EXAMINING "NORMS" IN MATHEMATICS EDUCATION LITERATURE: REFINING THE LENS

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INTRODUCTION

In the past 20 years, mathematics education researchers have come to examine mathematical learning with respect to the social interactions that take place in classrooms. In some cases, this direction was pursued as the result of finding psychological perspectives limiting when attempting to describe student learning within a classroom environment (e.g., Cobb & Yackel, 1996). In other cases, they were pursued because they were considered to be understudied in comparison to a psychological perspective or a subject matter analysis (e.g., Bauersfeld, 1988). Focusing on interactions led to investigating how these patterns might become "normative" in the sense that they were underlying what it meant to be in school and doing school mathematics. Social interactions among people are complex. In addition, it is not completely apparent how social interactions effect mathematical understandings. For at least these two reasons, studies related to social interactions in mathematics education will continue to be pursued.

In this paper, I review literature[1] on normative interactions in mathematics classrooms from multiple perspectives, including: a) sociological and emergent, b) epistemological/situated, and c) sociocultural (e.g., the cultural-developmental framework)[2]. In light of compiling and analyzing the literature, I have discovered that my own research focus needs to shift from not only accounting for micro classroom processes but also accounting for other communities of practice in which the participants are involved. Crossing sites (e.g., from home discourses to school discourses and vice versa) may shift teacher and student beliefs and values. Knowledge of these "sites" and how they influence teacher and student beliefs and values are imperative to broader understandings of this phenomenon.

One distinguishing factor among these perspectives includes what they emphasize in their analysis. For example, the sociological and emergent perspectives focus their lens more

[1] This review is selective rather than all encompassing. With the exception of one paper, I have chosen to limit my search to literature related to K-12 classrooms, while recognizing that some important contributions to this discussion have taken place in university mathematics and education courses. Focusing on social interactions and the kinds of normative behaviors in classrooms has become a growing force in mathematics education literature. Due to this fact, I limited my search in the ways I describe.

[2] With the exception of the epistemological/situated perspective, these labels have been given to each perspective by the authors themselves. I maintain those labels, while recognizing that there are common ideas across them. For example, "mathematical themes" in the "sociological perspective" are very similar to "classroom mathematical practices" in the "emergent perspective" and to "mathematical goals" in the "sociocultural perspective."

on the microculture of the classroom while nodding to the macroculture. The epistemological/situated and sociocultural perspectives recognize in more detail the macro and micro cultures, but in different ways. For the epistemological/ situated perspective, the macro-culture influences what the student believes his/her role should be in terms of how the institution of schooling “constructs” students; the micro-culture is more immediate and idiosyncratic and depends on the interactions between individuals. With a sociocultural perspective, the macro-culture is made up of layers of contexts that influence students in various ways. It is also fluid and is influenced by historical, political, and economic factors that are painted as background information to the analyses. When examining micro-culture, sociocultural perspectives focus on activities, practices and tools.

I begin the next section by looking at the range of terminology that has been used to discuss “normative” behavior in mathematics classrooms. In the third section of the paper, I look across the perspectives on “norms,” highlighting similarities and differences and discuss how “norms” are positioned with respect to ideas like “practice,” “culture,” and “interactions” within each set of literature. In doing so, I argue that both the macro- and micro- culture of the classroom should be examined carefully to construct a more complete picture of normative interactions in mathematics classrooms.

DEFINING “NORMS”

Studies of normative behavior in mathematics classrooms may use different terminologies, depending on the perspective one takes. In addition to using various terms for this idea, the word “norm” might be used but authors focus on different aspects of normative behaviours.

