the terms monolingual and bilingual as labels for *individuals* to using these as labels for the endpoints on a continuum of *modes*. Bilinguals make use of one language, the other language, or the two together as they move along a continuum from monolingual to bilingual *modes*.

Comparisons between bilingual and monolingual individuals reflect an assumption that monolingualism is the norm or standard for language skills:

Bilinguals have been described and evaluated in terms of the fluency and balance they have in their two languages; language skills in bilinguals have almost always been appraised in terms of monolingual standards (Grosjean, 1999, p. 285).

Recent work has moved away from comparison between monolinguals and bilinguals to instead study the details of bilingual communicative competence without comparing it to monolingual competence:

Researchers are now starting to view the bilingual not so much as the sum of two (or more) complete or incomplete monolinguals but rather as a specific and fully competent speaker-hearer who has developed a communicative competence that is equal, but different in nature, to that of the monolingual (Grosjean, 1999, p. 285).

One common misunderstanding of bilingualism is the assumption that bilinguals are equally fluent in their two languages. If they are not, then they have been described as not true, real, or balanced bilinguals and sometimes labeled as 'semilingual' or 'limited bilingual.' The concept of semilingualism has been discussed by several educational researchers and strongly criticized by many (for a review see Baetens Beardsmore, 1986 and MacSwan, 2000):

The obsession with monoglot norms of reference has led to the notion of *semilingualism*. Now the notion of *semilingualism* has led to considerable controversy and should be treated with great caution by anyone approaching bilingual studies (Baetens Beardsmore, 1986).

Semilingualism was first introduced by Nils Erik Hansegard (a Swedish philologist) in 1962 (without a theory of language) to conjecture that a period of "double semilingualism" occurs when an individual abandons her native language altogether in favor of a second language (MacSwan, 2000). In the U.S., Cummins (1976) used the term in describing the *Threshold Hypothesis*, the hypothesis that the level of linguistic competence attained by a bilingual child in a first and second language may affect his or her cognitive growth in other domains. Cummins originally defined "semilingualism" as low level in both languages (Cummins, 1979) to describe students who do not develop "native-like competence in either of their two languages" (Cummins, 1976, p. 20). This definition involves the conjecture that some children have limited or nonnative ability in the language or languages they speak (MacSwan, 2000).

Currently, most scholars in linguistics (even Cummins and Skutnabb-Kangas, two of the early proponents of this notion) have discarded the concept of semilingualism:

There appears to be little justification for continued use of the term 'semilingualism' in that it has no theoretical value and confuses rather than clarifies the issue. (Cummins, 1994, p. 3813)

I do not consider semilingualism to be a linguistic or scientific concept at all. In my view, it is a political concept. (Skutnabb-Kangas, 1984, p. 248) The grounds for discarding this concept range from objections to its nebulous nature, to arguments that it is a catchword of no use for unbiased research (Ekstrand, 1979, 1983), to the lack of empirical support and theoretical foundation for the notion:

Semilingualism does not exist, or put in a way which is non-refutable, has never been empirically demonstrated. (Paulston, 1982, p. 54)

Some critics of the concept of "semilingualism" argue that it confounds language proficiency (or linguistic competency) with academic register, formal schooling, SES (socio-economic status), or "language loss" (the shift in choice of language occurring across generations). The concept also confuses degrees of ability, levels of linguistic competence, and levels of language development with differences in experience with language varieties (dialects, registers, and discourses) or with school literacy (reading, writing, and other aspects of language use valued in school).

Perhaps the strongest argument against semilingualism is the empirical evidence that it is not possible to have limited or non-native ability in the language of one's own home community. Linguists agree that "all

normal children acquire the language of their speech community with some minor but ordinary degree of variation" and that "a native language is acquired effortlessly and without instruction by all normal children" (MacSwan, 2000, p. 25).[1]

Code switching

Research in mathematics education has drawn on work in sociolinguistics, sometimes using the concept of code switching, to inform studies in bilingual and multilingual classrooms. For example, studies in bilingual and multilingual mathematics classrooms have described the complex ways that teachers use multiple languages (Adler, 1998; Setati, 1998; Setati and Adler, 2001; Khisty, 1995) and that bilingual students communicate mathematical ideas (Moschkovich, 1999, 2002) during mathematics lessons.

Code switching has been used in sociolinguistics to refer to the practice of using more than one language in the course of a single communicative episode. Code switching has long been documented as stigmatized (Grosjean, 1999), particularly in classrooms (Valdés-Fallis, 1978). For example, in the U.S., teachers working with Latino students were documented to consider code switching as an unacceptable variety of language (Ramirez and Milk, 1986). Deficiency views of code switching continue in admonitions against code switching ("It's not good English" or "It's not good Spanish") or outright prohibition of using two languages.[2]

Views of code switching as a deficiency are connected to the notion of semilingualism.

It is now clear that switching is not simply a haphazard behavior due to some form of "semilingualism" but that it is, instead, a well governed process used as a communicative strategy to covey linguistic and social information (Grosjean, 1999, p. 286).

I distinguish between the terms 'code switching' and 'language switching' because these two terms refer to different situations. Research from a psycholinguistic perspective has used the term 'language switching' to refer to an individual cognitive phenomenon different from code switching. I use the term 'language switching' to refer to the use of two languages during solitary and/or mental arithmetic computation. This use is consistent with the terms proposed by Qi (1998) who used "language switching" for the language used when a person is individually engaged in an arithmetic computation rather than in a conversation. I reserve 'code switching' to refer to using two languages during conversations.

LANGUAGE SWITCHING DURING ARITHMETIC COMPUTATION

In this section I review several psycholinguistics studies of language switching during computation. I selected these because they are frequently cited as evidence in the literature on bilingual cognition (for example Bialystok, 2001). These studies examined whether bilinguals have a preferred language for calculation, whether this is the language of instruction, and whether there are significant differences between monolinguals and bilinguals in response times or error rates.

