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TEACHING MATHEMATICS IN MULTILINGUAL CLASSROOMS

ABSTRACT. In this paper we present the way in which language issues have become a relevant factor in research which aims to study the socio-cultural aspects of mathematics education in classrooms with a high percentage of immigrant students. Our research on language issues focuses on two aspects, namely the language as a social tool within the mathematics classroom and the language as a vehicle in the construction of mathematical knowledge. We introduce our problem within this area and provide a rationale for our research methodology, not avoiding its drawbacks, but rather giving examples of the kinds of difficulties we encountered. The paper highlights the integrated nature of the social, cultural and linguistic aspects of mathematics teaching and learning, and illustrates the fact that, even if the mathematical language can be considered universal, the language of 'doing mathematics within the classroom' is far from being universal.

THE RESEARCH WITHIN ITS CONTEXT

> 'Cal afirmar clarament que el llenguatge no sols és un tipus de comportament social, sinó que és, de fet, la major part del comportament social dels humans, tant que podríem dir que la societat humana és una societat lingüística.'
>
> (L. García, 1999, pp. 5–6)
>
> 'It has to be explicitly claimed that language is not only a kind of social behaviour, but, in fact, the greatest part of the social behaviour of human beings, to such a point that it could be said that human society is a linguistic society.'

In 1997, a project was commissioned to the first of the authors by the Ministry of Education in Catalonia, an autonomous region in northern Spain (capital Barcelona), concerning mathematics teaching in schools with large numbers of immigrant students. The project is funded by a Catalan private education foundation, Fundació Propedagògic, and is concerned with finding more appropriate ways to teach mathematics to immigrant students both in primary and secondary schools.

In recent years, there has been increasing immigration into Catalonia which has led to significant changes in the school population. The immigrant population in Catalonia in 1997 was about 1.4% of the whole population and reaches the 2.3% in Barcelona, where the project has been developed. This is significant because the realities of a multicultural class-

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room raise many questions related with issues concerning equity and justice (Keitel et al., 1989). It is important not only because in some schools the percentage of immigrant children is 90% but also because, since immigration is a growing phenomenon, more and more schools and communities will be facing a multicultural reality. Most of the immigrant pupils in our schools come from North Africa (Magreb), but also from other countries in Africa, North and South America, Asia and East European countries. This new situation has focused attention on the inadequacy of the educational provision in schools and classes which can be thought of as highly multicultural. In particular, our educational system is differentially effective for students depending on their social class, ethnicity, language background or other demographic characteristics, as it has been proved to be in other countries (Secada, 1992).

According to the educational administration, the main problem the immigrant students face is that of not mastering the Catalan language, a language with Latin roots and its own grammar. This language is spoken in Catalonia and in other small countries (Andorra) or regions in the world by about six million people. In Catalonia both Catalan and Spanish are official languages. The official language of instruction in Catalan schools is Catalan, but all Catalan people can speak both languages and use one or the other depending on their preferences, mainly related to their social context.

The Catalan educational administration considers that there is no possible regular access to the curriculum before having acquired a high competency in the official language of the learning. The Catalan language programme for non-Catalan students assumes that it is not possible to acquire the language while learning subject matter content. Therefore, to 'solve' the language problem of the newly arrived students the educational administration provides language lessons for 7 hours and a half per week, for six months after their arrival, in regular school hours. These Catalan lessons take place regardless of the fact that the lessons on other subjects keep going on. Therefore, when the students rejoin the regular classes, they may have missed previous sessions, making the required effort to follow the group even bigger. This may seem to the reader, as it seems to us, an inconsistent system.

From the point of view of the educational administration, not knowing the language is the *only* problem immigrant students face. From our point of view, being such a student within a mainstream school requires a more complex analysis than considering only language factors. In coming to Catalonia, and adjusting to Catalan schools, immigrant students and their families travel very long distances in a multidimensional sense: physical,

emotional and psychological. Many of these newly arrived immigrants encounter serious problems within our educational system, even more serious than what is involved in mastering the language. Their use of basic Catalan does not imply that they have understood or reached the values and the meanings linked to the new culture.

We agree with MacGregor and Price (1999) that proper analysis would imply taking strong political decisions. For example, most of the immigrants live in deprived neighbourhoods. As a consequence, permanent residents feel antipathy to the new identity of their neighbourhoods. The schools' populations have rapidly changed, especially those located in marginal sites: some public schools in Barcelona have nearly 90% of immigrant students. Some of these schools are near middle-class communities sites and the proximity from the residence to the school is the main criteria for allocating children to a public school. However, the Catalan families that can afford a private school take their children away from the public one if they feel there are 'too many' immigrant children.

Part of our research is developed in one of these 'ghetto' schools in Barcelona, a school with a high percentage of immigrant students, most of them being 'latest arrival immigrants'. We have worked with students aged 15–16, from 9 different countries, and, even if the students come from the same country, they may speak different languages. The range of main languages (i.e. "the language of greatest day-to-day use and facility for the speaker" (Adler, 1999, p. 48)) brought to the mathematics classroom under study are: Arabic, Tamazigh, Tarifit, Urdu, Punjabi, Tagalog, Catalan and Spanish. In addition, note that the Spanish spoken in Spain is different from that spoken in Latin American countries. Hence most of the students are being taught mathematics in their third or even fourth language. For instance, the first language of the students coming from Karachi (Pakistan) is Punjabi, the second is Urdu, the third is English, and they are learning Catalan and in Catalan, while improving Spanish (which is spoken by most of the local people where the students live).

THE TEACHING OF MATHEMATICS IN MULTILINGUAL SITUATIONS: TWO SIDES OF A 'PROBLEM'

When initiating the project, we were aware that it would require us to rethink what mathematics education could be, since we agreed with Cocking and Chipman (1988, p. 43):

> The research questions, the social class variables, the learning patterns, in fact, the basic assumptions about language and learning, seem to be changing as the educational population becomes more ethnically diverse.