Researchers in mathematics education who follow a “sociological” perspective[3] rarely used the term “norms,” (and do not define it when they do use it, see Bauersfeld et al., 1988) in their publications. Bauersfeld and his colleagues describe patterns that become ‘normative’ in nature when focusing on “patterns of interaction” (Bauersfeld, 1988; Voigt, 1985). These are described as the “underlying grammar” of the classroom because they are not explicit to the participants; they consist of networks of routines and obligations (Bauersfeld, 1988; Voigt, 1992). The patterns are pervasive in the sense that they continually exist to help reduce complexities associated with ambiguousness of meanings that occur when people interact. Some routines that have been described in this work include: “teacher’s use of ‘open’ questions to which one definite answer is expected, the suggestive hint, the decomposing of a solving process in small pieces of subsequent actions, the student’s routine of verbal reduction, i.e., restricting utterances to

[3] Bauersfeld and his colleagues propose a theoretical perspective that draws from symbolic interactionism (e.g., Mead (1934), Blumer (1969), & Goffman (1959, 1971, 1974), ethnomethodology (e.g., Garfinkel (1967), Cicourel (1973), Mehan, (1975, 1979) and Coulter (1979)), sociolinguistics (e.g., Cazden (1972), Cicourel (1974)), phenomenology (Shutz (1973)), to some extent from cross cultural studies (e.g., Cole et. al (1974), Cole and Means (1981)).and certain paradigms in cognitive science (e.g., Minsky (1975, 1980), Lawler (1981, 1985)) (Bauersfeld, 1988; Bauersfeld, Krummheuer, & Voigt, 1988). A basic assumption of this perspective is that cultural and social processes are essential to mathematical study (Voigt, 1995).

numbers or catchwords, the trial-and-error routine in order to meet the teacher's expectations, etc." (Voigt, 1992, p. 37).

While "patterns of interaction" could occur in any classroom, "thematic patterns (procedures) of interaction" are more specific to mathematics classrooms (Voigt, 1989). This latter type of interaction occurs when the teacher and students routinely constitute a theme around some related issues. People bring varying interpretations to mutual tasks (Neth & Voigt, 1991); sometimes misunderstandings occur over a long period of time and the participants are quite unaware of this (Krummheuer, 1983). To develop intersubjective meaning, then, a process of negotiation must take place. Negotiation is characterized as a process of shared adaptation during which the participants interactively create responsibilities for their activity (Voigt, 1985). Only mathematical meanings "taken to be shared[4]" can be produced through negotiation. From an observer's point of view, these constitute "mathematical themes" (Bauersfeld, 1988, p. 174). This occurs when people disregard that they could interpret something differently; yet, one can never be sure that two persons are thinking the same even if their interaction proceeds without conflict.

In their early research, Cobb and his colleagues drew from Much and Schweder's (1978) work and discussed five types of classroom norms: regulations, conventions, morals, truths, and instructions (Cobb, Wood, Yackel, & McNeal, 1992). In focusing on mathematics classrooms specifically, these researchers utilized mainly the last two categories and delineated two different classroom traditions (i.e., "school mathematics" and "inquiry mathematics").

They later described their perspective as an "emergent"[5] one, following the same theoretical grounding as the sociological perspective described above but coordinating a psychological perspective along with it (for a detailed description of this coordinated perspective see Cobb, Stephan, McClain, & Gravemeijer, 2001; or Cobb & Yackel, 1996). In building on the sociological perspective, they defined norms as characterizing "regularities in communal or collective classroom activity [which] are considered to be jointly established by the teacher and students as members of the classroom community" (Yackel and Cobb, 1996, p. 178). Some social norms they have described include: explaining and justifying solutions, listening to and making sense of each other's

[4] Bauersfeld (1995) relates the negotiation of taken-as-shared meaning to the development of "languaging" in social groups in which a "consensual domain" (Maturna & Varela, 1980) becomes a convention that emerges through social interaction.

[5] In a recent paper, Cobb et al. (2001) summarize the relationship between the social and psychological perspective as follows:

> [The coordination of the perspectives] implies that neither perspective exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective. … When we take a social perspective, we therefore locate a student's reasoning within an evolving classroom microculture, and when we take a psychological perspective, we treat that microculture as an emergent phenomenon that is continually regenerated by the teacher and students in the course of their ongoing interactions. … the coordination is between two alternative ways of looking at and making sense of what is going on in classrooms" (p. 122).

solutions, indicating non-understanding and posing clarifying questions when one does not understand, and explaining why they did not accept explanations that they considered invalid (McClain & Cobb, 1997, 2001).

