GABRIELE KAISER, MAIKE VOLLSTEDT

University of Hamburg

Faculty of Education

Von-Melle-Park 8

20146 Hamburg

Germany

gabriele.kaiser@uni-hamburg.de

Teachers' views on effective mathematics teaching – Commentaries from a European perspective

The previous papers of this issue discuss the views teachers from the United States, Australia, Hong Kong SAR, and Mainland China have on effective mathematics teaching and learning. Similarities and differences are found and a differentiation from West to East can be worked out in the order of the regions as listed above. The picture of teachers' views can, however, be differentiated when they are looked at from a European perspective. On the basis of the analysis of two comparative studies on teaching cultures in three European countries and a questioning of teachers, this commentary therefore tries to locate France, Germany, and England within this framework so that the East-West-contrast is distinguished in more detail.

1 Goals and intentions of the article

The following commentary on teachers' views on effective mathematics teaching comes from a European perspective and is based on two European comparative studies which include Germany, France and England, as well as a questioning of teachers in Germany. The studies are qualitatively oriented so that they do not claim representativeness. Also, they do not focus on the question of an effective mathematics teaching but on beliefs of mathematics and of mathematics teaching in general. They do, however, make clear that the contrast between East Asian and Western countries concerning effective mathematics teaching can be differentiated from a European perspective.

Therefore, to begin with, the results of a study on the mathematical beliefs of German teachers are presented. In contrast to the East Asian, Australian, and American teachers' views, in Germany, formalistic as well as schematic beliefs concerning mathematics as a discipline as well as concerning its teaching and learning clearly dominate compared to beliefs which are focussed on process or application. The two European comparative studies show that also within Europe, focussed on France – Germany – England, clear differentiations in the reconstructed practice of teaching and learning mathematics can be found. These shed light on the underlying beliefs about mathematics and mathematics teaching. Formal aspects of teaching mathematics dominate in France whereas, in contrast, pragmatic and applied beliefs are the main important ones in England. Germany can be located between these two poles. This makes it clear that among these Western countries strong differentiations have to be made as there are countries which are closer to the East Asian views of teaching and learning mathematics than others.

2 Reflections based on a study with German teachers

The following reflections describing the beliefs of German teachers concerning mathematics and mathematics teaching are based on a qualitative study carried out in 2002-2003 with 41 teachers from six schools in Hamburg, the second largest city in Germany (for details see Kaiser, Kornella & Ross 2005). The study was conducted in connection with the evaluation of a pilot programme of the German government and the federal states which was aimed at increasing the efficiency of mathematical and scientific teaching (so-called SINUS-programme). This innovative programme, carried out between 1998 and 2003, aimed at fundamental changes in mathematics teaching in two directions: (1) a change in tasks as practised in lessons, and (2) a change of the dominant learning and teaching structures (a stronger integration of applications and modelling examples).

2.1 Framework of the study and methodological aspects

The theoretical approach of this study refers to the discussion about beliefs as mental constructs that represent the codification of people’s experiences and understandings (Schoenfeld 1998). The teachers’ beliefs can be distinguished amongst others by whether they refer to the nature of mathematics as discipline or to mathematics teaching and learning (Thompson 1992). The study starts from the classification system of mathematical beliefs of students developed by Grigutsch (1996), which is elaborated by Grigutsch, Raatz, and Törner (1998) concerning teachers. Grigutsch, Raatz, and Törner (1998) categorise teachers’ beliefs mainly by four aspects of mathematical belief system which refer to the nature of mathematics as a discipline: From a dynamic perspective, mathematics can be understood as a science which mainly consists of problem solving processes (aspect of process), or as a science which is relevant for society and life (aspect of application). In contrast to these two dynamic views, there are two static perspectives on mathematics as a science. One comprises mathematics as an exact, formal, and logical science (aspect of formalism), the other views mathematics as a collection of rules and formulae (aspect of scheme). This theoretical approach differs from the one used by the study presented in the other papers of this issue, which refers to Ernest’s (1989) approach towards teacher’s beliefs. Clear similarities between the two approaches can be found, though. Ernest differentiates an instrumental view on mathematics, which understands mathematics as an accumulation of facts and rules. This instrumental view is related to the schematic perspective in Grigutsch, Raatz, and Törner (1998). On the other hand, Ernest’s category of Platonic view of mathematics, which focuses on the internal structure of knowledge itself, is related to the formalistic perspective. The additional beliefs of mathematics as process or application presented here enrich Ernest’s theoretical approach and enable more differentiated analyses.

