

Categories of Proof Schemes

The notion of "proof scheme" has been useful to us. Proving (or justifying) a statement includes two aspects: ascertaining (convincing oneself) and persuading (convincing others). An individual's proof scheme consists of whatever constitutes ascertaining and persuading for that person. Hence, a proof scheme is idiosyncratic and naturally can vary from time to time and from context to context, even within mathematics. It is important to note that "proof" as used in "proof scheme" need not connote "mathematical proof." The teaching experiments have had the intent to identify and alter students' proof schemes, and the interviews to test the sufficiency of the classification. The categories as currently conceived fall into three major classes. In a few cases the labels for the proof schemes are tentative, so the reader should rely not so much on the labels as on the brief descriptions and illustrations.

The External Conviction Proof Schemes

The earmark of the external conviction proof schemes is that justifications hinge on such external features as the endorsement of an authority (the authoritarian proof scheme), the form of the argument (the ritual proof scheme), or meaningless manipulations of symbols (the symbolic proof scheme).

The Authoritarian Proof Scheme. When students are not concerned with the question of the burden of proof, and their main source of conviction is a statement given in a textbook, uttered by a teacher, or offered by a knowledgeable classmate, they are exhibiting the authoritarian proof scheme. When asked how they might convince someone of a particular result, statements like "I would try to find it in a book" or "I think my professor said it, so it should be in my notes" would be offered under this proof scheme. The value of proofs may even be questioned, perhaps because in so much of the mathematics that the student has experienced the emphasis has been on the results, with little or passing attention to the reasoning processes used to arrive at those results. In the teaching experiments, where "why" is a routine expectation as well as "how," students have gradually become less unquestioningly accepting of assertions deliberately made by the instructor to test their willingness to accept the mere word of the "authority."

All this is not to say that accepting the word of an authority is all bad, of course. Even noted mathematicians are no doubt on many occasions willing to accept a result without examining the details of a proof. Rather, it is the attitude of helplessness in the absence of an authority, or the view that justifications are valueless, that handicap the students with an authoritarian proof scheme.

The Ritual Proof Scheme. *Martin and Harel (1989) examined whether students' judgments of an argument are influenced by its appearance in the form of a mathematical proof--the ritualistic aspects of proof--rather than the correctness of the argument.* They presented students with a false argument to a given mathematical statement and then examined the students' evaluations of that argument. They found that "many students who correctly accepted a general-proof