These authors further distinguished between general "social norms" and "sociomathematical norms," i.e., normative behavior related more particularly to the domain of mathematics. For example, a "social norm" they discussed focused on students' explanations and justifications of their solutions; when focusing on sociomathematical norms they included an examination of "what counts as an acceptable explanation and justification [which] deals with the actual process by which students contribute" (Yackel & Cobb, 1996, p. 461). Sociomathematical norms have been shown also to develop in technologically rich learning environments (Hershkowitz & Schwarz, 1999). The construct of sociomathematical norm was deemed significant in that it helped to understand: 1) students' progress in developing a mathematical disposition, and 2) students' increasing intellectual autonomy in mathematics (Yackel & Cobb, 1996).

Negotiating social norms appears to be necessary but not sufficient for productive mathematical learning. For example, Pang (2000; 2001) explored classrooms in Korea and the U.S. and showed that the social norms were similar across the classrooms. In addition, she illustrated that the sociomathematical norms were quite different and argued that the difference effected the type of learning the students were able to engage in. In support of this claim, Kazemi and Stipek (2001) have shown similar results. They analyzed four classrooms from three schools and showed that while all the classrooms had similar social norms, those that also focused on the kinds of sociomathematical norms that have been identified maintained a "high press" for conceptual thinking. These two terms have been taken up by many other mathematics education researchers and are probably the most commonly used terms in this body of literature.

Drawing from Cobb and his colleague's research, Herbel-Eisenmann (2000; 2002) focused particularly on the rights, roles, responsibilities, and expectations that were mutually negotiated between the teacher and students. The intent of that work was to examine how norms are embedded in and carried by the discourse of the classroom through focusing on pervasive patterns of talk. This work showed that the language forms (i.e., particular words and the ways they came together) teachers (unconsciously) used influenced the way students explored mathematical ideas and the way the teachers positioned themselves with respect to the external mathematical authority and the classroom learning community. A similar connection between authority patterns and student exploration of mathematics was made by Hamm & Perry (2002). This emphasis on discourse patterns associated with patterns of authority in mathematics classrooms describe a more specific set of norms rather than the broader category of "social norms" described in the previous paragraph.

Because Brousseau focuses on *didactique*[6] and situations, some researchers have labeled his ideas as being "situational" (see, for example, Kieran (1998)[7] and Pepin (1999) as

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[6] *Didactique* is the science related to the creation and articulation of knowledge. Brousseau (1997) states, "Knowing what is being produced in a teaching situation is precisely the object of *didactique*; it is not a

places that label his perspective as being situated, but at a more local level) and "epistemological[8]." Brousseau's (Brousseau, 1984, 1997; Otte & Brousseau, 1991) idea of "didactical contract" focuses on the normative nature of the interaction between the teacher and students. In short, he defined this term as the tacit, mutual understanding that the teacher knows the content and is expected to help the student learn it. Conversely, the student understands that the teacher knows the content and it is her job to learn the content that is set out for her. Through the contract "the teacher is obliged to teach, and the pupil is obliged to learn. …the very essential nature… of the teaching-learning situation" (Otte & Brousseau, 1991, p. 18). By this very arrangement, an implicit contract is formed by which each participant understands what s/he is supposed to do. The contract exists on two levels, one that is related to broader contexts of institutional mathematics (and the mathematics that is valued by the mathematical community) and schooling. The other level exists on a microlevel and plays out as the teacher and students interact in a learning situation (Herbst, In press). This is one difference between "social" and "sociomathematical" norms and the "didactical contract": the former are viewed to be mutually negotiated within the microculture of the classroom whereas the latter is considered to be both pre-existing and as existing in a particular situation.

When studying mathematical development using a "cultural-developmental" framework[9], Saxe (1999a; 1999b; 2001) describes collective practices and activity structures in classrooms that play a role in mathematical goals. Activity structures help interpret the goals that emerge for individuals in collective practices and include: a) routine phases or cycles of activities, b) norms and sometimes explicit rules for behavior, and c) emerging role relations between participants. While the term "norm" is not defined specifically in his work, Saxe (2001) describes norms that occur within the activity structures of a mathematics classroom. This idea is not as central to his work as the notion of "collective practices." Collective practices occur when individuals come together; they are historically situated and take shape in complex, fluid economic, social and political environments. More specifically, classroom practice may encourage enhanced goals

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result of observation, but one of analysis based on the knowledge of phenomena which define what they leave unchanged" (p. 29). *Didactique* draws on the assumption that "pupils construct their own knowledge, their own meaning … as a necessary response to [their] environment" (Balacheff, 1990, p. 259).

