examples (Medin, 1989). But as Sfard points out, mathematics students must become "sufficiently mature in the mathematical culture" to appreciate the role of definitions in mathematics (1992, p. 47). A similar maturity in the mathematical culture should lead to an awareness of the tentative nature of results suggested by examples.

Inductive Proof Scheme. When students ascertain themselves and persuade others about the truth of a conjecture by evaluating their conjecture in one or more specific cases, they are said to possess an inductive proof scheme. Every teacher has likely observed the dominance of this proof scheme among students, and research corroborates this observation. For example, Chazan (1993) has observed the existence of the inductive proof scheme among U.S. high school students. Martin and Harel (1989) found that more than 80% of their preservice elementary teachers considered inductive arguments to be mathematical proofs. Even with mathematics majors, who presumably are more sophisticated than the high school students or the preservice elementary teachers, the inductive proof scheme is common.

The Perceptual Proof Scheme. This proof scheme fits, for example, many geometric justifications that might be given by younger students. The perceptual proof scheme is based solely on visual or tactile perceptions. For example, a student may examine an isosceles triangle and decide that the base angles are congruent just by visual examination. Older students might be convinced that the medians of a triangle are concurrent by looking at several computer-generated examples, and they might attempt to convince others by showing them similar examples.

The Theoretical Proof Schemes

The Transformational Proof Schemes. The general characterization of these schemes is that students' justifications attend to the generality aspects of a conjecture and involve mental operations that are goal oriented and intended-anticipatory. They are the foundation for all theoretical proof schemes. Here is an example of transformational reasoning from a case study of a fourth-grader (by GH):

> I asked Ed to think of a triangle with two equal angles and describe what he thought the relationship between the sides opposite them. Ed responded almost instantly that the two sides must be equal. I asked Ed to explain to me how he had arrived at this conclusion. Using his hands to describe the triangle, Ed said something to the effect that if one angle (he puts one forearm horizontally and moves the second forearm diagonally to it) is equal to the other angle (switches between the forearms' positions), then the two sides (he puts the two forearms diagonally to form a triangle) are equal. When I continued to press Ed for more explanation, he went on to say: If you launch a rocket from this side (pointing to his right elbow and moving his right forearm diagonally to indicate the direction of the rocket) and at the same time you launch another rocket from this side (pointing to his left elbow and moving his left forearm diagonally to indicate the direction of the other