The differentiation of dynamic and static beliefs is pursued in the beliefs about the teaching and learning of mathematics. With relation to the psychology of learning, these beliefs are used in the study by Kaiser, Kornella, and Ross (2005) to differentiate between beliefs about a static and dynamic acquisition of knowledge. A static acquisition of knowledge is characterised by the perception of learning as acquisition of an inventory of knowledge, or a perception of receptive or reproductive learning respectively. Beliefs about dynamic forms of learning understand learning being as self-directed and discovering. On the other hand, the study described in the other articles of this issue refers to the approach by Ernest (1989), who differentiates the beliefs about the static perception of learning in mastery of skills and reception of knowledge. Dynamic ways of learning are differentiated in active construction of understanding and exploration and autonomous pursuit of own interest models.

To describe the perception of the teacher's role, the study described below reverts to a distinction between the traditional and extended role of the teacher: characteristics of a traditional perception of the role of the teacher are beliefs about the teacher being an expert who presents and imparts contents by recurring and thorough explanations. The extended role of the teacher sees the teacher as being responsible for the preparation and the organisation of stimulating learning situations. It also places emphasis on the communication with the learners. This distinction is a bit coarser than Ernest's (1989) classification of instructor, explainer, and facilitator used in the other papers of this issue. The roles of instructor and explainer hereby correspond to the traditional perceptions in the approach taken in this study; the role of facilitator corresponds to the other perceptions.

To conclude, the given theoretical approaches are very similar, they just differentiate some aspects to a different extent.

Methodologically, the study applied methods from qualitative social science. Furthermore, the applied empirical methods concerning the choice of sample, data analysis and data interpretation are based on the theoretical considerations of Grounded Theory (Strauss, Corbin 1998). The decision for a qualitative study was made on the one hand due to the small sample size which was available. On the other hand, this approach seemed to be especially appropriate as Grigutsch, Raatz, and Törner (1998) provide a study which worked with a quantitative, statistical design. The aim of this study, however, was to answer the question whether the beliefs described above can be reconstructed individually with every teacher or whether they occur as conglomerate. We will go into more detail below.

In this study, all teachers of the six participating schools who were involved in teaching mathematics to students from Year 7 and 8 were asked about their beliefs concerning mathematics as a discipline and the teaching and learning of mathematics. This was done twice: at the beginning of the project and after one and a half years. The questionnaires used consisted of both, open and closed items. The following aspects were covered in the questions:

beliefs about mathematics as a discipline,

beliefs about the nature of mathematics teaching and the underlying goals of mathematics teaching, and

beliefs about the learning of mathematics.

Altogether 41 teachers participated at the beginning but decreased to 29 at the second questioning. With 16 teachers who were chosen for certain theoretical criteria (see below), semi-structured interviews were carried out to enrich the data on the mathematical belief systems of the teachers. Eight teachers were interviewed at the beginning; another eight teachers were interviewed at the end of the study. Four interviews from the first interview session and four from the second interview session were then analysed in detail using methods from Grounded Theory.

To be precise about the procedure and the methods used, the written questioning was conducted with questionnaires which contained open as well as multiple-choice questions. For the evaluation, the theoretical approaches described above were taken as classification system. This means that a coding system was used which contained in-vivo codes as well as codes which were gained theoretically. With this coding system, the questionnaires were analysed, and every teacher was categorised in one of the approaches for every of the three perspectives. As this was often not distinct, primary and secondary beliefs were distinguished.

The choice of interviewed teachers was based on the results of the written questioning. This means that the sample was chosen according to the research interest so that teachers who convincingly represented aspects of formalism, schema, process, and application among the beliefs about mathematics as a discipline, as well as static and dynamic perceptions about the forms of teaching and learning were taken for the interviews.

Interviews were half-standardised and were conducted on the basis of an interview guideline. The use of half-standardised interviews for the reconstruction of beliefs is judged as especially appropriate in the literature (among others see Flick 1995).

The interviews were taped and transcribed. Afterwards, they were analysed according to the method of thematic coding. This means that, at first, the cases were interpreted as analyses of individual cases and, then, examined in contrastive comparison. The coding system used was based on in-vivo codes as well as codes which were gained theoretically. These codes were combined into categories is the course of the analyses.