[7] For example, Kieran quotes the following as evidence of Brousseau's perspective: "This know-how occurs within situations. It is not yet possible to take a question out of context and ask it independently. The results can not yet be depended upon as "acquired" knowledge, nor do the children identify them as such" (p. 204).

[8] Due to its focus on transformations of mathematical knowledge, *didactique* can be "characterized by an epistemological perspective focused on the knowledge that is at stake in that practice" (Herbst & Kilpatrick, 1999, p. 7). *Didactique* is not a theory of learning but concerns itself with organizing other people's learning, i.e., with the circulation and transposition of knowledge.

[9] Following constructivist treatments of cognitive development (Saxe, 1991; Saxe, Dawson, Fall, & Howard, 1996; Vygotsky, 1986; Vygotsky, 1978/1934), this framework takes for granted that:

> Children develop concepts and procedures as they construct mathematical goals and work to accomplish them. Within the constraints of their understandings, students create goals that emerge in and are supported by their participation in classroom practices, goals that they would not create on their own. In students' efforts to structure and accomplish emergent mathematical goals linked to their own understanding and classroom practices, students create possibilities for generating new learning keyed to instruction (Saxe, Gearhart, & Seltzer, 1999, p. 2).

through the kinds of opportunities children are afforded. The classroom practice is influenced by what the teacher and students value in terms of mathematical activity. For example, a teacher who values the kind of teaching/learning proposed by the *Standards* documents (NCTM, 1989, 1991, 2000) would have a different kind of practice than someone who values a procedural approach or someone who values a discovery approach (see, for example, Saxe, et al.'s (1999) example of three different classrooms using a fair-share problem to teach fractions).

In this section, I offered a range of terminology and foci for the study of normative behaviour in mathematics classrooms. In some cases, "norms" are central to the work (e.g., in the emergent perspective); in other cases, they are subsets of more encompassing ideas like "activities" and "classroom practices" (e.g., in the sociocultural perspectives). While there are many different terms used, there is convergence across the perspectives in the fact that normative behaviours can be related to more general social processes in the classroom or they may be directly associated with the specific content under study (i.e., mathematics). In the next section, I examine these ideas further to show how each perspective positions the "normative behaviours" in the mathematics classrooms. I show that we have learned a lot about normative aspects of mathematics classroom. However, I also argue that we need to consider more than just the classroom microprocesses in coming to understand mathematics classroom norms further.

MACROCULTURE, MICROCULTURE, PRACTICE, INTERACTION: WHERE ARE "NORMS" LOCATED?

Depending on the perspective, norms are located in either a central role in the analysis or they are not. In some cases, norms are described in a "snapshot" way or at a particular point in time; in other cases, emergence of the norms is pertinent. In this section, I attempt to tease out some of the differences with respect to each perspective as to how they view/use norms and where they are located with respect to the analysis. In doing so, I try to identify the affordances and constraints that each perspective has with respect to the study of norms.

Interactions and Themes

The sociological perspective mainly focuses on "interactions" that occur within the microculture of the classroom (Bauersfeld, 1988; Bauersfeld et al., 1988). By taking an interactionist perspective, these authors clarify the classroom microculture's dynamics and regularities. "Microculture" is used synonymously with "classroom culture" and is considered to be the "taken-as-shared" context (Voigt, 1995, 1998). The authors deem this important to study because of its hidden regularities; they claim these are more difficult to change than the macroculture of the classroom (e.g., general principles and teaching strategies, environment of the classroom situation). The microculture's characteristics depend on the hidden patterns, conventions and norms and they are difficult to change. In fact, changing the microculture is described as "evolutionary" (Voigt, 1995).