Explaining the difficulties of immigrant children in terms of cognitive deficit is questionable (Ginsburg and Allardice, 1984; Nunes, Schliemman and Carraher, 1993; Rasekoala, 1997). Moreover, this interpretation has social implications because it projects particular expectations onto concrete cultural groups. Our starting point was to consider the cultural contribution of ethnic minorities and different social groups as a source of richness to be maintained and shared. The research team did not see cultural and linguistic differences as a 'problem to be solved', but as a potentiality.

Mathematics education should be embedded in the comprehension of the social and cultural reality where it takes place (Oliveras, 1996). The difficulties immigrant students experience when learning mathematics is often linked to the distance from their different social and cultural frames of reference to the implicit ones within school (Crawford, 1986). From that point of view, we felt it was necessary to develop an approach that integrates, on the one hand, cognition, and on the other hand, attitudes and values considering all these components situated within the social and cultural context, as Abreu, Bishop and Pompeu (1997) suggested.

We were aware that in multilingual settings the teachers have to face two kinds of problems. First, the initial need for communicating with the students, to know more about their previous academic history, and their 'starting point' as mathematical learners. For instance, teachers have to distinguish between those students who had never attended school before and those who had regularly attended schools in their home countries. It could be thought that this problem may be solved just by providing adequate communication tools, e.g. by having adults present who know the minority students' languages. However, in the cases where the educational administration has promoted this kind of experience, it has proved to be of little help. For instance, in some cases the administration accepted the idea of ensuring that adults that could 'translate' were present during the examination which the student 'had to pass' to be allocated to a particular group. In such situations, our main argument with the administration was about whether giving the students a test was the best way to come to know about their previous school experience. However, we were never able to achieve what we really wanted: the permission to have adults from the different immigrant communities within the classroom to help the teacher, and ourselves, in communicating with the students.

Secondly, teachers have to face the problem of communication once the student is allocated to a class. The language practices that learners bring to school inevitably affect how and what they learn (Nieto, 1999). The students' level of mastery of the language of instruction is a crucial factor for the teacher's interpretation of their difficulties (Clements and Jones,

1983). What happens when there is no sharing of words or/and meanings? If students cannot comprehend either the language of the teacher or the meanings s/he attaches to the words, will they be able to participate in the mathematical discussion?

The use of several different languages by the students that neither the teachers, nor the researchers had mastered, appeared initially as an obstacle to our research: how could we study the interactions among peers if we could not understand their conversations? However, we soon realised that language issues were also to be considered as part of the social and cultural issues. We became aware of the importance of including the multifaceted nature of relationships among mathematics, school mathematics and language (Saxe, 1988) and of the need to consider a wide framework for interpreting language factors in mathematics learning (Ellerton, 1989).

Any mathematics classroom admits a study from the point of view of language and communication, whether or not it is multilingual or located in a deprived setting. However, in schools with a high percentage of immigrant children from poor countries, language conflicts appear more explicitly. In these cases, two distances emphasise the communication gap: the objective distance because there is no common language; and the social distance in the meanings of the messages once a common language has been acquired. It is difficult to imagine what it means for youngsters to arrive at school without knowing the official language while they are expected to get used to the norms that regulate the school dynamics and the societal habits. However, it is not only a matter of reaching a survival competency in the official language but also that of succeeding in the use of the social and cultural communicative tools. To acquire communicative competency in a new environment means constructing a bridge between the meanings of the own context and those of the new one.

Our research on language issues focuses on two aspects:

- *The language as a social tool* in the process of sharing meanings within the mathematics classroom. This led us to try to find answers to questions such as: Are cultural conflicts in the mathematics classroom linked to language issues? What are the beliefs and expectations about the individuals involved in a multilingual situation? Are the learners aware of their potentialities as mathematical contributors despite their lack of linguistic competency? What are the values and meanings associated with one language or another in this particular context?
- *The language as a vehicle in the construction of mathematical knowledge*. This led us to try to find answers to questions such as: To what extent the written symbolic system of mathematics language can be considered as the common language in a multilingual setting? How

the absence of a common language can interfere with the teacher's interpretation of the learning process of his/her students? What are the tools teachers can use to communicate within the mathematics classroom when there is no common language? What decisions concerning language do they take in order to facilitate their students' learning processes?

SITUATING THE PROBLEM IN THE WIDER LITERATURE

Language factors in mathematics learning can appear under many faces. Our focus is the lived realities of immigrant children in classrooms where the learners and the teacher do not share the language, nor their culture and values. Our theoretical approach considers language as a wide notion where social, cultural, linguistic, emotional and cognitive factors are intertwined.

Language has to do with communication at different levels. Our understanding of the language factors in mathematics education avoids a reductionist approach and emphasises as central notions both communication and culture as stated by Ellerton and Clarkson (1996), considering 'language as a social tool' (Abreu 1998), since we agree with García (1999, p. 5):

> against segregating language from the other human behaviours, it must be repeated, once and again, that languages do not exist physically, but those that speak a language. . .

When minority language students join a mathematics class, they often find different norms, regulating both the social dynamics of the mathematics classroom and the mathematical practices. Discontinuities in understanding new words and new meanings can turn into a wide variety of cultural conflicts and disruptions of the learning process. In order to overcome these drawbacks, many teachers try to be more transparent in their use of language. Throughout our work, the constructs *norms, discontinuity, cultural conflict* and *transparency* have proved useful.