Bilinguals sometimes switch languages when carrying out arithmetic computations. Anecdotal evidence, self-reports from bilinguals (including myself), and reports during interviews (Marsh and Maki, 1976; McLain and Huang, 1982; Tamamaki, 1993) support the claims that adult bilinguals have a preferred language for carrying out arithmetic computation and that the preferred language is usually the language of instruction.[3] Kolers (1968) first suggested that bilinguals perform arithmetic operations in the language of instruction and nearly all of the bilinguals in his studies reported they carried out arithmetic operations in the language of instruction. For example, Spanish is my first language and I learned mathematics through the 7th grade in Spanish. I carry out addition and multiplication of whole numbers in Spanish (either "in my head" or in a mumbled whisper).[4]

Several psycholinguistic experiments explored response time and error rates for bilinguals' performance on arithmetic operations (Magiste, 1980; Marsh and Maki, 1976; McLain and Huang, 1982; Tamamaki, 1993). The results of these studies are contradictory and difficult to summarize.[5]

One early study (Marsh and Maki, 1976) found that adult bilinguals performed arithmetic operations more rapidly in their preferred language than in their non-preferred language and that switching from one language to another during an experimental session slowed reaction time. The difference between performance in a preferred language and a non-preferred language was slight (0.2 seconds). Performance (in English) showed a slight but statistically significant difference of about 0.49 seconds for mean response time between monolinguals and bilinguals who preferred English to Spanish. The overall error rates for monolinguals and bilinguals did not differ significantly.

A later experiment (McLain and Huang, 1982) compared response times in the preferred and non-preferred languages for Chinese and Spanish bilinguals. The researchers used two experiments to present addition problems auditorily in either the preferred or non-preferred language and concluded that solution time was, on average, about 0.2 seconds faster in the preferred language. Performance and error rates for monolinguals and

bilinguals using their preferred language did not differ significantly. This study showed that allowing bilinguals to choose the language decreased solution time and requiring bilinguals to change from one language to another within an experimental session slightly increased solution time. The researchers concluded that if bilinguals are required to use only one of their languages during an experimental session, the preferred language "advantage" was eliminated.

In summary, and as suggested by other reviews of this research, all we can safely say at this time is that "retrieval times for arithmetic facts *may* be slower for bilinguals than monolinguals" (Bialystok, 2001, p. 203). Retrieval, response, or solution times may be slower when adult bilinguals are not using their preferred language or are asked to switch from one language to another. Are these possible slight differences in retrieval times relevant to learning mathematics in K-12 classrooms? All the studies described here were conducted in experimental settings rather than classrooms, homes, playgrounds, or other naturalistic settings. Four of these studies were conducted with adults and only one problematic study with adolescents. Therefore, we cannot conclude that these reported small differences in response time would appear in natural settings, among young children, or across bilingual student populations of different ages or educational backgrounds.

The differences in calculation times reported in these four studies ranged from 0.2 s to 0.5 s (when average response times ranged between 2 and 3 s). Such differences seem negligible on the scale of time for oral interactions between students and teachers in classrooms. The results obtained by McLain & Huang (1982) showing that if bilinguals are required to use only one of their languages the "preferred language advantage" can be eliminated, seem relevant to mathematics classrooms. This study supports classroom practices that allow bilingual students to choose the language they use for arithmetic computation in the classroom.

Because these studies focused on computation, they say little regarding language preference during more conceptual mathematical activity, for example, the role of translating arithmetic operations from one language to another while solving word problems. Qi (1998) provides a case study of one adult bilingual who switched to her first language for simple arithmetic computation while solving word problems. The study concludes that these switches were swift and highly automatic and that language switching facilitated rather than inhibited solving word problems in the second language.

Overall there seems to be strong evidence suggesting, "language switching does not affect the quality and integrity of thinking at the conceptual level in second language production (Cumming, 1989, 1990, as cited in Qi,

1998). In the words of one researcher summarizing work on bilingualism and mathematical performance:

The results of these studies present a complex picture and appear in some instances to contradict each other. The most generous interpretation that is consistent with the data is that bilingualism has no effect on mathematical problem-solving, providing that language proficiency is at least adequate for understanding the problem. Even solutions in the weaker language are unhampered under certain conditions. (Bialystok, 2001, p. 203)

In sum, how can the findings on preferred language and speed inform classroom practice? If assessments of mathematical proficiency focus on the speed of simple arithmetic computation in a bilingual's non-preferred language, it is possible that bilingual mathematics students might be assessed as less proficient in computation if they do not use their preferred language or are required to switch languages. A teacher might ascribe slight delays in responding orally or longer delays on written assessment to students "not knowing their math facts" when, in fact, these students do know their math facts. Considering these findings, instruction should, when possible, allow bilingual learners to choose the language for carrying out arithmetic computation and to take more time on timed tests of arithmetic computation.

Studies comparing monolinguals and bilinguals seem to have focused on studying those differences that are disadvantages for bilinguals rather than the reported differences that favor bilinguals and may be relevant to learning mathematics. For example, one difference that may favor bilinguals is the role of selective attention. After reviewing research on the cognitive consequences of bilingualism, Bialystok (2001) concluded that some bilinguals develop an "enhanced ability to selectively attend to information and inhibit misleading cues" (p. 245). This conclusion is based, in part, on the advantage reported in one study that included a proportional reasoning task (Bialystok and Majunder, 1998) and another study using a sorting and classification task (Bialystok, 1999). An interesting direction for future research on mathematics cognition with bilingual learners would be to explore the role that selective attention plays when bilinguals solve mathematics problems, particularly problems involving sorting, classifying, and proportional reasoning. Another skill that is highly developed in bilingual children is translation. Bilingual children are regular "language brokers" (Orellana, 2003) and use high level sophisticated translation skills between two languages (Malakoff and Hakuta, 1991). These sophisticated translation skills should be examined in relation to learning mathematics.

CODE SWITCHING DURING CONVERSATIONS

The psycholinguistic research on language switching described above does not address the use of two languages during mathematical conversations. Since mathematical conversations are a subset of conversations in general, the study of code switching during mathematical conversations should be framed and informed by previous work on code switching during conversations, particularly in classrooms, even if this research was not specifically about mathematical conversations. I first review sociolinguistics research on how children use two languages during conversations about general topics, especially in classrooms, and then consider how this research can inform the study of mathematical conversations.