2.2 Results of the study

The analysis of the written questionnaire from the beginning of the study shows a clear dominance of static beliefs about the nature of mathematics, i.e. for most teachers mathematics meant exact mathematical thinking and exact ways of working as described in the formalism-oriented approach. Likewise, beliefs about the nature of mathematics teaching can be classified as static with both, formalistic and schematic perceptions dominating. Beliefs concerning the objectives of mathematics teaching are schematic, dominated by the teaching of rules and formulae. While beliefs concerning the teaching of mathematics are dominated by static aspects, the dynamic aspects prevail with beliefs concerning the learning of mathematics. This is a surprising result. It makes clear that teachers notice that learning is an active process in which the learners independently have to build cognitive structures. On the other hand, in contrast, they persist in their traditional view about the role of the teacher in which the teacher directs and presents the contents via recurring and diligent explanations. Other views about an extended role of the teacher in which the teacher takes the responsibility to prepare suitable learning situations in which the learners can work constructively and creatively are hardly found.

These results form a certain contrast to the results of the study by Grigutsch, Raatz, and Törner (1998), which states a stronger emphasis on the aspects of application and process in contrast to the schematic and formalistic aspects concerning the perception of mathematics as discipline. They especially state that the aspect of schema is strongly rejected especially by teachers teaching at a *Gymnasium*, i.e. the school type in the tripartite German school system for the highest achieving students. Restrictively it has to be stated that the study was conducted with 310 teachers during a conference on mathematics education. As there are no other studies concerning beliefs of German teachers, there are no consistent quantitative results.

In this context, it is important to know more explicitly the perceptions of teachers who represent a tendency towards schematic, formalistic, process-oriented, or application-oriented beliefs about mathematics as discipline, as well as their perceptions about teaching and learning processes in mathematics. It is possible to give some statements from the interviews done with teachers in the study by Kaiser, Kornella, and Ross (2005), which are presented below. In each case, we concentrate on one teacher respectively and describe their perception in more detail:

A *formalistic* teacher – Mr. A – describes his view about mathematics as follows: Mathematics is at first a 'formal language': In contrast to colloquial language, it is 'not redundant', 'precise', and 'logical/consistent'. According to this teacher's opinion there is only a weak relationship between mathematics and everyday life: 'For me mathematics is … not always, sometimes yes, … has also a relation to life.'

For this teacher, learning means a 'learning sequence' in which the contents are sequenced according to their mathematical content. They are based on each other and have to be learnt one by one before transfer can be made. The teacher thereby is, in his view, primarily a lecturer, who conveys knowledge to the learning group: 'The amount of subject matter is so big that one really has to work high-power and there is no chance of doing anything else additionally.' Typical metaphors for teaching are therefore: 'to implement', 'to push something through', 'to introduce', or 'to work high-power'. That this teacher's concept of the teacher's role focuses on the teacher is also reflected in his metaphors for learning: 'to comprehend', 'to grasp', 'to follow a work instruction'. Active and developing metaphors about learning like 'to be able to play with concepts' are expressions which this teacher seldom uses. If used, they chronologically come after comprehension.

From a methodical point of view, problems have a high significance in the lessons for Mr. A. Learners are supposed to learn the 'know-how' of the mathematical language with them. This means that the careful reading of work instructions and the training of clean and precise presentations are trained. In this context, Mr. A is concerned about 'technical practice in the sense of consolidation of handicraft'. It becomes obvious that thorough practice of mathematical skills is a central goal of his teaching so that it inheres in great significance and meaning. But not only rules and theorems are in his focus. Mr. A also points out structural mathematical relations which the students are supposed to gain an eye for by practicing.

The position of the teacher who is classified as *schematic* – Mr. B – shows some similarities to a formalistic view of mathematics: For him mathematics is 'the logical sequence of formulae' so that it is reduced to the accumulation of rules and formulae. Non-mathematical applications do not form a constitutive part of mathematics. In mathematics lessons, students learn 'the basic conditions of mathematics', 'and everything else comes from the other subjects, there one continues to calculate.'