Though Cobb, Wood and Yackel (1993) have focussed on monolingual mathematics classrooms, their construct of norm is useful for the comprehension of the dynamics of any classroom: the different interpretation of the same norm in a classroom community, due either to the different languages or to cultural backgrounds, often implies a lack of communication. We find useful the notions of social norms and sociomathematical norms (Yackel and Cobb, 1996). There are plenty of meanings and values attached to mathematics teaching and learning by the different participants

and, in particular, to the social norms that regulate the classroom dynamics, e.g. the legitimised ways of participating, the role of the textbook, or the role of the teacher. Also there are different meanings associated with the norms of mathematical practice: the criteria for validating a mathematical process, the way of participating in mathematical discussion, the different uses of algorithms, or the role of out-of-school mathematical knowledge. Our research (Gorgorió et al., 2001) has shown that, in a multiethnic classroom, the possible interpretations of the norms by some of its participants are often difficult to understand by the others, and are, therefore, a potential source for cultural conflicts that may interfere with the learning process.

When minority language students have an intermediate level of the official language, they are supposed to understand the more fundamental words and therefore the meanings of the context messages. Nevertheless, to have a full comprehension of the meanings and values of a social situation is a further step beyond the learning of the words (Sapir, 1970). The more distant the meanings from the different worlds, the more need for making explicit those of the new situation. The variety of dissonances minority language students experience during their transition process often result in a wide range of failure manifestations: disruptive behaviours, 'silent autism' behaviours, absenteeism or cognitive and emotional blockages. This often leads to failure within the school system.

When having difficulties in structuring the new meanings in the new context, the learners may experience a gap in the coherence and the continuity of their living experiences, appearing as cultural discontinuities (Nieto, 1999) and hence as cultural conflict (Bishop 1988). The idea of 'learning as resolution of cultural conflict' (Bishop 1998) has helped us to organise and co-ordinate our research activities. Bishop's (1998) interpretation of cultural conflicts as being unavoidable, avoidable or minimizable is also appropriate for linguistic conflicts, which may be:

- unavoidable: inherent in the distance between the student's main language and the official one;
- avoidable: linked to the lack of information about 'the language problem' or to narrow values associated to linguistic diversity; or
- minimizable: arising from the choice of specific classroom dynamics and subject content that interfere with communication.

Much research having to do with language and mathematics has focussed on unavoidable conflicts (Zepp, 1989). However, focussing on linguistic conflicts that can be minimised or avoided, and trying to facilitate continuity can have important educational implications.

Moschkovich (1996, p. 27) refers to different types of discontinuities in language minority classrooms, "from first language to second language, from social talk to academic talk, and from the everyday to the mathematics register". While these are useful, of greater importance are the linguistic barriers that create obstacles to mathematical learning. The mathematics register is not the same as the everyday language register, the main language is not the same as the official language in the classroom. The mixture arising from this double discontinuity results in code switching being difficult for the learners in many different ways.

Code switching goes beyond mere translation, and further than switching from the first language to the language of instruction. It implies changing not only words, but also their meanings and the ways they are used. It means being able to activate a different communication system, with new symbols, new figures and new words, and also with words and figures which exist in both systems but which may also represent different things depending on the system. It means a radical change in the conventions governing the use of the system, and a change of the norms that regulate the activity in the context where the system is used.

Ellerton suggested a multidimensional framework for interpreting language factors in mathematics learning which portrays the centrality of teachers, the mathematics classroom, and curriculum in addressing these issues (Ellerton and Clarkson, 1996). Following Ellerton's idea, we considered it crucial to know more about the teacher's knowledge of the linguistic situation and how to link it with the curriculum. In particular, it is important to take into account the relationships between the educational aspirations of minority language students and their teachers (Coughlan, 1995).

The construct of transparency (Lave and Wenger, 1991) is central in the literature concerning the different teaching strategies used to face the failure of minority language students. Adler's notion of transparency (Adler, 1997 and 1999) is developed as involving a dilemma. Teachers need to make explicit the norms that regulate the dynamics of the mathematics classroom and of mathematical practice. This is even more important if learners come from different linguistic and cultural backgrounds. However, this care in focussing on the language in order to minimise cultural conflicts may produce a loss in the mathematics problem under consideration (Adler, 1999). Or, even worse, there may be no place left for a mathematical conceptual focus (Valdes, 1999). In searching for transparency teachers must not reduce the whole mathematical practice to visible talking. Moreover, when using visible talking, teachers should avoid an incomplete and confusing language of learning.

Teachers in multilingual settings usually address their students using a simplified language register (Fillmore Wong, 1982). Teaching in a simplified form of the official language does not guarantee that learners have better access to the mathematical content, but may add an obstacle because it interferes with the acquisition of rich mathematical concepts, by obscuring them (Adler, 1997; Valdés, 1999). It is necessary to reach a point where the language of learning helps the acquisition of school mathematics and vice versa. Even if learners have difficulties in verbalising a mathematical process, the teacher can promote the mathematical thinking by distinguishing the talk from the thinking (Adler, 1999). For example, they may allow learners to use their main language to face the mathematical problem, before using a shared language publicly in the classroom. However, while many teachers in multilingual classes themselves use a simplified language, they ask for correct use of the official language by the minority learners. This does not promote participation (Moschkovich, 1999).

DATA COLLECTION

The main goal of the project being to 'promote changes' in a school context, we considered the best approach to be action-research involving a team of university researchers and in-service teachers. Such an approach builds bridges from educational research to intuitive educational acts. The most adequate research approach was a collaborative one under a qualitative and interpretative paradigm. The research team included primary and secondary mathematics teachers, already linked with research at university, and who had a partial release of their teaching hours to devote time to the project. Only through the perspectives of teachers could we properly address the practical issues of the research. Teachers have a better knowledge of the context, know more about the other practitioners who shape mathematics education, and know much more about real possibilities for change. Moreover, teachers' questions and explanations derive from a knowledge domain that is distinct from, and complementary to, that of researchers from university. The presence of teachers as full members of the research group, legitimates and facilitates the contact and the communicating process with other teachers, and also helps with finding ways to disseminate the research findings and the innovation proposals.