Because researchers within sociolinguistics do not use "code switching" to refer to one single agreed upon language practice, I first provide some brief definitions, using examples from Latino communities in the U.S., and consider whether and how these subtle (and contested) distinctions are relevant to mathematics cognition and learning.

Definitions of Code Switching

Work in sociolinguistics (Auer, 1984; Gumperz, 1982; Zentella, 1981) describes code switching not as an individual phenomenon but as a complex and evolving social activity and language practice tied to an individual speaker's community or communities. Gumperz (1982) defines code switching as "the juxtaposition within the same speech exchange of passages belonging to two different grammatical systems or subsystems." There are several distinctions among types of code switching and some disagreement on when each definition applies. Some researchers in sociolinguistics (for example, Torres, 1997 and Zentella, 1981) distinguish between "code mixing," –transferring linguistic units from one code to another, –and "code switching," –the alternation of one language to the other that corresponds to a change in participants, social situation, and so on. Zentella reserves the term "code mixing" to refer to switching for immediate access to an unknown term, usually a single noun or noun phrase, and uses "code switching" to refer to changing completely from one language to the other at major boundaries.

Some researchers use code switching to refer to switches within one speaker turn, others to switches within one conversational episode, and others to within sentence switches.[6] The use of a single word from one language in an utterance that is in another language is a difficult phenomenon to classify; While some researchers consider using single words in another language as cases of code switching (Sanchez, 1994, p. 169), other

researchers prefer to call these loans. Loans can be loosely defined as single words from one national language used by many members of a bilingual community, for example, using the word "troque" for "truck" instead of the Spanish word "camión" within an utterance in Spanish.[7]

These subtle distinctions illustrate the complexity of code switching as a linguistic phenomenon and theoretical notion. Many researchers in mathematics education (for example, Adler, 2001; Khisty, 1995; Setati, 1998; and Moschkovich, 2002) have used one term, code switching, to refer to what sociolinguists might call code mixing, code switching, or borrowing. This seems reasonable since these distinctions may not be relevant to the issues we study in mathematics classrooms. As mathematics education researchers, we are more concerned with how language use relates to mathematics learning and teaching than with making subtle distinctions among different language practices. However, it may be important to consider whether these distinctions might be relevant to a particular study, classroom, situation, or age group, and if so, how.

For example, the subtle distinction between code switching and code mixing may be relevant when considering mathematics conversations among children of different ages. For example, Zentella (1997) reports that a study by McClure with Mexican-American children found that older children (ages 9–13) "code switched" more than younger children, and younger children (ages 2–9) "code mixed" more. This study reminds us that young children and adolescents may participate in different language practices and that the distinction between code switching and code mixing may be relevant when comparing children of different ages. This study might also lead us to expect this difference to be evident during mathematical conversations.

Are other distinctions made by sociolinguistics relevant to mathematical talk? Future research in mathematics education should explore whether the distinctions among extra-, intra-, and inter-sentential code mixing are useful for analyzing talk in mathematics classrooms. For example, do different patterns in talk emerge if we make these distinctions? Are these patterns related to the mathematical content? Do different patterns emerge for different languages, for example those with more and less extensive mathematics registers? The only way to decide which distinctions are relevant is to empirically examine their relationship to mathematics cognition and learning.

Bilingual Children Code Switching

Several conclusions from sociolinguistics research on bilingual children code switching seem relevant to mathematics classrooms.[8] First,

sociolinguists have concluded that, overall, young bilinguals (beyond age 5) speak as they are spoken to. A bilingual child's choice of language seems to be most dependent on the person addressing them. The language ability and language choice of the person addressing a bilingual child are "recognized as the most significant variable to date in determining the child's language choice" (Zentella, 1981, p. 110). We can assume that in mathematics classrooms children will also speak as they are spoken to, depending on the language ability and choice (including monolingual or bilingual modes) of the person addressing them. Second, research in sociolinguistics also shows that children's code switching is a social language practice that is connected to a community's norms. While code switching activity has an improvised quality, individual children's code switching is also guided by the norms of their community (Zentella, 1997).[9]

Sociolinguistic research on code switching among adults shows that this practice can be a reflection of stylistic switches to add color, emphasis, or contrast; random switches of words or phrases that appear in talk with high frequency items; as well as switches related to language dominance, memory, routines, and automatic speech. Code switching occurs in response to multiple aspects of a situation that work in complex ways in conversation. There are several aspects of a speech situation that are relevant to consider such as "the characteristics of the speakers, the context of communication, and the semantic objectives of the communicative act" (Torres, 1997, p. 99). One researcher concludes, "it is impossible to compile a comprehensive inventory of the functions of code switching since the number of possible functions is infinite" (Martin-Jones, 1995, p. 99). Instead of examining functions, she recommends examining how participants exploit codes in specific types of communicative encounters.

Lastly, sociolinguistics research reminds us that the debate regarding whether code switching demonstrates linguistic confusion or a controlled form of expression was settled long ago. Researchers concluded that "code switching is not an ad hoc mixture but subject to formal constraints and that for some communities it is precisely the ability to switch that distinguishes fluent bilinguals" (Zentella, 1981). One consistent finding regarding code switching that is relevant to bilingual mathematics learners is that code switching is not a reflection of a low level of proficiency in a language or the inability to recall a word (Valdés-Fallis, 1978). If, as Grosjean proposes (Grosjean, 1999), we view bilinguals as moving along a spectrum of modes, then code switching is a normal language practice in the bilingual mode and bilingual language competence is simply different from monolingual competence (Bialystok, 2001; Cook, 1997).

A common misunderstanding about code switching is that it is a reflection or consequence of a missing word in the speaker's lexicon. It may

seem reasonable to conclude that a word in language A in the middle of an utterance in language B means that the speaker does not know or cannot retrieve that word in language B. This is not, in fact, the best explanation of code switching. Because bilinguals use two languages depending on the interlocutor, domain, topic, role and function, researchers in bilingualism caution us against using someone's code switching to reach conclusions about their language proficiency, ability to recall a word, or knowledge of a particular technical word.