According to the view of this teacher, the teacher has the main responsibility for learning. Learning works in the way that learners 'get something tangible, something fixed' which has to be comprehended. The implementation of learnt calculation methods is more important than to reason oneself. Against this background, 'basics just like that' have to be 'memorised'. The teacher therefore sees himself as 'crammer' who trains basics and who functions as controller for systematic working.

Tasks are of very high significance in this perception of mathematics teaching. They primarily serve for practice of core mathematical contents. Hereby, 'the mere function of practice' is important, which means that 'they [the students] practice the same again and again with little variation'. In doing so, the relation to daily life is important, the tasks, however, do not have to be necessarily realistic.

For Mr. C, the teacher with *process-oriented* beliefs towards mathematics, mathematics is understood as an intellectual exposition of problems. Thinking mathematically means for him 'to develop logic', 'to develop stringency', or 'to proceed logically and coherently'. This goes along with the fact that only a weak relation is seen between mathematical subject knowledge and the real world. In the interview, the teacher explains that mathematics might even be replaced by playing chess because mathematics is aimed at developing thinking abilities. So the teacher describes basic mathematical skills as being useful for real life but, the more complex mathematics becomes from the subject's systematics, the more it loses this link.

This dynamic process-oriented view towards mathematics goes hand in hand with a strong orientation to students' learning. The talk-and-chalk teaching approach is therefore not appropriate. On the contrary, to let the students deal with mathematics individually, which also implies that the teacher holds himself back, is more meaningful to him. Therefore, he says: 'Supposing I stiffly stick to the textbook, then it is hardly the case that a student on his own departs to, departs to somehow independently ask questions or bring independent thoughts into play because basically they, then, follow the structure'. The aim of such a learning process is that the teacher becomes 'superfluous'. It is hereby important that the teacher does not continuously comment on the thinking processes of the students but that they learn to write down their own thoughts during the investigation of mathematical problems. The teacher views himself as operator of problems in the lessons who 'sometimes also does not see the solution' due to his openness towards the approaches of his students.

The tasks preferred by this teacher are usually open questions which serve 'to discover mathematical structures'. Questions which practice mathematical methods are not rejected, but neither do they play a decisive role.

Mrs. D, the teacher for whom the aspect of *application* plays a central role, makes clear in the interviews that the aspect of application has a fundamental meaning for her: 'What shall I do with mathematics, if I cannot apply it somehow to my life?' However, not its profits but the training of 'critical questioning' is as important as the training of thinking abilities for her. The performance of exercises, which does only serve to stabilise basic mathematical skills, is seen as only meaningful to a limited extent. The teacher stresses the experimental character of mathematics and that the goal of teaching mathematics must be 'to get the hand of the seeming objectivity of mathematics'.

This dynamic view of mathematics means that the teaching and learning processes of mathematics do not consist of memorising rules. Mechanical learning like 'calculating lots and lots of … questions' or 'to put into formulae' has for her only limited significance. For this teacher, learning is primarily an actively discovering process. The students must 'make the things accessible on their own'. The goal of her teaching is 'to take the responsibility for one's own learning'. She is aware of the problems this approach causes when it is realised in the lessons. Altogether, the teacher sees herself as ‘manager of the learning’ who organises the learning process along ‘meaningful contexts’.

Overall it became clear that for most of the teachers who represent the formal or schematic view of mathematics static views of the learning processes are emphasised. According to the views of these teachers, instruction is supposed to be geared to the instructor- and explainer-model, i.e. the subject matter is presented by the teacher and the quality of the lessons depends on the teacher’s ability to explain. Beliefs about the role of the teacher which come close to the role of a facilitator in the meaning of Ernest (1989), or beliefs in which the teacher totally holds himself back and sees him- or herself as organiser of the learning processes are rather exceptional.

It is interesting to see that process-oriented dynamic beliefs about the learning of mathematics, which stress the responsibility of the individual student for his or her learning process, and the static belief of the teacher’s role, which is rather oriented towards a traditional view, diverge but, still, occur together. The useful side of mathematics is, however, seen in the belief about mathematics as discipline, as well as in the beliefs about the teaching and learning of mathematics. They do, however, not play a decisive role for the teachers participating in this study.