The research group has followed two paths. One of them is studying teachers facing multilingual classrooms, teachers whose only connection with the project is being 'an object of study'. They were not required, or expected, to adhere to the goals and principles of the study. The second route

is to study with teachers, through an action-research model, analysing their teaching strategies agreed upon by the members of the research team using the theoretical framework. Some of our methodological choices, similar to those used by Abreu (1995) and Presmeg (1997), have been:

- to study in-service teachers belonging to the first group to identify their understanding of the language problems they face when teaching foreign students by tape-recording and analysing interviews with several teachers, and video-recording mathematics lessons;
- to study in-service teachers using an action-research model to identify teaching strategies that could help to overcome the linguistic barrier by videotaping mathematics lessons which, a priori, were considered to contain useful strategies to overcome the linguistic barrier; and
- to observe students systematically during mathematics lessons to identify their problems regarding language and interviewing them for insight into their problems.

Problems

We had to face several challenges and problems in the process of data collection, due to both educational administration constraints and to methodological issues caused by individuals' lack of willingness to co-operate with the project.

The first challenge has been to achieve the full acceptance of the project by the educational administration. The project was initially the result of a request from the administration, but the team's understanding of the cultural and linguistic reality in schools goes far beyond that of the officials. The initial request was to create 'ready-for-use' materials to be given to the teachers having immigrant students with an incomplete knowledge of Catalan language. Behind this request are hidden assumptions such as the idea that once immigrant students have reached a practical level of Catalan, there would be no problem getting them to 'reach the same level' as local students.

Since the goals of the research project were not fully appropriated by the educational administration, we had to argue many points. For instance, they were reluctant to accept the need for that part of our study which was focussed on ghetto schools, to provide help for overcoming linguistic barriers or for gathering general data about the students. We also had to convince them of the need for videotaping mathematics classes, and for a partial release of the teachers involved.

The methods used within a qualitative research whose 'object of study' are individuals obviously require their willingness to co-operate. School

principals saw our project as something against the well established tradition, and teachers felt themselves at risk. Therefore, the teachers interviewed were only those who were willing to accept, and were a group who showed some level of sensitivity towards the difficulties of teaching immigrant students.

We also needed the co-operation of students, and therefore we had to convince them of 'our good intentions', to overcome some practical matters related to different cultural traditions or personal situations (especially with respect to video-recording), and to overcome the linguistic barrier (e.g. finding ways to analyse interactions when they took place in a language unknown both to the teacher and the researcher). Moreover, due to the unstable social and economical position of the students' families it became difficult to carry through some case studies when students disappeared during our study.

The teachers on the team had to face problems working under an action-research model. On the one hand, they had to continue to do their jobs within the constraints of the school administration, including finding time to work on a research project while being a devoted teacher. On the other hand, there were tensions arising from researching their own teaching, for example the tension between the teachers' responsibility to the students and to the research. The research team explicitly agreed that the responsibility as teachers was superior to that as researchers, even if that meant a 'loss' for the study.

We had also to face the risk of bias when interpreting data obtained from a classroom where the roles of teacher and researcher were played by the same person (Robinson, 1998). Discussing and contrasting the different points of view within the research team, having an observer in the classroom, and documenting and analysing the development of lessons through the video recording and the teacher's diary have all helped to control the biases.

Despite all the tensions and challenges, after three years we are convinced that the understanding of the situation we have obtained through a collaborative and action-research approach, under a qualitative and interpretative paradigm, were worth the effort of overcoming all the constraints and conflicts we had to face.

RESULTS AND INTERPRETATIONS

Language issues and cultural conflicts

We have documented examples where there was a lack of communication even after the immigrant students had learnt the basics of Catalan vocabulary and grammar. These examples illustrate that language is a communication tool that goes beyond the translation of words. In many of the cases language issues cannot be disentangled from cultural and social facts. The following example shows that endowing meanings to words requires knowing the norms of the context where they are used.

Ramia, a girl aged 15, has a reasonable understanding of Catalan, however she seems not to understand what is going on during the mathematics lessons. When working in small groups, her understanding seemed to improve. However, when the teacher addressed the whole class, Ramia seemed lost. After observing her in class, we concluded that, even if she had a communicative competence on the Catalan language on daily matters, she had problems understanding the register of language of her mathematics teacher. Through the interview with Ramia, we concluded that she never told her teacher that she could not understand her because that would have conflicted with her cultural patterns of behaviour. According to her values, it is not good manners to tell the teacher that one can not understand her, nor to talk with her friends while the teacher is talking. She preferred to 'become lost' than to behave improperly, according to her values. It was difficult for the teacher to convince Raima that if she could not understand it was not necessarily her fault, and that asking for clarification was essential for her learning.

Other examples, like the following one, show that it is not only the case that cultural values interfere with communication by preventing students asking for help, but also they refuse help because of them.

After the students have been working in small groups on solving a problem, a Pakistani boy, Sheraz, aged 15, shows on his worksheet a solution different from those generated by other students:

Teacher:	Sheraz, could you please explain us, what you have done?
Sheraz:	I can't, it is too difficult.
Teacher:	Too difficult? But it is your solution, it cannot be difficult for you, you show it to us, and we will see if it is too difficult for us.
Sheraz:	It is too difficult to explain.
Teacher:	To explain?
Sheraz:	In Catalan.

Teacher:	Could you, Sajid, help him with the explaining?
Sajid:	It is too difficult!
Teacher:	(getting nervous with so many difficulties) Sajid, it is not too difficult, you speak good Catalan.
Sajid:	Not the Catalan, the maths!!!! I do not understand what he has done, . . .
Teacher:	Well, Sheraz and Sajid, how are we supposed to know what Sheraz has done?
Saima:	Miss, if you want, I can try to help.
Sheraz:	(very upset) No!! Not Saima, she is an Indian girl!!! (Saima begins to cry).