Code Switching in Mathematics Classrooms

To describe bilingual students' choice of language in mathematics classrooms, research needs to consider many aspects of a situation. We can start with some of the aspects suggested by Zentella (1981) and Torres (1997): setting, social roles, topics, addressees, and markers of identity. This means considering the place, the purpose, the topic, the participants, and the social relations among them. Important questions to ask are who the student is addressing, especially whether the speaker is addressing a bilingual or monolingual person, whether the setting is private or public, what social roles participants play (is the speaker addressing a teacher, another student, an aide, an elder, a child, etc.), what topics are being discussed (is the conversation about family history, an exchange of cooking recipes, a school topic, an academic subject, etc.), and whether oral or written modes are involved.

When focusing on mathematics, we should also consider:

a) What are the mathematical aspects of the situation? For example, is a student doing computation or engaged in more conceptual activities? What is the mathematical topic (algebra, geometry, etc.)?

b) What are the student's experiences with each language in and out of school, in particular, past experiences with mathematics instruction in each language?

The type of mathematics problem and the student's experience with mathematics instruction can influence which language a student uses. For example, some students may choose to use their first language when working alone on arithmetic computation. After completing a computation, a bilingual student may or may not translate the answer to the other language, depending on who else is involved in the conversation. On the other hand, if bilingual students have not been exposed to mathematics instruction in a particular topic in their first language, it seems reasonable that they would talk about that topic primarily in their second language. In other situations, students might switch between two languages. Students have had varied

experiences with the mathematics register (Halliday, 1978; Pimm, 1987) and mathematical discourse in each language. A student who may be less proficient in the vocabulary for a specific topic in mathematics in one language may be proficient in another aspect of mathematical discourse in that language such as making comparisons between quantities and presenting a mathematical argument (for examples of mathematical discourse practices, see Moschkovich, 2002). These examples point to the importance of considering the specifics of each situation in understanding the relationship between mathematical activity and a student's choice of language.

The sociolinguistic work reviewed here examined code switching in two communities in the U.S. (Mexican Americans and Puerto Ricans). Although there may be some consistencies in language practices across different settings, comparisons across speech communities are difficult since "the linguistic function and social meaning of code switching vary in each bilingual speech community" (Zentella, 1981, p. 109). There are differences among classrooms in the U.S. with bilingual students, classrooms in the U.S. with students from several different language communities, and classrooms with bilingual (or multilingual) students in other countries. One crucial difference to consider across national languages is the mathematics register in students' native language. For example, while mathematical terms exist in languages such as Spanish or Arabic, this may not be the case in the home languages of students in other settings such as South Africa (Setati and Adler, 2001), in the case of Australian Aboriginal languages (Roberts, 1998), or in Maori (Barton, Fairhall, and Trinick, 1998). Nevertheless, views of code switching as a complex language practice rather than a deficit seem to apply across multiple geographic settings and national languages.

ILLUSTRATION: CODE SWITCHING DURING A MATHEMATICAL CONVERSATION

In this section I use a mathematical conversation between two bilingual students to illustrate how a sociolinguistics perspective can inform analyses of bilingual mathematical conversations. The goals of this example are to illustrate how sociolinguistics can complicate our view of code switching, provide alternatives to seeing code switching as a sign of deficiency, and suggest how code switching can provide resources for communicating mathematically.

This transcript was analyzed in detail in a previous publication (Moschkovich, 2002). Although that analysis did not address code switching, when I use this excerpt in presentations it consistently generates

questions, conjectures, and explanations regarding why this student switched languages. Some conjectures I have heard reflect a view of code switching as a deficit itself or as a sign of a deficiency in mathematical knowledge. Such conjectures regarding code switching can lead to over simplified assessments of students' linguistic and/or mathematical competence. In contrast, a sociolinguistic perspective of code switching provides interpretations that are both more complex and more grounded in empirical research. Below, I first expand on the previous analysis by considering multiple explanations of why and how code switching occurred. I then revisit the analysis of resources for mathematical communication provided in Moschkovich (2002).

The excerpt is taken from an interview after school. In this discussion two ninth-grade students used Spanish and English to clarify the mathematical meaning of a description. These two students had been in mainstream English-only mathematics classrooms for several years. One student in this example, Marcela, had some previous mathematics instruction in Spanish. Before working on a set of problems connecting linear equations and their graphs, the interviewer had drawn lines with different slopes on the blackboard, described the lines in Spanish and English, and asked the students to identify which line was steeper ($m = 3$) and which line was less steep ($m = 2$).

The two students were working on the problem in Figure 1. They had graphed two lines on their paper (see Figure 2). They were discussing

8a. If you change the equation y=x to y=-0.6x, how would the line change?
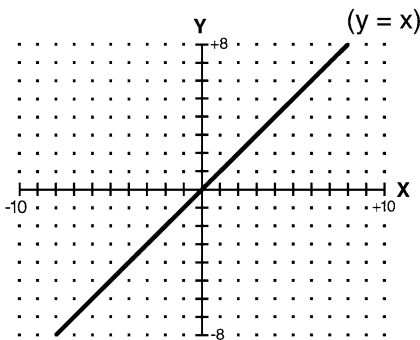
A. The steepness would change.
   Why or why not?

_____NO _____YES   ___ STEEPER
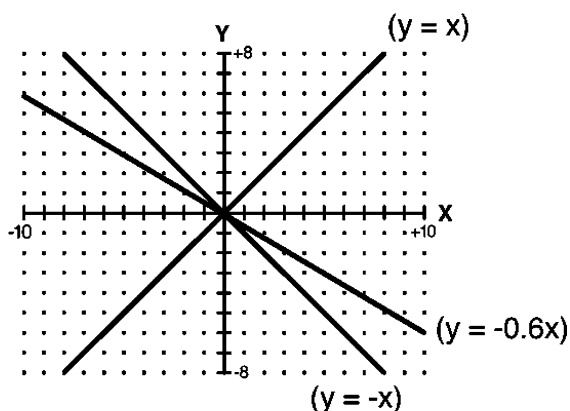   ___ LESS STEEP

Figure 1. Problem.

