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SURVEY PAPER

The Role of Mathematics in interdisciplinary STEM education

Katja Maass[1] · Vince Geiger[2] · Marta Romero Ariza[3] · Merrilyn Goos[4]

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Abstract
In times of rapid technological innovation and global challenges, the development of science, technology, engineering and mathematics (STEM) competencies becomes important. They improve the personal scientific literacy of citizens, enhance international economic competitiveness and are an essential foundation for responsible citizenship, including the ethical custodianship of our planet. The latest programme for international student assessment results, however, indicate that even in economically mature countries such as those in Europe, and the USA and Australia, approximately 20% of students lack sufficient skills in mathematics or science. This trend serves to highlight the urgent need for action in relation to STEM education. While it is widely acknowledged that mathematics underpins all other STEM disciplines, there is clear evidence it plays an understated role in integrated STEM education. In this article, we address an element of this concern by examining the role of mathematics within STEM education and how it might be advanced through three interdisciplinary approaches: (1) twenty-first century skills; (2) mathematical modelling; and (3) education for responsible citizenship. At the end of the paper we discuss the potential for research in relation to these three aspects and point to what work needs to be done in the future.

Keywords STEM education · Mathematical Modelling · Education for responsible citizenship · Twenty-first century skills · Socio-Scientific Issues · The role of Mathematics in STEM · Numeracy

1 Introduction

There is momentous societal and economic change being experienced across the globe due to the rapid rate of technological innovation, a development that is shifting economics from manufacturing to information and knowledge industries. These changes have had significant impact on the labour market and led to major demographic and associated social changes (European Commission 2018a). As a result, it is likely that many children entering school today will be employed in occupations that do not yet exist (ACOLA 2013; European Commission 2017; Partnership for twenty-first century skills 2002). As Binkley et al. (2012) point out:

> No longer can students look forward to middle class success in the conduct of manual labor or use of routine skills—work that can be accomplished by machines. Rather, whether a technician or a professional person, success lies in being able to communicate, share, and use information to solve complex problems, in being able to adapt and innovate in response to new demands and changing circumstances, in being able to marshal and expand the power of technology to create new knowledge, and in expanding human capacity and productivity (p. 17).

At the same time, societies now face global challenges that must be addressed: large scale poverty; increasingly virulant communicable diseases; climate change; food and water shortages; lack of energy security; and mass migration. Thus, the disruption faced by contemporary society is not just digital but social, political, environmental and economic. The outcomes of these different forms of disruption have consequences for well-being and financial prosperity at personal, national and global levels. As a consequence, questions related to social justice, values of freedom and the health of the very fabric of our societies are now topics of vigorous debate on the global political stage.

✉ Katja Maass
katja.maass@ph-freiburg.de

1 International Centre of STEM Education, University of Education Freiburg, Freiburg im Breisgau, Germany

2 Australian Catholic University, Brisbane, Australia

3 University of Jaen, Jaén, Spain

4 University of Limerick, Limerick, Ireland

In response to these parallel but interconnected issues, stakeholders from different sectors, such as education, policy and business are emphasizing the urgent need to identify and improve the competencies young people will require to meet the demands of their futures—personal, civic and workplace. The European Union, for example, "encourages Member States to better prepare people for changing labour markets and active citizenship in more diverse, mobile, digital and global societies and to develop learning at all stages of life" (European Commission 2016, p. 4). These competencies have been identified as lifelong learning competencies (e.g., Organisation for Economic Co-operation and Development [OECD] 2005), key competencies (e.g., European Commission 2018a) and twenty-first century skills (e.g., NSTA 2011). While there is considerable acceptance of the need to develop the skills of young people in preparation for all aspects of twenty-first century life, there is limited practical advice to teachers about how to promote these capabilities.