3 Reflections based on international comparative studies on teaching processes

In the following section, results of international comparative studies concerning teaching research are to be presented. These results give insight into the actual practice of teachers in the European context from which conclusions can be drawn about the beliefs represented by these teachers. These studies, however, did not contain methodically controlled questionings of teachers but, on the other hand, methodically controlled observations of lessons and teachers respectively. Therefore, these studies are restricted to the question which picture of mathematics and the teaching of mathematics can be reconstructed from the teacher’s usual (inter)action in the classroom. In the following, it is assumed that there is a close relationship between the respective teaching culture of a country and the beliefs of the teachers teaching there (so-called ‘filter function of beliefs’ see Pehkonen 1994).

3.1 Results from a meta-study

Altogether it can be stated that the studies which compare the teaching culture in Europe show that the diversity of beliefs about mathematics and mathematics teaching can also be found at European level, although with a different bias. On a meta-level, the study by Kaiser, Hino, and Knipping (2006) analyses the teaching culture in three European countries, namely France, Germany, and England (Japan, which is also analysed in this study, is left aside here due to the focus of this paper). The study is based on several bi-national comparative studies which carried out extensive classroom observations. The first study on mathematics teaching in England and Germany is an ethnographic study embedded in a qualitatively oriented paradigm of the social sciences, which used the method of participant observation. Several hundred lessons were observed in both countries. The study refers to the approach of the ‘ideal typus’ developed by Weber (1904), and describes idealised types of mathematics teaching reconstructed from the classroom observations in both countries. This means that typical aspects of mathematics teaching are reconstructed on the basis of the whole qualitative study rather than on one existing empirical case (for details see Kaiser 1999, 2002). The second study compares mathematics education in Germany and France and is based on processes of argumentation and proof. The observations in six French and six German classes were documented with audio-tape recordings and photos from the blackboard writing and drawing. Analyses were carried out based on theoretical reflections and involved functional analysis of argumentations. Based on the construction of prototypes, ideal-type characterisation of proving processes were developed (for details see Knipping 2001, 2003).

The meta-study based on these two studies shows that there are vitally different directions of mathematics teaching in the European teaching traditions. Starting from a different significance and a different concept of mathematical theory, beliefs about the teaching of mathematics in France and England can be reconstructed as contrasting poles. Thus, French mathematics teaching can be seen as an ideal type characteristic of a scientific understanding of theory. This means that in France there is a strong focus on the subject structure of school mathematics, and that theoretical mathematical considerations are very important. Therefore, concepts, theorems, and formulae are explicitly used so that theory is made explicit in the lessons.

England, on the other hand, can be described as an ideal type of a pragmatic understanding of theory. Mathematical theory is therefore treated in a practical and purpose-oriented way. The differences in the pragmatic and scientific understanding of theory in England and France respectively can be seen on various levels such as the way new concepts are introduced, the meaning and importance of proof, and the significance of rules and exact mathematical language.

The situation in Germany is characterised by its focus on the subject structure of mathematics and on mathematical theory. The situation is, however, very different in the different school types of the tripartite school system. The *Gymnasium* shows a strong dominance of theoretical subject-related reflections. In the lower achieving school types, however, teachers often reduce theory to rules and algorithms so that theory as such often only occurs as comments by the teacher or remarks in the textbooks.

Concerning rules and working with examples as well as generalisations it can be stated that they play a different role in these European countries, which points out, that there are remarkable differences concerning the role of memorisation and the importance of exercising.

German mathematics teaching can be characterised by its rule-orientation, manifested by the exact execution of algorithms, especially arithmetic and algebraic ones. It could be observed that many teachers expect from the students that they are able to execute central algorithms with certainty by heart. However, significant differences in the tripartite school system have to be taken into account. Especially in the lower types of the secondary school, teachers expect that pupils know the central algorithms such as calculation of percentages or formulae for solving equations by heart. For many teachers this memorisation serves as substitute for a deeper understanding, which would, in the opinion of these teachers, ask too much of the weaker students. *Gymnasium* teachers expect a more competent usage of the formulae, although in practice even there, algebraic transformations are often reduced to calculations and practised with plenty of exercises. But, as Kaiser, Hino, and Knipping (2005) emphasise, besides this orientation towards execution of algorithms by heart, some teachers emphasise a content-related understanding of formulae and the ability to develop such formulae by themselves (as an example for such a type of teacher see teacher C in the description in section 2.2).