Figure 2. Lines drawn by students.

whether the line $y = -0.6x$ was steeper or less steep than the line $y = x$. In the preceding discussion, Giselda had alternated between proposing that the line was steeper and less steep and Marcela had repeatedly asked Giselda if she was sure. In the excerpt below, Marcela proposes that the line is less steep and she explains this choice to Giselds. (Transcript annotations are between brackets. Translations are between brackets and in italics directly below an utterance in Spanish).

1. Marcela: No, it's less steeper . . .
2. Giselda: Why?
3. Marcela: See, it's closer to the x-axis . . . [looks at Giselda] . . . Isn't it?
4. Giselda: Oh, so if it's right here . . . it's steeper, right?
5. Marcela: Porque fíjate, digamos que este es el suelo.
                [*Because look, let's say that this is the ground.*]

                Entonces, si se acerca más, pues es menos steep.
                [*Then, if it gets closer, then it's less steep.*]

                . . . 'cause see this one [referring to the line $y = x$]

                . . . is . . . está entre el medio de la x y de la y. Right?
                      [*is between the x and the y*]
6. Giselda: [Nods in agreement.]
7. Marcela: This one [referring to the line $y = -0.6x$] is closer to the x than to the y, so this one [referring to the line $y = -0.6x$] is less steep.

There are several instances of code switching in this transcript. One is the use of "steep" in line 5. Another is the use of "right" as an extra-sentential English tag at the end of an utterance in Spanish (end of line 5). Another is the pronunciation of the letters "x" (as "ex" rather than "equis") and the letter "y" (as "why" rather than "y griega") in English within a Spanish utterance at the end of line 5.

Let us look at Marcela's use of "steep" in line 5. Two frequent interpretations of Marcela's use of the word "steep" line 5 are that a) Marcela does not know the word for "steep" in Spanish and/or b) Marcela is struggling with the concept of "steepness" and her switch to English signals this struggle. A conjecture that this switch reflects forgetting or not knowing the Spanish word for steep, "empinada," implies that code switching is a sign of a deficiency in her vocabulary or linguistic knowledge. The second interpretation, that the code switching signals a struggle with the concept of steepness implies that code switching is a sign of deficiency in her mathematical knowledge.

Invoking "semilingualism" to describe this student as not a full, true, or fluent bilingual because she does not use the word "empinada" in Spanish confuses proficiency in a first language with fluency in the register of school mathematics. In general, a bilingual student's language use should not be compared to that of individuals who have received formal instruction in mathematics where Spanish was the medium of instruction. Instead, "school-related loans reflect the lack of Spanish-language instruction in the public schools for many, many years" (Sanchez, 1994). Without mathematics instruction in Spanish on particular mathematical topics, it is not surprising that some Latino bilinguals might lack the more technical and formal styles of standard Spanish (MacSwan, 2000).

Sociolinguistics suggests we go beyond simplistic views of why individuals code switch and instead consider the details of the communicative situation: the speakers, the context of communication, and the semantic objectives of the conversation. One way to explain Marcela's code switching to "steep" without invoking deficiencies is that this word was used in the written materials, which were provided only in English and were right in front of the two students during this conversation. In contrast, the word "empinada" was not used in the written materials. (The interviewer did use the word "empinada" briefly during an oral introduction to the materials, describing lines on a backboard immediately preceding their discussion.)

It is not possible at this point to know how much exposure to or use of the word "empinada" either of these students had during previous instruction in Spanish, at home, or in school. More extensive data on language and instructional background would be needed to explore such questions. Further analyses of bilingual mathematical discussions

need to provide a fuller account of student experiences with their first and second languages in multiple settings. Future studies can provide this fuller account of language experiences by including ethnographic data such as interviews with students or observations in school, at home, and with peers.

Furthermore, there are several alternative interpretations of the code switching in this excerpt that are well grounded in empirical sociolinguists findings on code switching in Latino communities. One way to understand the code switching in this example is to consider how using "steep" connotes familiarity, in contrast to the more formal choice in Spanish, "empinada" (the Spanish word for "steep"). Using "empinada" would have been more formal than using the English word "steep" because "empinada" is a formal school term. This interpretation of Marcela's use of "steep" is consistent with empirical research on Chicano discourse (Sanchez, 1994). Bilingual Chicanos in the U.S. have been documented using English words that are less formal than the Spanish. In many working class Latino communities English is used for formal and technical domains and Spanish tends to be preserved for use in informal or intimate situations, especially in the home and neighborhood. Some utterances with a loan can connote "familiarity, while the standard (Spanish) expressions connote distance or coldness and in some cases pedantry" (Sanchez, 1994, p. 126).

The switch to "steep", then, could be interpreted as a switch to technical English, a less formal and more familiar choice, than "empinada" for the goal of communicating with another student who is also looking at a written work sheet where the word "steep" is used. Another example of a switch to connote familiarity is from an earlier exchange where Giselda switched to Spanish to ask Marcela to "Look it over, then" (Revisalo, pues"), making her request more friendly and familiar by using the language of home and family. Inducements and jokes have also been documented as calling for brief code switching episodes (Sanchez, 1994).

Marcela's explanations in lines 3, 5, and 7 are examples of intersentential code switching. Here switching between the two languages may be serving as a transitional device that allows for repetition of a point already raised, another documented use of code switching in Latino communities (Sanchez, 1994). This interpretation parallels analyses of conversations among bilingual Latinos who were recorded switching for elaboration, first expressing propositions in English and then giving expansions, additional information, or details in Spanish (Sanchez, 1994). Marcela's explanations are an example of how code switching from one language to another can serve as a resource for elaborating ideas while expanding, repeating or adding information for another speaker.

Marcela's explanations in this excerpt are also examples of how code switching can provide resources specifically for *mathematical* communication. During this excerpt Marcela used code switching back and forth between Spanish and English as a resource to communicate mathematically and participate in mathematical discourse practices. She used two phrases in Spanish that are common in the school mathematics register, "let's say this is . . ." and "if ——, then ——." Marcela also used the metaphor that the x-axis is the ground, uttered in Spanish, *"Porque fíjate, digamos que este es el suelo"* [*Because look, let's say that this is the ground*] as a resource for supporting her claim that the line was less steep.