One way that has been identified to achieve this goal is through investment in the development of science, technology, engineering and mathematics (STEM) competencies in order to improve the personal scientific literacy of citizens and increase international competitiveness, through greater engagement with STEM based industries, in a world economy now driven by innovation (European Commission 2016). While business, in particular, points to the importance of responding to the needs of labour markets now and even more dramatically in the future (cf. Archer et al. 2013; STEM Alliance 2017), it is also increasingly recognized that Science, Technology, Engineering, and Mathematics (STEM) education is an essential foundation for responsible citizenship and the ethical custodianship of our planet (e.g., Bybee 2010; European Commission 2018a, b; STEM Alliance. 2017). The latest programme for international student assessment (PISA) results, however, show that even in economically mature countries such as those in Europe, and the USA and Australia, approximately 20% of students lack skills sufficient for these purposes in mathematics or science (OECD 2016). These results are particularly concerning as, in many cases, they have settled at approximately this level in the long term, or worse, are continuing to increase. For example, between 2012 and 2015, the trend for the EU as a whole does not appear likely to meet the 2020 maximum benchmark of 15%, with mathematics remaining stable around 22% and science worsening from 16.6–20.6% (European Commission 2018a). This trend serves to highlight the urgent need for action in relation to STEM education.

Arguments placing workforce and economic competitiveness at the front of the STEM movement, at the expense of other aspects of human and societal development, are attracting increasing criticism (e.g., Zouda 2018). This discussion has contributed to the identification of the challenges associated with the development of both policy and action needed to promote effective STEM education, including how to promote a well-informed and critical citizenship. In this paper, we emphasize how STEM education in general, and mathematics education in particular, can contribute to preparing individuals better for twenty-first century challenges, including dealing with socio-scientific issues and responsible research and innovation in the search for societies that are ethically acceptable, socially desirable, and sustainable.

Currently, there is no widely accepted agreement on whether STEM education refers to the promotion of knowledge within individual subjects or to an interdisciplinary approach to instruction (see Sect. 2). This debate continues to take place against a context in which it is well understood that most applications of STEM in real-life and in the world of work are interdisciplinary in nature. This 'reality' provides a strong argument for considering interdisciplinary or integrated approaches to STEM education. Such an approach will need to recognize the interdisciplinary connection of the four STEM subjects as well as how STEM can be linked to other subjects, as Hazelkorn et al. (2015) comment, as follows: "Greater attention should be given to the value of all disciplines and how inter-disciplinarity (STEAM [Science, Technology, Engineering, Art, Mathematics] rather than STEM) can contribute to our understanding and knowledge of scientific principles and solve societal challenges" (p. 9). Thus, consistently with this argument, in this paper we interpret STEM education as interdisciplinary in nature.

While interdisplinary approaches offer the advantage of presenting the knowledge utilization as integrated rather than discipline-specific when solving real world problems, it is yet to be established that this is beneficial to all students. There is evidence, however, that interdisciplinary approaches can support the learning of low achieving students (European Commission 2013, p. 5).

There are also challenges inherent in conducting research into STEM education. First, STEM education is a very new field within which understandings of what constitutes effective practice, and how this can be supported through curriculum, are still in their infancy. As English (2016a) points out, "Research that targets STEM integration is an embryonic field with respect to advancing curriculum development and various student outcomes" (p. 2). Second, there is as yet no clear understanding of the role of individual subjects within integrated STEM. To date, the main focus appears to have been on science, which has led to some commentators pointing to the under-representation of the other disciplines, especially in the case of mathematics (e.g., Fitzallen 2015). Although interdisciplinary approaches to STEM teaching are sometimes included in the curriculum (see, e.g., Ministerium für Kultus, Jugend und Sport, Baden-Württemberg 2016), implementation as day-to-day teaching practice remains difficult. The difficulty associated with integration appears to be related to three primary factors: first, the process of integration in itself is challenging (Honey et al. 2014); second, teachers may lack confidence or specific discipline knowledge—the "capacity gap in STEM teaching" (e.g., Australian Council of Learned Academies (ACOLA) 2013); and third, preparation in initial teacher education programs has tended to focus on one, or at most two, areas of disciplinary expertise (and usually not mathematics), which has the potential to narrow perspectives on the importance of other disciplines (Venville et al. 2002).