After the class the teacher had a conversation with Sheraz, and offered him the opportunity to present his solution the next day, but he kept on refusing. Sheraz, the son of a family that has left a position of high social class in Pakistan, lives his schooling in Catalonia with anxiety. He feels he is loosing possibilities and wasting his time here. He expects to return to Pakistan, and therefore does not make much effort to learn Catalan or Spanish. He does not accept his classmates, even those, like Sajid, who also come from Pakistan. In one of the interviews, when the teacher said: "I see, you are from Karachi like Sajid", he answered very crossly: "I am here because war, he is here because he is poor".

Saima, an Indian girl aged 15, the oldest daughter of a very poor family, is an extremely clever and beautiful girl, and dresses in Indian traditional clothes. She participates in the class, and has got friends in the school. She has difficulties communicating with boys. She has needed to strongly negotiate with her family to get the permission to attend school, and she knows that if her brother, who comes everyday to wait for her after school, sees her talking with a boy, this would mean the end of school for her. She wants to become a teacher: "as you miss, but only a girls' teacher".

The 'language problem' as seen by teachers

Since we believe that many of the linguistic conflicts that we have observed can be either avoided or minimised through the way communication takes place, we consider that teachers play an important role. Therefore we wanted to know more about how the 'language problem' was seen by teachers.

We asked teachers to say whatever they wanted about their multilingual mathematics classes, and the strategies they employ to improve both their teaching practice and their students' learning process. The following is part

of an interview of one teacher and shows most of the issues that we have observed repeatedly with the other teachers.

Interviewer: Do you need any special resources to cope with multilingual classes?

Teacher: Well, what can you do when half of the class does not understand the language and the other half understands the language but does not understand maths?

Interviewer: It seems a great challenge...

Teacher: Yes, it is. I try not to use very long presentations when introducing a maths problem, just to facilitate, as far as possible, the understanding of the content with no interfering with the form... but then the instructions remain unclear... I am absolutely convinced they are able to think in their main language, I have no doubt at all, but as they must speak in Catalan and their thinking is in Urdu, or whatever... I have no access to their thinking...

Interviewer: So, what can we do as teachers in order to gain explicit evidence of their thinking?

Teacher: Actually, I don't think it is a good idea to have children with no Catalan proficiency in math class... you know, math requires a sophisticated use of language.

Interviewer: Then... there is no possible dynamics to face multilingual classes...

Teacher: My experience tells me they should first have a [period of] language immersion before entering the maths class. If they were good at maths in their countries of origin, they are going to lose their self-esteem by remaining silent in an autistic way. If they were bad at maths, they are going to reinforce dramatically their academic burden.

Interviewer: But as soon as they get some proficiency in the Catalan language they could be in the mainstream classes, couldn't they?

Teacher: Well, I do not refuse to work with non-Catalan-speakers, but even if they are intermediate learners of Catalan, I claim that the mathematics they are going to learn while being taught in Catalan is going to confuse them. I wish I could speak all the languages represented in my class!

Interviewer: What do you mean by confusing them?

Teacher: For instance, a Muslim student knows how to do a subtraction, but then you use an algorithm for subtraction on the board different from the one he has used, and you want to explain to him that it is just a different way to do the same thing. However, he cannot properly understand your words and then he starts to think he is wrong doing the subtraction his way...

Interviewer: Maybe other students can help...

Teacher: The Muslim student will hide his subtraction from then on...

First of all, we can see how well-qualified mathematics teachers project a resistance to having minority language students in their classes. On the one hand, the teacher feels she cannot know about the thinking processes of their students because of the linguistic distance. Nevertheless, she is quite convinced that her students, who are aged 16, have developed higher order skills such as reasoning. However, she believes that she will not have any possibility of gaining direct evidence of them until the students have full proficiency in Catalan. On the other hand, she seems respectful of the cultural diversity in her classes, and acknowledges different mathematical practices linked to other school cultures. She is familiar with other forms of algorithmic representation, and seems sensitive to the richness that may arise out of the different norms of mathematical practice. According to her colleagues in the school "she can put herself in her students' place ". Moreover, she acknowledges the fact that minority language students may develop low self concepts due to the difficulties they have with communication.

The teacher says that trying to use a simpler language to communicate mathematical ideas to her students has proved to be of little help. She cannot find resources beyond verbal language to explain, for instance, the validity of both subtraction algorithms. She argues, as do many other teachers, that it is better for the minority language learners not to attend mathematics lessons until they master the Catalan language. She stresses the fact that sharing a common language in class is crucial for teaching in multilingual settings.

However, there is no acknowledgement of the maintenance of the cultural conflict beyond the acquisition of common words. Many of the teachers we have interviewed expressed the view that once a common language is reached there is no need to worry about cultural conflicts: "as soon as they learn our language, there are no more significant differences ... may be, in the social sciences, but certainly not in the mathematics classroom" (from a teacher interview).

We have also documented several cases where teachers believe that their students have a greater capacity for understanding their oral messages

than they actually have. This seems to be especially relevant when the immigrant students come from countries, other than Spain, where Spanish is also the official language. For example, there was a Dominican student, Joel, for whom, in most mathematics lessons we recorded, there occurred a moment where he seemed, all of a sudden, astonished, and stopped attending to what was being said in class. The following is from his interview:

Interviewer:	What about math? Is it difficult for you?
Joel:	Yes, but it is not my fault!
Interviewer:	What do you mean?
Joel:	I am Caribbean!
Interviewer:	And. . . what do you mean by that?
Joel:	I am not Catalan, and I am not Spanish. When teachers talk to me they think I am Spanish because I speak similar to them. But you see, I am completely negro!
Interviewer:	Don't you speak Spanish in Santo Domingo?
Joel:	Yes, we do, but it is a different Spanish, we only have the same words, that's all!
Interviewer:	So. . . what happens in the math class?
Joel:	The teacher, she is very nice. . . when she looks at me she stops speaking Catalan and starts speaking Spanish. Sometimes she says very strange things and she doesn't realise at all. . . believe me! I just cannot concentrate!