As described in Moschkovich (2002), mathematical discourse is more than vocabulary. During conversations in mathematics classrooms, students participate in valued mathematical discourse *practices* such as describing patterns, making generalizations, and using representations to support claims. Marcela's explanations in Spanish during this excerpt are examples of important mathematical discourse practices. First, Marcela explicitly stated an assumption, a discursive practice valued in mathematical discourse, when she said: "Porque fíjate, digamos que este es el suelo" [*Because look, let's say that this is the ground*]. Second, she supported her claim by making a connection to mathematical representations, another valued discursive practice. She used the graph, in particular the line y = x (in line 5 in Spanish) and the axes (in line 5 in Spanish and in line 7 in English), as references to support her claim about the steepness of the line.

Current understanding of bilingualism and empirical studies of code switching do not support views of code switching as a deficit itself or as a sign of deficiency in mathematical knowledge. It is important for analyses of bilingual mathematical conversations to avoid interpreting code switching as a deficiency and instead explore how code switching can be a resource for mathematical communication. Sociolinguistics research suggests that we should not expect bilingual students to switch into their first language only to provide a missing English vocabulary term. While some students may sometimes use their first language in this way, other students will use their first language to explain a concept, justify an answer, describe mathematical situations or elaborate, expand and provide additional information, as Marcela did in the example above. In general, code switching has been documented as a resource for elaborating on a point that is repeated, without repeating the initial utterances word for word. In particular, code switching can provide resources such as phrases from the mathematics register in two languages and multiple ways to participate in mathematical discourse practices.

Code switching may seem like the most salient difference between bilinguals and monolinguals. However, other aspects of bilingual learners'

language practices and features of talk may be relevant to learning mathematics, yet are less salient to the untrained ear, and should be explored. Two other important features of conversations to attend to are intonation and the use of gestures. For example, intonation patterns vary across languages and among dialects:

Perhaps the most prominent feature distinguishing Chicano English from other varieties of American English is its use of certain intonation patterns. These intonation patterns often strike other English speakers as uncertain or hesitant (Finegan and Besnier, 1989, p. 407).

Perceiving a student as uncertain or hesitant because of intonation patterns may have an impact on how researchers and teachers perceive student contributions in mathematics classrooms. Bilingual students' use of gestures to convey mathematical meaning has been documented in several studies (for example, Moschkovich, 1999, 2002). Further exploration of the use of gestures during mathematical discussions would provide more detailed descriptions of the role of gestures in how bilingual mathematics learners communicate.

Conclusions

Psycholinguistics and sociolinguistics can inform practice and research on bilingual mathematics learners as they use two languages in mathematics classrooms. Psycholinguistic experiments, although limited in scope, support observations and reports that adult bilinguals have a preferred language for carrying out arithmetic computation. These findings suggest that classroom instruction should allow bilingual students to choose the language they prefer for carrying out arithmetic computation, either orally or in writing. Research on classroom assessment should explore how bilingual learners are assessed on timed arithmetic computation.

Future research on bilingual mathematics learners should consider the cognitive advantages of bilingualism reported by psycholinguists that may be related to mathematical thinking, such as selective attention and translation skills. Research on bilingual mathematics learners, however, need not focus on comparisons between bilingual and monolingual individuals (or between monolingual and bilingual classrooms). Instead, work in sociolinguistics suggests that research focus on describing how bilingual learners communicate mathematically, grounding analyses of classroom discussions in ethnographic observations of classroom interactions. The relationship between previous instructional experiences in mathematics and current proficiency in mathematical discourse practices is crucial for a full understanding of how bilingual learners communicate

in mathematics classrooms. Sociolinguistics also suggests that analyses of classroom communication should be informed by data on students' experiences, building profiles of students' language history, educational background, and attitudes towards bilingual communication for students, peers, teachers, and parents.

Sociolinguistics research provides a complex view of code switching during conversations. Sociolinguistic researchers have long agreed that code switching is not a deficiency or a sign of semilingualism. In fact, they have set aside the notion of semilingualism because it lacks theoretical clarity and is not supported by empirical evidence. Instead, researchers see code switching as a hybrid language practice (Gutierrez, Baquedano-Lopez and Alvarez, 2001) that is the mark of fluency in two languages (Pfaff, 1997). Code switching will continue to seem "odd" only if it is compared to a monolingual norm, to some imagined set of "pure" or "normal" language practices, or to an ideal monolingual speaker-hearer who functions in a homogeneous monolingual speech community. Research in sociolinguistics suggests that rather than viewing code switching as a deficiency, research and practice with bilingual mathematics learners should focus on documenting the resources that bilingual mathematics learners use to communicate mathematically.

NOTES

1. While setting aside the notion of semilingualism, some researchers agree that variation exists in students' proficiency in educationally relevant aspects of language, in the formal language skills in one or more languages, and that bilingual learners need to develop what is currently called "academic English." The topic of academic English, however, is beyond the scope of this article (for a discussion of academic English see Cummins, 2000).

2. There are many bilingual students in mathematics classrooms around the world today (as well as in the past) that are not allowed to use anything other than the dominant language in the classroom.

3. There is one piece of contradictory evidence regarding the preference for the language of arithmetic instruction from one study of adolescents in a bilingual program and their preferred use of the language of instruction for arithmetic (Magiste, 1980). This study reported that only 41% of the adolescents in the study reported carrying out written arithmetic problems in the language of instruction.

4. Some bilinguals (myself and a researcher in an anecdotal report in Kolers (1968) also report that they perform different types of calculations in different languages depending on the language of instruction for different topics (arithmetic, algebra, or calculus). For example, I carry out arithmetic calculations in Spanish, algebra in either language, and calculus in English.

5. One problematic study (Magiste, 1980) compared adolescent monolinguals (14 German dominant) and bilinguals (32 German-Swedish) on a series of written responses to arithmetic problems. This study reported differences in both response time and errors between the two groups and concluded that bilinguals took more time and made more errors than monolinguals. However, these conclusions seem exaggerated since not all group differences in response time reached significance and there was no report of whether differences in average error rates between monolinguals and bilinguals were significant or not. Another problem with this study is that it reports the "time required for each block of 20 problems", making it difficult to make comparisons to other studies where response times were recorded for only one problem.