The purpose of this paper is to address the second of these challenges by advancing understanding of the role of mathematics within integrated approaches to STEM education.. Thus, the research questions that guide the direction of our argument in this paper, and the whole special issue are as follows:

What is the role of mathematics within interdisciplinary STEM approaches?

How can we further advance the role of mathematics within interdisciplinary STEM approaches?

Detailed responses to the first and second research questions are presented in Sects. 2 and 3 respectively. Below, by way of overview, we examine three perspectives on the role of mathematics within interdisciplinary STEM approaches and how it may be advanced.

First, we look for links between interdisciplinary STEM teaching and the notion of twenty-first century skills. Supporting students in developing twenty-first century skills or key competencies is also a significant policy agenda internationally. Key competencies are seen as "those which all individuals need for personal fulfilment and development, employability, social inclusion and active citizenship" (European Commission 2018b, p. 1) and are seen as vital underpinnings for both young people and adults. Common to both STEM education and twenty-first century skills are capabilities such as critical thinking, problem solving and analytical skills (e.g., European Commission 2018b), each of which can be enhanced through the use of mathematics as a source of evidence.

Second, we explore the vital role of mathematics in understanding and making predictions about the world—a fundamental aim of STEM education. Thus, experiencing real life applications and modelling real world situations mathematically is central to effective STEM education (e.g., for quantitative reasoning, for using graphs and formulas to describe how things grow, move and so on, using mathematical models to describe phenomena). The importance of STEM approaches based on applications of the relevant disciplines within real world contexts is also emphasized within policy documents (see for example, European Commission 2013). This approach, done well, is seen as more likely to engage the entire student population, from low to high-achieving students, as it makes STEM more tangible and relevant to their lives and the world of work. This in turn can raise their interest in STEM related careers.

Third, we argue for the vital role of mathematics within STEM with regards to responsible citizenship. Education for responsible citizenship is also a significant policy agenda internationally that has a history of being linked to science education and applications of science (Bybee 2010; Hazelkorn et al. 2015). This agenda emphasizes the necessity of ensuring that young people acquire social, civic and intercultural competencies by promoting democratic values and fundamental rights, social inclusion and active citizenship, and by enhancing critical thinking and media literacy (Eurydice 2016). As responsible citizens, students need to be able to understand discussion on global and societal challenges (such as climate change, food provision, waste reduction) and to make informed decisions (Maass et al. 2019), which are also subject to ethical, moral, cultural and economic influences. Many of these issues are reported and discussed in the media in mathematical formats that involve numerical data, graphical representations or symbols—all of which require competence with mathematics.

In the following, we first discuss the nature of STEM education in more detail and elaborate more explicitly on our three suggested ways of advancing the role of mathematics within STEM education. In doing so, we not only highlight why mathematics is important, but also what mathematics has to offer in promoting understanding of global challenges related to STEM and twenty-first century skills.

2 The nature of STEM education

In the 1990s the National Science Foundation united science, technology, Engineering and mathematics to coin the acronym STEM. In actuality, the disciplines were initially unified using the acronym SMET (Science Mathematics Engineering and Technology) but this was eventually changed to STEM for phonetic reasons. This initiative was in response to the perceived need for improving STEM competencies of young people and adults in order to maintain national economic competitiveness. This was also a strategic approach adopted by scientists, technologists, engineers and mathematicians to create a stronger unified political voice. An early outcome of this approach was the creation of the the first Degree in STEM Education in 2005 (Martín-Páez et al. 2019) by the Virginia Technology University as a way of highlighting the role of education in ensuring the delivery of appropriate STEM training (Caprile et al. 2015). From this point the acronym started to gain increasing international attention within the field of education. While this term is now firmly established, there have been continuing calls for the need to clarify what is meant by STEM education and to reach some consensus about what characterizes this approach in contrast to the teaching and learning of the individual disciplines. In response to this quandary, Akerson et al. (2018) argued that STEM is a socially constructed label that emerged in response to economic and global pressure, while advocating that it should be addressed through its composite disciplines at the same time as identifying areas where they could influence and build on each other. In order to make the case for STEM integration, Akerson et al. (2018), raise the following questions: *Does calling what we do STEM significantly change what we are doing?* and *Does calling what we teach STEM change what and how we are teaching?*