The negotiations that take place during interactions are viewed as mediating between cognition and culture (Bauersfeld, 1988). Across time interactions become somewhat stable to help reduce the complexity of the interchange so that ambiguities of meaning are not constantly interrupting communication. The patterns establish as turns are taken and develop from expectations, interpretations, obligations and relations (Bauersfeld, 1988; Voigt, 1985). Although the participants are seen as actively involved in the negotiation, these routines are hidden and participants are not conscious of them (Bauersfeld, 1988; Voigt, 1992).

Mathematical understanding is studied through examining the mathematical themes that emerge. The evolution of mathematical themes, which are improvisations of thematic patterns, it is argued, seem to correspond to the students' cognitive development (Voigt, 1995). The mathematical theme is not fixed but as a topic of discourse it is interactively constituted and it changes through the negotiation of meaning. Voigt (1995) writes:

> When the interactional concept of theme is applied to teacher-student interactions, it mediates between two theoretical perspectives. One perspective stresses the experiential situation as a person subjectively constructs it. The individual's conceptual operations are of interest in this perspective. The other perspective views the global cultural context, as stabilized and institutionalized by a community (or mathematics teacher and other persons) over a longer period of time. Because the theme is interactively constituted, it would not exist without the teacher's and the students' contributions. It is related to the students' individual thinking processes as well as to the mathematical and educational claims of a global context that the teacher represents (p. 176).

Because these researchers focus on a sociological analysis, they reconstruct the themes *between* people. They do not consider the cognitive processes occurring within one individual person.

Norms are related to the thematic patterns of interaction. However, these authors are more concerned with describing the themes (in terms of both the social interactions and the mathematics) rather than how the "norms" are established. In that sense, their discussions of norms are more "snapshot" and seem to be the result of particular routine patterns.

Social and Sociomathematical Norms

While drawing on the sociological perspective the emergent perspective has further elaborated the microculture of the classroom to include three analytic categories: social norms, sociomathematical norms, and classroom mathematical practices. (See Cobb et al. 2001 or Cobb & Yackel, 1996 for detailed discussions of these constructs. A summary of their coordinated perspective is given in Table 1 below). The development of each of these ideas has been chronological, beginning with social norms then moving to articulating sociomathematical norms. Most recently, their publications have focused on classroom mathematical practices (see, for example, Bowers, Cobb, & McClain, 1999;

Cobb et al., 2001). Mathematical practices are considered to be a backdrop for mathematical activity and mathematical communication (or discourse or dialog). This idea was adopted from the sociocultural notion of "cultural practice." The practice is seen to be part of human activity and is viewed as emergent. Within the classroom community, normative practices are constituted by the teacher and students through their interactions.

Social Perspective	Psychological Perspective
Classroom Social norms	Beliefs about own role, other's roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and reasoning (formerly "mathematical conceptions and activity")

Table 1

An interpretive framework for analyzing
communal and individual mathematical activity and learning

Instead of the negotiations always being implicit (as described in the sociological perspective), the teacher is seen to sometimes make the negotiation of social and sociomathematical norms explicit to the students in her role as an authority in the classroom (Cobb, Yackel, & Wood, 1993). They point out that these explicit comments ("talking about talking mathematics") sometimes emerge in the course of the interaction without prior intention. In fact, this type of explicit talk contributes to shifts in the discourse that contribute to mathematical development (Cobb, Boufi, McClain, & Whitenack, 1997).[10] In their more recent work in classrooms, however, the teachers intended to establish certain norms and went in with that aim (see, for example, Cobb et al., 1997; McClain & Cobb, 1997, 2001).

As with the sociological perspective, they view the patterns and regularities as being taken for granted and as making possible smooth collective activity. The patterns of interaction reveal the implicit social norms which are identified by regularities in the patterns of social interaction. These norms are fairly transparent except when a breech occurs (Cobb, Yackel & Wood, 1993). The emergence of certain norms has been the focus of some of their more recent work, including how some sociomathematical norms appear to have been afforded by others (e.g., the notion of 'efficient' grew out of what made something mathematically different). In addition, they have shown how specific activities seemed to encourage particular sociomathematical norms to emerge (i.e., recording student thinking on the board seemed to allow students to understand what it meant for solutions to be mathematically different (McClain & Cobb, 1997).