Throughout the interview, and also during the observations in class, we can identify particular emotional moments when the linguistic becomes overwhelmingly important. Joel assumed he was failing in mathematics because of his origin. As a consequence, he regards himself not as the one responsible for his learning but as a victim. He attributes his failure to the lack of a common language and uses linguistic difficulties to excuse himself for not understanding. During a mathematics lesson, Joel asked his teacher for help, saying that he could not understand the activity because "I have only a few hours of Catalan per week". The point here is that the teacher was explaining the problem not in Catalan but in Spanish!

Teaching and learning mathematics in the absence of a common language

In this section we present results about three different aspects concerning teaching and learning mathematics where there is no mastery of the language of instruction.

Not knowing everyday language interfering with work on mathematical activities

When involved in a mathematical discourse, many minority language students experience difficulties in understanding some everyday words in the statement of the mathematical activities. Mostly we observed that they do not ask immediately, and sometimes they never ask, for an explanation or a translation, because they think they are getting the general message.

The following example shows a student's misunderstanding of a word in the statement of a problem preventing the student following the thinking process and the global discussion of the solution. The students were working on the following rephrasing of a well known problem:

A farmer has 3 sons. In his will he gives his sons 17 cows. The oldest one must receive 1/2 of the cows, the second 1/3 and the third 1/9. How many cows will each of them receive?

When introducing the problem, Aftab, a Pakistani student asked the teacher what the word 'will' meant. However, Aftab seemed not to understand the explanation, and nobody in the classroom was able to translate the word into Panjabi, Aftab's main language. Seeing that he had not understood, the teacher, searching for transparency, finally said: "Okay, Aftab, it is a present the father gives to his children". Later on, during the discussion of the possible problem-solving processes we recorded the following dialogue:

Jossua:	Is the father there at the moment?
Teacher:	Why do you need to know that?
Jossua:	(working on the idea of a 'present') If the father is still alive, then he will need some cows or maybe he can buy more cows. . .
Ramia:	(working on the idea of a 'will') The father is dead!
Aftab:	(Shouting angrily) Why do you want to kill him?! What has he done to you?!

Aftab went on to say that he did not agree with killing the father, and if the class decided to do so, he would not come back to the mathematics lessons. Aftab was so disturbed by the context of the reasoning that he was not able to pay attention to the problem-solving process. The search for transparency turned into a misunderstanding, and a single word was enough to obstruct his process of thinking.

Teachers finding difficulty understanding students thinking processes
The following example is one of the many that illustrate the difficulties teachers can encounter as they try to understand their students' thinking processes in the absence of a full communication. The teacher asked their students to solve the following problem:

> Two friends, Maria and Mohammed, go down the stairs by jumping. Both of them begin with the left foot. Maria jumps 4 steps each time. Mohammed jumps 3 steps each time. How many steps do the stairs have to have at least, if there is a step on which Maria and Mohammed have to land with the same foot? Will the foot be the rigth foot or the left foot?

The teacher's intention was to lead the students to use a visual thinking process, and to help those experiencing communication problems to illustrate their solution with a diagram. The students were working in small, linguistically homogeneous groups. The group whose main language was Urdu did not succeed in solving the problem, the students looked very worried and, finally, told the teacher that the problem was not solvable. On their sheet of paper, there was a diagram with an attempt to solve the problem, but it was crossed out, and they where working with numbers, by trial and error. The teacher could not understand the obstacle for them. Their first attempt would have led them to the right solution, and in fact, they actually had the solution. When the students were asked what was the matter with the diagram, they only answered "it is impossible". The teacher's interpretation was that they did not accept a solution reached by a visual method, that they had used it only to have an idea of the problem statement. When the solving process was shared on the blackboard, they kept on refusing to accept it. The following dialogue shows how, finally, the teacher came to understand their refusal:

Teacher:	Why don't you like the solution?
Sheraz:	It must be something wrong. It is impossible!
Sheraz:	It has too many stairs, and that is not a staircase.
Teacher:	What do you mean by that?
Sheraz:	You cannot carry stairs so long!!!!

At that point, the teacher realised that they were thinking of a stepladder, which in Catalan is 'escala' as it is stairs. In their main language there are different words for stepladder and for stairs, and they only knew the translation for the first one. Since all the problems they had solved until then had meaning in the real world, their visual solution was in contradiction with common sense and the problem had no solution.

Students experiencing difficulties with the meaning of mathematical words or symbols

Immigrant students belonging to cultures with other alphabets and codes often experience difficulty in adapting to new ones. Murshed, a 16 year old Bangladeshi boy seemed to have difficulties while adding and subtracting. When interviewed by his teacher he said "To add here, I do not know, to add in Bangla (his main language) I know". In particular, Murshed had difficulties in interiorising the symbols for the numbers. On the interview he said: "My six is your seven". When he showed his teacher the mathematics book he brought from Bangladesh, she could see that, in fact, the symbol for the digit 6 in Bangla is very similar to the way we write 7.

Another example concerned Sergev, a Bosnian boy aged 15 who spoke neither Catalan nor Spanish, but understood a little English. Sergev attended school regularly, seemed to enjoy doing mathematics, and became actively engaged in problem-solving tasks. In one of the mathematics lessons some students, in attempting to solve a problem, used an equation of second degree. A volunteer wrote down the problem's solution on the blackboard, writing only the positive solution, '3', without any sign, explaining that the negative solution had no sense in that particular context. At that moment, the bell rang. Sergev remained in his seat. When the teacher asked whether he wanted to go to the playground, Sergev said nothing, but went to the board and pointed at the number 3:

Teacher:	Have you understood the problem?
Sergev:	In my school this number (pointing at the 3 written on the board) is always positive. Here this number is positive and negative
Teacher:	What do you mean, Sergev?
Sergev:	The number with no sign is two numbers, the positive and the negative. The number with sign plus is positive, and the number with sign minus is negative. I understand here the numbers. In my school is different. . .
Teacher:	What do you mean?
Sergev:	Here the mathematics are different, you write different, but it is easy to understand . . .