Taking a different perspective, Zollman (2012) suggests that the integration of the individual disciplines is important for developing students' STEM literacy—the ability to understand and apply content from the STEM disciplines to solve real problems. To do so effectively requires that interdisciplinary approaches be employed in order to orient students towards problem-based learning. Consistent with this view, Baran et al. (2016) also see the need to address STEM education through interdisciplinary teaching methods that integrate STEM subjects, a position Martín-Páez et al. (2019) also endorse in defining "STEM learning as the integration of a number of conceptual, procedural and attitudinal contents via a group of STEM skills for the application of ideas or the solving of interdisciplinary problems in real contexts" (p. 5). In the same vein, the California Department of Education (2014) highlights that STEM education:

> …is much more than a convenient integration of science, technology, engineering and mathematics; it is an interdisciplinary and applied approach that is coupled with real-world, problem-based learning. STEM education integrates the four disciplines through cohesive and active teaching and learning approaches. We now understand that these subjects cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce. (p.11).

This commentary, however, does not address how interdisciplinary approaches to STEM teaching can be achieved, an issue we now address by looking at recommendations that have emerged from general discussion about STEM education as well as the specific role of mathematics within STEM.

There is broad acceptance that the use of real world contexts is central to STEM education as an important aim is better to prepare individuals to deal with complex real world problems that require the application of knowledge and skills from different disciplines in a critical and creative way. This aim leads to contextualized and situated learning that enhances understanding and meaningful acquisition of new concepts and skills (Sevian et al. 2018). Thus, STEM education should establish connections between the academic contexts in which subjects are taught and the real contexts in which we live. At the same time, an effective STEM education should offer opportunities to address complex phenomena or situations via tasks that require students to use knowledge and skills from multiple disciplines (Erdogan et al. 2016; National Academy of Sciences 2014; Martín-Páez et al. 2019). In an aligned but different view, Nikitina (2006) suggests that students should examine specific concepts from the different disiplines while connecting to real world applications. For example, the relationship between surface area and volume could be explored from a physical chemistry perspective by examining how sugar dissolves in coffee, or from a biological view, how penguins in cold areas are bigger than in warmer areas. Both perspectives should address the underpinning mathematic concept of surface area to volume ratio.

As discussed earlier in this paper, the term STEM is used to signify that there is potential to work with the content (concepts, procedures, tools, skills, values) of several individual STEM disciplines in combination, in order to address a real world problem. However, in spite of arguments in favour of integrated STEM approaches, some studies within STEM education show that this integration is not always the case. For example, Martín-Páez et al. (2019) found that 30% of publications under this label were not really integrated activities and were sometimes better identified with one specific discipline. Such studies add weight to the argument promulgated by some authors that the word might be used simply to sound more innovative or to increase funding opportunities (Akerson et al. 2018). Based on these considerations many papers (e.g., Satchwell and Loepp 2002; Shahali et al. 2017; Stump et al. 2016; Thibaut et al. 2018) stress the importance of genuinely trying to integrate at lease two disciplines.