[10] For further articles related to student's mathematical understandings and beliefs, see, for example, Cobb, Wood, Yackel & Perlwitz (1992), Cobb et. al (1997), Nicholls, Cobb, Yackel, Wood & Wheatley (1990), and Yackel, Cobb & Wood (1991).

When studying mathematical understandings, the emergent perspective states that they coordinate both the sociological and the psychological perspectives. However, it appears that most of the articles and book chapters that they have published work within individual cells of Table 1 but do not look more specifically across the rows of the Table. For example, they have not studied "sociomathematical norms" and coordinated that sociological analysis with a study of "mathematical beliefs and values." In articles where they have discussed the coordination, they argue from a theoretical basis and acknowledge that the relationship is a "conjectured" one (Cobb et al., 2001; Cobb & Yackel, 1996).

Because both the sociological and the emergent perspectives focus centrally on the microculture of the classroom, the broader macroculture is not really addressed. While the latter group has argued that their perspective is more fitting for the phenomena they are studying (Cobb, 1994; Cobb & Bowers, 1999), the former has not investigated the effects of the broader culture on the classroom interactions (in the readings that I have done). In fact, although the emergent perspective has described the broader macroculture as providing background for their work (see, for example, Cobb, Wood, & Yackel, 1993, for an instance of a more detailed account of this), they see it as not being central to what they are trying to understand: student understanding of mathematics (Cobb, Wood et al., 1993; Cobb & Yackel, 1996). However, as some of the sociocultural perspectives have argued, at least two components seem to be important to understanding mathematics classrooms and are not addressed: 1) students are part of other communities of practice that influence how they come to participate and be part of the schooling system (or not) (Hodge & Stephan, 1998; Meira, 1997); and 2) teachers typically know a lot about the context in which they teach and this knowledge effects what they do/say in their classrooms (Atweh, Bleicher, & Cooper, 1998). In addition, Herbst (1997) points out the need to attend to "relations between power, knowledge, and discursive practices" (p. 59), and that the theory needs to acknowledge the basic intentionality of education , helping the participants to see sociomathematical norms as mathematical norms. This latter point is also supported by Hodge & Stephan (1998).

Didactical Contract

Brousseau's "didactical contract" appears to have characteristics in common with both the sociological's notion of "themes" and the emergent perspective's "social norms," "sociomathematical norms" as well as "classroom mathematical practices." The didactical contract, like themes, has to do both with ways of functioning socially and of functioning mathematically. The processes of adaptation utilized by the student in a given teaching situation are not all mathematical; the student also relies on knowledge of "the teaching system, its norms and customs, and guesses about the expectations of the teacher" (Artigue, 1999, p. 1378). In addition, these aspects of classrooms are considered to be implicit and beyond a level of consciousness for the participants, yet they are tacitly understood and cannot be ignored (Brousseau, 1997).

Because the didactical contract cuts across the mathematical and social, it carries with it aspects associated with "social norms" as well as "sociomathematical norms." Also, it appears to be centrally tied to particular mathematical ideas being studied (Brousseau, 1997), which makes some of its characteristics similar to that of "classroom mathematical practices." The didactical contract is different in that it is described as existing on two contextual levels (both the macro (institutional) and the micro (classroom)) (Herbst, In press). So, the participants are seen as having brought particular ideas with them to the context and this influences what happens in the classroom interactions. However, in reading work by Brousseau, exactly how the macrolevel factors influence the microlevel interactions was not clear.