6. One set of distinctions that sociolinguists do seem to agree on and have explored in great detail are the distinctions among extra-sentential, intra-sentential (within sentences), and inter-sentential switching. Extra-sentential switching involves the insertion of a tag in one language into an utterance in another language. For example, adding "you know," at the end of an utterance in Spanish, or adding "Mira (look)," "Estábien (it's OK)" at the end of an utterance in English. Inter-sentential switches occur at a clause/sentence boundary, with one clause in one language and the other clause in the other. Intra-sentential switching describes switching of different types that occur within the clause boundary (including within word boundary as in the loan blend of an English verb with a Spanish infinitive ending as in "check-ear" or "save-ar"). This set of distinctions seem to be especially relevant for the sociolinguistic study of code switching concerned with describing the grammatical rules governing code switching, but are not apparently relevant to mathematics cognition and learning.

7. Although some sociolinguists make a distinction between switching and borrowing, I will not. Borrowing can be described as "the adaptation of lexical materials to the morphological, syntactic and usually (but not always) phonological patterns of the recipient language" (Hamers and Blanc, 2000). Distinctions can be made between loan words used by bilinguals (for example using the English inspired "troque" in Spanish rather than the Spanish word "camión"), loan words used by monolinguals who do not code switch (for example tortilla), and "nonce-words" or nonce borrowing, an idiosyncratic use limited to one speaker on one occasion. "An English word in Spanish discourse may be a code mix, an idiosyncratic use limited to one speaker on one occasion (nonce borrowing), or a bona fide loan word used by many community members" (Torres, 1997, p. 64). Hamers and Blanc (2000) propose that borrowing and code switching are phenomena at either end of a continuum. Because this distinction does not seem to have direct relevance to mathematics learning situations, I will leave it aside.

8. Because this paper focuses on bilingual children learning mathematics, I will not describe research documenting teachers code switching in classrooms. For examples, see Khisty (1995), Setati (1998), Setati and Adler (2001), Valdés-Fallis (1976), and Zentella (1981).

9. Zentella found that conversational strategies that allowed speakers to realign their footing, to clarify or emphasize their messages, and to control their interlocutors were particularly important. Children manipulated conversational strategies in two languages in keeping with *el bloque* norms, the communicative objectives of the moment, and the unequal positions of the majority and minority language groups in the national economy. (Zentella, 19997, p. 113)

REFERENCES

Adler, J.: 1998, 'A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom', *For the Learning of Mathematics* 18(1), 24–33.

Adler, J.: 2001, *Teaching Mathematics in Multilingual Classrooms*, Dordrecht, The Netherlands: Kluwer Academic Press.

Auer, P.: 1984, *Bilingual Conversation*, Amsterdam: John Benjamins.

Baetens Beardsmore, H.: 1986, 'Definitions and typologies', in *Bilingualism: Basic Principles*, Clevedon: Multilingual Matters, pp. 1–41.

Barton, B., Fairhall, U. and Trinick, T.: 1998, 'Tikanga Reo Tatai: Issues in the development of a Maori mathematics register', *For the Learning of Mathematics* 18(1), 3–9.

Bialystok, E.: 2001, *Bilingualism in Development: Language, Literacy and Cognition*, Cambridge: Cambridge University Press.

Bialystok, E.: 1999, 'Cognitive complexity and attentional control in the bilingual mind', *Child Development* 70, 636–644.

Bialystok, E. and Majumder, S.: 1998, 'The relationship between bilingualism and the development of cognitive processes in problem solving', *Applied Psycholinguistics* 19, 69–85.

Cazden, C.: 1986, 'Classroom discourse', in M. C. Wittrock (ed.), *Handbook of Research on Teaching*, New York: Macmillan.

Cazden, C.: 1993, 'Vygotsky, Hymes, and Bakhtin: From word to utterance to voice', in E. Forman, N. Minnick and C. A. Stone (eds.), *Contexts for Learning: Sociocultural Dynamics in Children's Development*, NY: Oxford University Press, pp. 197–212.

Cook, V.: 1997, 'The consequences of bilingualism for cognitive processing', in A. de Groot and J. Kroll (eds.), *Tutorials in Bilingualism*, Mahwah, NJ: Erlbaum, pp. 279–299.

Cumming, A.: 1989, 'Writing expertise and second language proficiency', *Language Learning* 39, 89–141.

Cumming, A.: 1990, 'Meta-linguistic and ideational thinking in second language composing', *Written Communication* 7, 482–511.

Cummins, J.: 1976, 'The influence of bilingualism on cognitive growth', *Working Papers on Bilingualism* 9, 1–43.

Cummins, J.: 1979, 'Cognitive/academic language proficiency, linguistic interdependence, the optimum age question, and some other matters', *Working Papers on Bilingualism* 19, 197–205.

Cummins, J.: 1994, 'Semilingualism', in *Encyclopedia of Language and Linguistics* (2nd edn), Oxford: Elsevier Science.

Cummins, J.: 2000, *Language, Power, and Pedagogy*, Buffalo, NY: Multilingual Matters.

De Avila, E. and Duncan, S.: 1981, 'Bilingualism and the metaset', in R. Durán (ed.), *Latino Language and Communicative Behavior*, Norwood, NJ: Ablex Publishing Corporation.

Finegan, E. and Besnier, N.: 1989, *Language: Its Structure and Use*, NY: Harcourt Brace Jovanovich.

Garcia, E. and Gonzalez, R.: 1995, 'Issues in systemic reform for culturally and linguistically diverse students', *Teachers College Record* 96(3), 418–431.

Gee, J.: 1996, *Social Linguistics and Literacies: Ideology in Discourses* (3rd edn). London, The Falmer Press.

Gee, J.: 1999, '*An Introduction to Discourse Analysis: Theory and Method*, NY: Routledge.