Previous studies have demonstrated that students do not spontaneously integrate knowledge and practices from different disciplines (Pearson 2017; Thibaut et al. 2018). Therefore, if an aim of STEM education is to support students in connecting key ideas across disciplines, it is vital that the origin of knowledge from different disciplines be made explicit. At the same time, some have sounded a warning that attempts to bring together different disciplines to bear on a real world problem should remain meaningful and purposeful and that more integration is not necessarily better (e.g., Guzey et al. 2016a, b; Pearson 2017; Thibaut et al. 2018). Thus, as Thibaut et al. (2018) emphasizes, "integrated STEM education should also focus on learning goals and standards in the individual STEM subjects, so as not to inadvertently undermine student learning in those subjects" (p. 6).

While STEM education has the dual aims of promoting individual discipline knowledge and practices and, at the same time, using integrated approaches when addressing real world problems, the balance of how different disciplines are represented within STEM activities has emerged as a significant issue within the field (English 2016b). This issue has been acknowledged by the National Academy of Sciences (2014), who observe that the integration of the STEM disciplines usually results in the dominance of one discipline over another. For example, engineering appears as a dominant discipline in the study of robotics (e.g., Barak and Assal 2018), engineering design (e.g., Shahali et al. 2017) and engineering-based problems (e.g., Toma and Greca 2018). In contrast, mathematics appears to be a discipline that, while acknowledged as an underpinning of science, technology and engineering, is often underrepresented in integrated STEM based activities (Martín-Páez et al. 2019). To illustrate this position, English (2016a) provides an analysis of the 141 papers presented at an important conference in the STEM field, which took place in July 2014 in Vancouver (https://stem2014.ubc.ca/). During the conference 45% of the papers were about science, 12% about technology, 9% about engineering, 16% about mathematics and 18% were classified as "general", with several papers in this category addressing two or more of the STEM disciplines. This situation only serves to reinforce the perception that the role of mathematics within the field is under-represented and must be strengthened (e.g., English 2016b; Fitzallen 2015; Marginson et al. 2013) as mathematics plays a major role in all STEM fields.

3 Advancing the role of mathematics in interdisciplinary STEM education

As outlined earlier, we see three possibilities for advancing the role of mathematics within the field of STEM. These involve connecting learning in STEM with the following: (1) the acquisition of twenty-first century skills; (2) meaningful inclusion of mathematical modelling in school education; and (3) education for responsible citizenship. In the following we elaborate on these aspects in more detail.

3.1 The notion of twenty-first century skills

The notion of twenty-first century skills has developed out of a need to respond to economic and societal problems that are emerging due to the rapid pace of change in our world. Engaging higher-order learning skills and connecting to different disciplinary knowledge is seen as key in responding to twenty-first century problems (National Research Council 2012).

Existing alongside frameworks that have been developed to identify twenty-first century competencies are different approaches to their incorporation into educational programs. Such approaches propose that twenty-first century competencies can be: (a) added to the already existing curriculum as new subjects or as new content within traditional subjects; (b) integrated as cross-curricular competencies that both underpin school subjects and place emphasis on the acquisition of wider key competencies; or (c) part of a new curriculum in which the traditional structure of school subjects is transformed and schools are regarded as learning organizations (Gordon et al. 2009; Voogt and Roblin 2012).

Recent studies of PISA results (e.g., McConney et al. 2014) have raised concern about what is emphasized in existing curricula and teaching practices. Moreover, the interpretation of PISA results in some countries has resulted in a greater emphasis on basic literacy and numeracy skills and high stakes testing practices (Au 2011). These responses suggest that policy-makers do not seem to link the findings from the PISA study to the need to restructure curricula in order to realize twenty-first century competencies (Voogt and Roblin 2012).