Another distinction can be made between these ideas. All of the research reported by Cobb and his colleagues grew out of their work in classrooms and in collaboration with teachers. They are more practical and useable to the practitioner audience. In contrast, Brousseau (1997) recognizes the difficulty in making the didactical contract useful for teachers. Instead, it is viewed as an analytic tool for understanding the teaching of mathematics. As with the study of "norms" in the emergent perspective, the contract only becomes apparent when a breach has taken place. It is at that moment that negotiation takes place. Because of the lack of transparency, the idea of a didactical contract may help the teacher understand his or her practice, but it is not a tool for "acting on that practice" (Herbst & Kilpatrick, 1999, p. 9). In addition, the didactical contract is recognized to exist in any kind of mathematics classroom, not just in inquiry based or teacher development environments. In fact, it is rare that the contract can be described, rather it is taken for granted and is used to track the negotiation that takes place when meanings are being made.[11]

Goals and Norms

It seems that some sociocultural perspectives take mathematical understandings, the microculture of the classroom, the broader macroculture, etc. all into account. One line of research that seems to encompass these levels is the "cultural-developmental" framework used by Saxe. However, this type of detailed analysis seems to have been done more thoroughly in his early work when he was focusing on out-of-school mathematics learning in Brazil (e.g., Saxe, 1988; 1998) as well as the effects of schooling on arithmetical and measurement understandings in children in Papua New Guinea (e.g., Saxe, 1982; 1985). More recently, when he has applied his framework to analyze the mathematical development of students in the U.S. (e.g., Saxe, 1999a, 2001), the macro-levels seem to be described more as background information rather than as informing the detailed analysis in more profound ways.

Saxe's analyses focused on "goals" at various levels. At the most macro-level, he focused on how collective practices and activity structures were historically situated and that they take shape within a fluid, complex organization of economic, social, and political circumstances. These are termed "official goals" and are determined by a cultural

[11] I would like to recognize the contributions of discussions with Patricio Herbst to this series of distinctions I am making.

analysis. The learner's goals are teased out through a developmental analysis that focuses on microgenetic development, sociogenetic development and ontogenetic development. The shifts and developments of the goals are traced through a social interactional analysis. Within all of these areas, Saxe (2001) claims that social interactions figure prominently in the development of goals because the goals emerge through social interaction in ways that they would not be able to without the interactions. Routine social activities are seen to be constituted by patterned ways individuals participate and include "activity structures." Norms are not as central to this work because they are only one integral aspect of activity structures. When Saxe (e.g., 2001) applies his framework to classrooms in the U.S., norms are described in an instant of time rather than in a developmental way.

When analyzing collective practices, Saxe attends to two strands: 1) children's emerging mathematical goals; and 2) the development of children's mathematics. When analyzing the first, Saxe attends to: "the way in which goals emerge in relation to the structure of the activities[12], social interactions, artifacts, and the prior understandings that students bring" (p. 277). "Activity structures" help researchers make sense of goals that arise for individuals in collective practices. Saxe (2001) points out that the classroom in this paper has activity structures that reflect reform-based practices. For example, the teacher acts as a facilitator and problematizes student offerings in inquiry-oriented activities. He also states: "A norm that is central to the reform-oriented activity structure in [this classroom] …is that student contributions must, in fact, display reasoning" (p. 278). He then uses this norm to show how it is accomplished and how it is central to the activity structure of the class and to the mathematical goals the children generate. Social interactions are imperative in leading to new mathematical goals, goals that may not have emerged on their own. Artifacts (e.g., the graphs generated on a graphing calculator) are created by humans and when they are used in a collective practice, some become valued and others do not. Artifacts can effect and influence the kinds of goals that emerge during interactions. Prior understandings are the "ground from which children create particular goals" (Saxe, 2001, p. 288).

In the second strand to which Saxe attends, three levels of developmental processes are considered: a) microgenesis, b) sociogenesis, and c) ontogenesis. While all three of these processes are established in activity as individuals use them, they target different kinds of developmental process:

> Microgenesis is concerned with how particular forms (like the display) and the functions that forms afford are turned into means to accomplish emerging goals in activities. Sociogenesis is concerned with the appropriation and spread of forms in communities, a social process that occurs as individuals appropriate one another's efforts. Ontogenesis is concerned with the shifting relations between the forms used and the functions that they serve in individual activity over an individual's development (p. 290).

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[12] The patterning of routine social activities constituted by the way that individuals participate with one another are the "activity structures." These include "routine phases or cycles of activity, norms and sometimes explicit rules for behavior, and emerging role relations between participants" (p. 278).