The situation in France is quite different: teaching of mathematics is characterised by its focus on exercises and the justification of solutions within the studied theoretical frame of mathematics. The exact and precise processing of algorithms is considered to be important but shall serve as basis for solving more complex problems. According to Kaiser, Hino, and Knipping (2005), teachers emphasise that sticking to exactly prescribed procedures and routines is regarded as being important but seem to be not always sufficient in order to solve complex problems. The teachers usually expect a high engagement of the students in exercises.

In English mathematics teaching, rules and standard algorithms are of minor importance, which goes along with the low importance of generalisations and general solving schemes. Many teachers work with example-bound explanations and do not emphasise the standard algorithms. So, in contrast to German teaching, the performance of manipulations and algorithms by heart and time-consuming exercises can rarely be observed. But in contrast to French teaching, only seldom do teachers emphasise general structures and theoretical reflections on a general level.

3.2 Results from a three-country-study

These results, which are gained from lesson observations, are also confirmed by a comparative study by Pepin (1997), in which she shadows twelve teachers (four teachers per country) for a few weeks in order to develop an understanding of their beliefs concerning teaching and learning, as well as their classroom practices. Pepin reports from this study that there are three lines of perception about the nature of mathematics: mathematics as a tool, as training the mind with its logic, as a criterion for selection. She describes that most teachers see mathematics as a tool. Some English teachers point to the skill side and describe its function for other subjects. The more transcendent nature of mathematics as training of the mind has a high priority particularly for French teachers, who feel that logic is the principal element of mathematics. German teachers, especially those teaching at a *Gymnasium*, also emphasise logic as the principal element of mathematics. In the two lower school types, however, theory is less important and often reduced to rules and algorithms. English teachers, on the other hand, mention the necessity to promote mathematical reasoning, which is in strong contrast to their teaching, in which reasoning is hardly asked for. Teachers seem to assume that logical skills will be learnt from activities such as investigational work, where reasoning is necessary.

In another study, Pepin (1999) describes three dimensions that underpin teachers' practice and which show the great diversity within mathematics teaching and the conceptions about it in Europe. These dimensions are the coherence of mathematical concepts, the teacher's orientation on process vs. product, and the coherence of the students' mathematical experience.

Pepin describes that in France, teachers (and inspectors) repeatedly emphasise the reasoning and training-of-the-mind aspect and that this conviction can also be observed in practice. The reasoning can, for instance, be seen in the fact that teachers make the students reason their results (sometimes with precise proof) and often give them problem-solving activities to engage them in the act of finding mathematical concepts themselves. The focus here lies on the process rather than on the product. On the other hand, the training-of-the-mind is done by the focus on developing mathematical thinking. French teachers are concerned about the best way to teach mathematics so that all students can learn the whole mathematics which is taught at the respective level, and that they improve their understanding. After giving cognitively challenging problems to the students, they for instance collect the students' ideas for solutions. Then, two things can be established: the connections between the mathematical notions which could be developed while working on the tasks, and, on the other hand, the ideas, skills, and investigations made. Therefore, the three dimensions underpinning French teachers' behaviour are, firstly, their picture of mathematics which is characterised by their perception of the different elements of mathematics, namely the strict organisation of mathematical concepts, process orientation, and their entitlement. Secondly, their teaching is influenced by different cognitive approaches to get a grasp of the structure of mathematics as well as the relation of concepts and methods. It is, thirdly, shown that French teachers have an egalitarian view of teaching practise as they expect the whole class to make progress together. Therefore, great importance is put on mathematical reasoning. The German teachers' view of mathematics can be described as a relatively formal understanding of mathematics, which is characterised by logic and proof. The teacher's role is to explain the structure of mathematics and to present it in an interesting context. German teachers desire to teach every topic as deeply as possible so that they spend a substantial amount of time on each topic. This is mostly done in the dominant teaching style, the chalk-and-talk approach together with and interactive conversational style. Links between concepts can be elaborated on in their lessons as topics are not fragmented but an extensive amount of time is spent on each topic. Pepin could not reconstruct an orientation on process in the lessons. Teachers are concerned to pass on their knowledge to the students as successfully as possible.