Sergev was trying to adapt what he knew about mathematical symbols to understand why there was only one number written as a solution. He had not understood the explanation why only the positive solution had been written, nor did he ask for it. Since he knew there should be two solutions, he reinterpreted the number 3, with no sign accompanying it, as a way of representing both the positive and negative solutions.

In search of a strategy: Monolingual groups working on a problem solving

There seems to be an agreement about the influence of grouping methods on the learner's performance at school. The specific characteristics of each class community make the grouping decisions particular to those characteristics. In our case, the variety of languages and the lack of proficiency of many students in the language of instruction were regarded as crucial aspects to consider in grouping decisions. One of the teaching strategies that we have implemented to try to throw light on how to improve the learning processes of minority language students has been to get students to work in monolingual groups on problem-solving tasks. One of the secondary teachers within the project experimented this approach.

Among the students speaking a particular minority language, there was usually one with a better proficiency in Catalan. Therefore, we had two reasons for creating small groups that were linguistically homogeneous: to have groups where the students could use their main language in their discussions; and to guarantee that in each group there was one student who could represent the group's point of view in the official language in class discussions.

At first, not all the students welcomed this criterion for creating groups. However, during the sessions the students began to realise that this approach facilitated communication. Initially, it was not possible to allow for mixed-ability grouping in an explicit way, since it was difficult to combine the two aspects – language and ability – with the same emphasis at the same time. However, in fact, it turned out to be the case that the groups were mixed-ability groups.

When deciding upon the teaching strategies that we would implement in the class, we assumed that working within a problem-solving 'ambiance', where the activities were situated in contexts near the students' interests and realities, could help minority language learners to communicate and to engage in collaborative work. By posing problems and promoting group discussion, we were able to observe some language issues concerning learning processes.

First, in 4 of the 5 groups, the student with a better proficiency in Catalan developed the role of translator, even before the teacher suggested the idea. One such student explained that he concentrated especially hard during the mathematics lessons because he felt responsible for the other members of his group. In fact, that did not necessarily mean that he concentrated on the mathematical content or arguments – rather he seemed to pay attention to content vocabulary and to differences in mathematical conventions.

The 'translator' students in the different groups had some common features. Because of their role, they often had low participation in the mathematical discussions, reducing their classroom participation to communicational mediator. They seemed to assume that there were no further mathematical expectations on them. Translators liked to have a supportive role to their cultural group and felt quite important for doing so, "I told the Imam at the Mosque, and he felt very proud of me!". However, they tended to confuse classroom participation with mathematical participation, "I work a lot in the math class, I have to translate a lot".

Our observations made it clear that the learners who benefited from the translation generally liked being in linguistically homogeneous groups. The minority language students that we observed remained silent in class discussion, but participated in the small group discussion and, through the help of the translator, could partially follow the whole group discussion. Most of minority language students developed a positive attitude towards the possibility of using their main language during mathematics lessons. In general, minority language students did not become over-dependent on their 'translators', but relied on them only for getting the meanings, but not the answers or the solutions for the problems.

The problem solving 'ambiance' demonstrated that language mediated the sharing of the norms that regulated both the dynamics of the mathematics classroom and mathematical practice. We have observed elsewhere (Gorgorió et al., 2001) that cultural backgrounds and personal histories of students greatly influence the different interpretations of the norms. Since the teacher was aware that most of the students were not used to classroom environments aimed at facilitating problem-solving, she insisted on making norms as explicit as possible. We observed that, in some cases, there was no real sharing of the norms, despite her explanations. It was not a matter of misunderstanding the words but the meanings of those words in that context.

For example, a Pakistani boy, Imram, with good academic marks, remained silent and passive after a problem had been posed at the beginning of one of the sessions, though this behaviour was unusual for him. He had an intermediate proficiency in the Catalan language and, in fact, was the one who usually translated the teacher's instructions to the Pakistani students. His behaviour did not seem to the teacher to be a question of a lack of understanding. In fact, however, Imran was one of many students in the class who did not understand the non-directive role adopted by the teacher. Despite the fact that the teacher had given several explanations so that the learners would understand the teacher's role, Imran felt nervous because

the mathematics lesson *'never began'*. He was expecting the teacher to write down the solving process on the blackboard.

OVERVIEW AND SOME RECOMMENDATIONS

The project, as a whole, highlights the integrated nature of the social, cultural and linguistic aspects of mathematics teaching and learning. It is very difficult to disentangle the social and cultural conflicts of a multiethnic mathematics classroom from the language issues. Considering language as a tool to share ideas, mathematical or not, can help us to understand cultural and social conflicts. Our research shows that the conflicts experienced by language minority learners are not always due strictly to a language obstacle, understood on a restrictive way, but to a wider communication obstacle. Behind most of social and cultural issues in the mathematics classroom are the meanings people attach to a particular situation within the context of its social dynamics. Since words are connected with meanings, being aware of the misunderstandings that can appear in a multilingual situation can help teachers and researchers to understand both social and cultural conflicts.

After three years work, we claim that the discontinuities associated with the forms of communication and with the cultural conflicts are neither positive nor negative. What is important is the approach to and treatment of them developed by the teacher and the 'adaptative strategies' (Bourguignon, 1979) that minority students develop. Often teachers associate the discontinuities in communication with the linguistic transition process only and, in search for transparency, tend to use a simplified language even though they regard it of little use. Our data shows that transparency is not always a good strategy for dealing with the difficulties minority language students face during their mathematics lessons. More is required than a simple change in the vocabulary, or 'a patient attitude'.