An assumption that underlies the idea of microgenesis is that objects only take on meaning within a particular activity, in this case mathematical activity. Artifacts are organized by individuals to accomplish mathematical goals. Sociogenesis occurs when new and valued forms and their associated functions spread across the community involved in the practice. For example, students may pick up and use the same (or similar) word form(s). These words may be used in the same way or could serve quite different purposes. In contrast to sociogenesis, ontogenesis occurs over a period of time in which children either develop new functions for the forms in their activities or pick up new forms. Saxe (2001) claims that the "interplay between the forms and functions over the course of the children's developments is a process of ontogenetic change and key to understanding children's developing mathematics in collective practices" (p. 295). Ideally, in understanding the ontogenesis of children's development one would follow them over the course of a number of years.

In the sociocultural perspectives (e.g., the work of Saxe and Atweh, et. al), norms are described in a "snapshot" way: that is, they are described as existing and are not treated from a process perspective that might trace how they emerge and change over time. This "snapshot" approach seems to be due to the fact that the focus of these analyses is more on practices or situations rather than on "norms" as an entity in itself. The detailed tracing across time focuses more on the practices that are taking place.

All of these authors recognize the teachers as a representative of the broader mathematical community in some form or another. For example, Brousseau discusses the idea of "institutionalization" in which the teacher situates a student's contributions and ideas with respect to what is culturally and scientifically relevant. Cobb and his colleagues describe mathematical learning as including an "enculturation" or "acculturation" component. However, how this effects the learning environment is not really taken up as an issue. In some cases, such representation could override student options for contributing in particular ways because, for instance, they may think they need to align their contributions with what the teacher approves. In addition, this aspect of teaching needs to be carefully considered. For example, Atweh et al. (1998) shows that the levels at which teachers consider this aspect of their teaching may effect whether students can be acknowledged as part of the mathematical community or not. If teachers do not consider how this might influence the student's options for being recognized as part of the mathematical community, they may, in effect, be contributing to cultural reproduction by limiting access.

SUMMARY

In this paper, I have discussed how "norms" appear in mathematics education literature. Because researchers draw from different theoretical perspectives, the level of attention to "norms" as an idea varies: it is especially essential in the emergent perspective, but is not as central to the sociological and sociocultural perspectives. While each perspective has offered information that is helpful to understanding norms in mathematics classrooms, there are also limitations associated with the way that each perspectives views norms. For example, the sociological and emergent perspectives do not take into account the other practices in which participants take part and do not examine the way in which teachers use their knowledge of context in their interactions with students. In addition, some sociocultural perspectives (e.g., sociolinguistic) do not try to make claims about the developing mathematical ideas that occur in the interactions. Other sociocultural perspectives, e.g., Saxe, seem to take the broader macroculture into consideration as well as the microculture of the classroom by focusing on varying levels of goals.

SOME QUESTIONS RAISED

To date, we have learned a lot about "normative interactions" in classrooms from the perspectives who have explored this notion. However, unless we turn toward a more encompassing understanding of the micro and take into account an historic tracing of these interactions, we may be limiting our understandings of this idea. People enter the classrooms having many experiences, beliefs, values, etc. from which to draw. To truly understand what becomes normative and why, we must also understand some of these aspects of the participant's lives.

Some questions that may be raised as a result of this literature review include:

- When is a norm stable? When is there enough evidence to call something 'normative'?
- For whom do the norms become stable? And when?
- Do some students implicitly pick up on the norms more quickly than others? If so, who? If so, why?
- Are the students aware of the implicit and explicit negotiation of norms? Does making them explicit allow access to the "underlying grammar" of school for more students?
- How do other aspects of the participant's lives (e.g., their values, beliefs, goals, experiences) effect the ways in which they participate in the classroom? How do they effect what the participants take away from the classroom context?
- Methodologically, how can we account for all of these aspects of classroom life?
- How are the observer's interpretations of classroom "norms" different from the teacher's? From the student's?
- What role does the context of schooling play in effecting the norms?
- What role does power and authority play in effecting the norms?

These are a sampling of the questions this literature review raises. They indicate that we still have much to learn about the microculture of the classroom.

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