The picture in England is a very different one. In contrast to France with their focus on process and mathematical reasoning, the emphasis in England lies on results and the development of mathematical skills. Although logical reasoning and proofs are named as aims of the English teachers, they are rarely found in their teaching. Teachers are busy with trying to cover the content of the curriculum, which might be a reason why they spend only little time with the explanation and illustration of a certain mathematical notion or skill to the class. Then, the students are asked to work by themselves on exercises during which time the teacher consults individual students. Cognitively challenging questions with, for example, more than one solution are only rarely used and mostly kept for 'investigation' lessons. Therefore, usually only high achieving students are taught notions like justification and proof. Secondly, process and content are separated because exploration and content are usually taught at different times. Exploration or cognitively challenging questions like problem-solving tasks have their own position in English lessons. The students are then rather taught how to 'behave mathematically' than to determine mathematical structures or patterns.

4 Concluding remarks

When comparing these results with the results found in the studies about teachers' beliefs in Mainland China, Hong Kong SAR, Australia, and the U.S. (see Cai; Perry; Wang & Cai; Wong; Cai, Bryan, Wang & Perry; and Wong in this volume), one can see that the European perspective clearly differentiates the picture drawn beforehand. The categorization of the beliefs in 'East' and 'West' is, on the one hand, still important but can be distinguished even further when France, Germany, and England are also taken into consideration.

Following Ernest (1989), teachers from Hong Kong and China hold a 'Platonic view' of the nature of mathematics which focuses on the internal logical structure of mathematics. It is gained and purified by abstraction from real life problems. The other pole on a continuum is found in the 'functional view' of mathematics held by U.S. and Australian teachers. Here, mathematics is treated as a language which describes and explains physical phenomena. The European perspective is not that easily classified on one side. On the contrary, France lies rather on the East Asian side with a scientific understanding of mathematical theory and a strong focus on logic as principal element. England, on the other hand promotes a pragmatic understanding of theory, i.e. a rather practical and purpose-oriented view which is in line with the Australian/U.S. perspective. Germany can be located somewhere in the middle, though with a strong tendency towards the formal approach, which is predominantly applied in the highest achieving school type of the tripartite school system. The lower types of secondary school, however, do not have this strong emphasis on theory so that they rather tend towards the English/Australian/U.S. side of the continuum.

When it comes to the role of the teacher, all teachers agree on the fact that they have to understand the needs of the students. The perception of how a 'good lesson' is taught, is, however, very different. In China, Hong Kong, and France, effective teachers are instructors who provide their students with the skills needed for correct performance and which enable them to find mathematical notions on their own. In Australia and England, the beliefs about effective teachers are somewhere between facilitator and explainer as teachers encourage their students to solve mathematical questions on their own and, on the other hand, explain how the concepts are related to each other. In the U.S., teachers engage students in problem-solving activities and are, therefore, facilitators. Interestingly, German teachers’ beliefs are located somewhere between the East Asian/French and the Australian/English ones as the subject matter is usually presented and explained by the teacher. The quality of the lesson depends on the teachers’ ability to explain. Therefore, the beliefs concerning the role of the teacher as instructor and explainer can be found in Germany.

There are quite big differences between the beliefs about the learning of mathematics and the engagement of the students in the learning processes. Beliefs from the U.S. and Europe often encompass group activities in the classroom as well as active engagement of the students verbally as well as physically. This means that the students have to actively participate in the lessons by either answering or also, as it is the case in Germany, developing the contents dealt with in the lessons in an interactive-communicative style. On the other hand, they are also invited to actively use hands-on manipulatives. These devices are also used by East Asian teachers but to a different extent and for a different purpose. Hong Kong teachers for instance only use them for demonstrations. Chinese teachers, on the other hand, rather engage students verbally in the lessons. Here, one can see a rather teacher-led view in the East and a student-centred view in the West. Germany can, again, be seen as in an intermediate position as, on the one hand, lessons are often teacher-dominated with chalk-and-talk-approach but, on the other hand, learning is process-oriented and seen as an active cognitive process which is fostered in the interactive-conversational style. France also occupies this middle position of the continuum although, once again, French teachers show a strong tendency towards East Asian approaches.

The perspective of teachers’ beliefs and their view of effective mathematics teaching can be broadened with remarks from a European perspective. The European countries presented here show significant differences concerning their teachers’ beliefs so that the overall picture can be differentiated. It is probably most interesting that the continuum U.S. – Australia – Hong Kong – China can be extended with England, Germany, and France. When located in this continuum, England takes a close relation to Australia and the U.S., whereas France shows close similarities with the East Asian beliefs. The German position is, again, somewhere in between these composite groupings.

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