Our research also highlights the communicative nature of mathematics teaching and learning. In the mathematics class, the linguistic barrier goes further than barriers arising in simple everyday communications: teachers feel that the absence of a common language amongst students is a barrier for their mathematics learning. However, it is difficult for teachers to find resources beyond verbal language. They stress the fact that sharing a common language in class is crucial for teaching in multilingual settings, while ignoring alternative possibilities such as using visual language.

Multilingual situations raise the question of the language base of mathematical thinking. Most mathematics learners practise symbol manipulation without reference to the meaning of the symbols (Resnick, 1987).

However, there are additional obstacles for minority language students, since they have to construct meanings for the symbols on the basis of a language of instruction that they do not fully understand. A single word can be enough to obstruct a student's process of thinking.

In the search for a successful teaching strategy, working on problem-solving in small monolingual groups was tried. It proved to be useful, both from the point of view of the research where it allowed us a closer approach to language difficulties for the purposes of our study, and for the students. Linguistically homogenous small groups has shown us that some conflicts and linguistic discontinuities can be thereby either minimised or avoided. Problem-posing activities were a powerful approach for encouraging students' mathematical thinking, and a useful tool for identifying learners' language difficulties. They allowed us to realise the importance of language as a factor mediating the sharing of the norms.

Since we believe that many of the linguistic conflicts that we have observed can be either avoided or minimised, we consider that teachers play a crucial role. An important result of the present study is that teachers feel that students' lack of communicative abilities is the real problem. Many teachers consider that once they can communicate with their students, in a social sense, there will be no further problems, because mathematics is 'a universal language'. We want to expose the negative implications of the widely accepted myth that associates the extent of language acquisition with academic achievement and academic potential. Minority language learners are often supposed to be handicapped academically. However, the insights we got through developing our project, lead us to note that the greatest effect language may have on the academic achievement has essentially to do with "the way in which teachers and schools view students' language" (Nieto, 1999, p. 195). We support an approach that assumes that the students' languages and different cultural backgrounds can be seen as a resource in their learning.

We finish the paper with a consideration of how our position towards language has changed since the beginning of the project. Using the metaphor of 'signal to noise ratio' as in electronics, initially the social and cultural aspects were the 'signals', i.e. the focus of our attention, and we regarded language as a 'noise', because it was disturbing the research procedures. After three years work, however, language has become, for us, the 'signal', the focus of our attention.

Language issues are to be considered as crucial components in the process of constructing mathematical knowledge within the classroom. However, it could be argued that this kind of statement applies to any teaching and learning situation. The main point is then: what is particularly

interesting about the possible relationships between language factors and the social construction of *mathematical* knowledge within the classroom?

The connections between language and mathematics reinforce the idea of mathematics being a social and cultural product and its teaching being a social and cultural project. If language is the main carrier of a culture, then the 'language of the mathematics class' conveys the culture of the classroom as a social group doing mathematics, with their norms and its legitimated roles. The 'language of the mathematics being taught' in the classroom, being mathematical language, aims to introduce the students into the culture of mathematics – or, at least, of school mathematics.

The analysis of language issues within the mathematics classroom illustrates that, even if the mathematical language can be considered universal, i.e. shared by all those doing mathematics, then the language of 'doing mathematics within the classroom' is far from being universal. The language of the exploratory talk is certainly not universal: we observed that most of the time students were using their main language. The discourse-specific mathematical talk, the mathematical talk and writing taking place in the language of instruction, was often understood and used in a non-univocal way.

It appears clear to us that during the process of constructing, developing or expanding a particular concept or procedure the mathematical language associated with it is not necessarily shared, or understood in an agreed way. This fact has implications of great importance in our particular context, where mathematics, as a school subject, is still mostly considered to be "the best subject to facilitate the rapid incorporation of latest arrival immigrant students within the school culture and into the classroom environment".

Teaching mathematics should aim at facilitating the students' move from exploratory talk to discourse-specific talk, and the knowledge of how this move takes place is still incomplete. Communication always has some breaks in it. However, when lacking the basis for a 'minimum' communication within the classroom the teacher cannot guarantee that discourse-specific mathematical talk takes place and promoting it is a hard task when the exploratory talk is already a 'broken' talk. More research is needed to help to clarify how mathematical language can be taught and to investigate the relationships between the 'language of the mathematics class', mathematical language, and the process of constructing mathematical knowledge.

In our research, we could not find any clear evidence of a fluent mathematical 'conversation' when working with a whole class. We have no doubt that the students observed were able to produce, for instance, a chain

of statements for logical deduction. However, either we could not 'hear' it, because it was taking place within the small group interactions in a language strange to us, or the students had difficulty showing it using the language of instruction, partially alien to them.

From that point of view, language was a 'noise', not only to the study, but also to the students' learning process. We understand that this fact was caused not by working in a multilingual situation but by extreme characteristics of this situation. Nevertheless we are convinced of the importance of continuing to study the teaching and learning of mathematics in multiethnic and multilingual situations. To continue it we need to work in multilingual contexts with less extreme characteristics and should have the help of language experts.

The need for continuing the study can be justified not only in terms of equity and social justice, but also in terms of the richness of the research ground. In monolingual mathematics classrooms the borderline between the exploratory talk and the discourse-specific mathematical talk is often vague. However, in a multilingual situation the code switching between common talk about mathematics, exploratory talk, and the mathematical writing and talking is often associated to the switching between the students' own language and the language of the instruction. Therefore it is more obvious, even if more complex. Moreover, there are other reasons, as Adler (2000, p. 20) suggests:

> I believe that the insights into mathematical learning in contexts where communication cannot be taken for granted has much to offer. In particular, I don't believe there is any classroom where linguistic capital is equitably distributed. In consequence, what might appear as extremely 'different' settings that do not speak to mainstream practice, in fact are illuminating of communication issues in all classrooms.

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