Grosjean, F.: 1999, 'Individual bilingualism', in B. Spolsky (ed.), *Concise Encyclopedia of Educational Linguistics*, London: Elsevier, pp. 284–290.

Gumperz, J.: 1982, '*Discourse Strategies*', New York: Cambridge University Press.

Gutierrez, K. , Baquedano-Lopez, P. and Alvarez, H.: 2001, 'Literacy as hybridity: Moving beyond bilingualism in urban classrooms', in M. de la Luz Reyes and J. Halcon (eds.), *The Best for Our Children: Critical Perspectives on Literacy for Latino Students*, New York: Teachers College Press, pp. 122–141.

Hakuta, K. and Cancino, H.: 1977, 'Trends in second-language-acquisition research', *Harvard Educational Review*, 47(3), 294–316.

Hakuta, K. and McLaughlin, B.: 1996, 'Bilingualism and second language learning: Seven tensions that define research', in D. Berliner and R.C. Calfe (eds.), *Handbook of Educational Psychology*, NY: Macmillan.

Halliday, M. A. K.: 1978, 'Sociolinguistic aspects of mathematical education', in M. A. K. Halliday, *The Social Interpretation of Language and Meaning*, London: University Park Press, pp. 194–204.

Khisty, L. L.: 1995, 'Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students', in W. G. Secada, E. Fennema and L. B. Adajian (eds.), *New Directions for Equity in Mathematics Education*, New York: Cambridge University Press, pp. 279–297.

Kolers, P.: 1968, 'Bilingualism and information processing', *Scientific American* 218, 78–86.

MacSwan, J.: 2000, 'The threshold hypothesis, semilingualism, and other contributions to a deficit view of linguistic minorities', *Hispanic Journal of Behavioral Sciences* 22(91), 3–45.

Magiste, E.: 1980, 'Arithmetic calculations in monolinguals and bilinguals', *Psychological Research* 42, 363–373.

Malakoff, M. and Hakuta, K.: 1991, 'Translation skill and meta-linguistic awareness in bilinguals', in E. Bialystok (ed.), *Language Processing in Bilingual Children*, Cambridge, UK: Cambridge University Press, pp. 141–166.

Marsh, L. and Maki, R.: 1976, 'Efficiency of arithmetic operations in bilinguals as a function of language', *Memory and Cognition* 4, 459–464.

McLain, L. and Huang, J.: 1982, 'Speed of simple arithmetic in bilinguals', *Memory and Cognition* 10, 591–596.

Mehan, H.: 1979, *Learning Lessons: Social Organization in the Classroom*, Cambridge, MA: Harvard University Press.

Moschkovich, J. N.: 1999, 'Supporting the participation of English language learners in mathematical discussions', *For the Learning of Mathematics* 19(1), 11–19.

Moschkovich, J. N.: 2002, 'A situated and sociocultural perspective on bilingual mathematics learners', *Mathematical Thinking and Learning*, Special issue on Diversity, Equity, and Mathematical Learning, N. Nassir and P. Cobb (eds.), 4(2 and 3), 189–212.

Orellana, M., Reynolds, J., Dorner, L. and Meza, M.: 2003, 'In other words: Translating or "para-phrasing" as family literacy practice in immigrant households', *Reading Research Quarterly*, 38 (1), 12–34.

Paulston, C.: 1982, *Swedish Research and Debate About Bilingualism*, Skoloverstyrselsen: National Swedish Board of Education.

Pfaff, C.: 1997, 'Contacts and conflict: perspectives from code switching research', in M. Putz (ed.), *Language Choices: Conditions, Constraints and Consequences*, Amsterdam: John Benjamins, pp. 342–360.

Pimm, D.: 1987, *Speaking Mathematically: Communication in Mathematics Classrooms*, London: Routledge.

Qi, D.S.: 1998, 'An inquiry into language-switching in second language composing processes', *Canadian Modern Language Review* 54(3), 413–435.

Ramirez, A. and Milk, R.: 1986, 'Notions of grammaticality among teachers of bilingual pupils', *TESOL Quarterly* 20(3), 495–513.

Roberts, T.: 1998, 'Mathematical registers in aboriginal languages', *For the Learning of Mathematics* 18(1), 10–16.

Sanchez, R.: 1994, *Chicano Discourse: Socio-Historic Perspectives*, Houston, TX: Arte Público Press.

Setati, M.: 1998, 'Code-switching and mathematical meaning in a senior primary class of second language learners', *For the Learning of Mathematics* 18(1), 34–40.

Setati, M. and Adler, J.: 2001, 'Between languages and discourses: Code switching practices in primary classrooms in South Africa', *Educational Studies in Mathematics* 43, 243–269.

Skutnabb-Kangas, T.: 1984, *Bilingualism or Not: The Education of Minorities*, Clevedon: Multilingual Matters.

Tamamaki, K.: 1993, 'Language dominance in bilinguals arithmetic operations according to their language use', *Language Learning* 43(2), 239–262.

Torres, L.: 1997, *Puerto Rican Discourse: A Sociolinguistic Study of a New York Suburb*, Mahwah, NJ: LEA.

Tse, L.: 2001, Why don't they just learn English? Separating fact from fallacy in the U.S. language debate. NY, NY: Teachers College Press.

Valdés-Fallis, G.: 1978, 'Code switching and the classroom teacher', *Language in Education: Theory and Practice*, Volume 4. Wellington, VA: Center for Applied Linguistics.

Valdés-Fallis, G.: 1979, 'Social interaction and code switching patterns: A case study of Spanish/English alternation', In G. D. Keller, R.V. Teichner, and S. Viera (eds.), *Bilingualism in the Bicentennial and Beyond*, Jamaica, NY: Bilingualism Press.

Zentella, A. C.: 1997, *Growing up Bilingual: Puerto Rican Children in New York*, Malden, MA: Blackwell Publishers.

Zentella, A. C.: 1981, 'Tá bien, You could answer me en cualquier idioma: Puerto Rican code switching in bilingual classrooms', In R. Durán (ed.), *Latino Language and Communicative Behavior*, Norwood, NJ: Ablex Publishing Corporation, pp. 109–130.

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