In response to these concerns, bodies such as the National Science Teacher Association (NSTA 2011) have acknowledged the need for and importance of twenty-first century skills and their promotion within the teaching of core disciplines. In doing so, NSTA also recognizes the key role of the following: core subject knowledge; learning and innovation skills; information, media and technology skills; life and career skills; adaptability; complex communication/social skills; non-routine problem solving; self management/self-development; and systems thinking. Consistent with the position of NSTA, the OECD has proposed that twenty-first century skills, rather than being distinct from the traditional school curriculum, are instead highly relevant to effective learning in all knowledge domains—including mathematics (Schleicher 2012). Thus, we argue for the promotion of twenty-first century skills through a high quality education that consistently integrates the acquisition of content knowledge and the development of key skills and values in a balanced way.

While there is no definitive catalogue of so-called twenty-first century competencies, core features common to different versions of this construct can be clearly identified. For example, the Assessment and Teaching of twenty-first century skills project (ATC21S 2009) is based on four broad categories:

1. Ways of thinking: Creativity, critical thinking, problem-solving, decision-making and learning.
2. Ways of working: communication and collaboration.
3. Tools for working: information and communications technology (ICT) and information literacy.
4. Skills for living in the world: citizenship, life and career and personal and social responsibility.

Alternatively, a recent survey of OECD countries conceptualised twenty-first century skills as activities grouped within three dimensions:

1. Information: Searching, selecting, evaluating and organising information; transforming information through Creativity, problem solving and decision making.
2. Communication: Collaboration and critical thinking.
3. Ethics and social impact: Developing social responsibility and awareness of social impact.

A further example can be found in the *Partnership for 21st century skills* (P21) *framework* that places an emphasis on the acquisition of key competencies listed under the following three categories:

1. **Learning and innovation skills** critical thinking and problem solving; communication and collaboration; creativity and innovation.
2. **Information, media and technology skills** information literacy, media literacy, ICT (information, communications & technology) literacy.
3. **Life & career skills** flexibility and adaptability; initiative and self-direction; social and cross-cultural skills; productivity & accountability; leadership and responsibility.

Further conceptualisations of twenty-first century skills can be found in the JRC science policy report, the entrepreneurship competence framework and the DigComp 2.0 Frameork of the EU.

Thus, adaptable and flexible critical thinking and reasoning, the capacity to search for and interpret information, to communicate and collaborate with others within the contexts of cultural, and social and ethical obligations, are key aspects of twenty-first century skills.

In the discussion about how mathematics education might prepare students for the society of the future, Gravemeijer et al. (2017) argue that twenty-first century skills should be adopted as goals of mathematics education. These authors pay special consideration to the position of mathematics in a digitized society and to specific mathematical forms of argumentation and communication to ask the question of what mathematics we should teach, when digital technologies can do most of the mathematics we teach at the moment. The authors argue that the twenty-first century digitalization of society requires a focus on mathematical competencies that complement the work of computers. In this respect, individuals must be able to understand the mathematics underlying the mathematical work computers take over. The authors highlight that students need conceptual understanding on a generic level to make connections to the key ideas underlying the relevant mathematics.

Gravemeijer et al. (2017) particularly recommend using real world contexts for supporting the development of twenty-first century skills. In this respect, they discuss a four-step procedure: (1) recognizing where mathematics is applicable; (2) translating practical problems into mathematical problems; (3) solving the mathematical problem; and (4) interpreting and evaluating the outcomes. This proceedure is very close to the modelling cycle as discussed in Sect. 3.2.

English (2016b) notes the potential for advancing the role of mathematics within STEM education through attention to twenty-first century skills. She argues that twenty-first century skills should be taught in all core subjects and thus can provide a link between all STEM subjects.

3.2 Mathematical Modelling

English (2016a, b) makes strong connections among interdisciplinary STEM education, modelling real world situations, and twenty-first century skills. “Modelling is a powerful vehicle for bringing features of twenty-first century problems into the mathematics classroom” (English 2016b, p. 10). These connections can be made for two main reasons: (1) real world problems for which mathematical modelling is a valid approach are most often interdisciplinary in nature (see, e.g., Mascil 2013; Compass 2010); and (2) the competencies needed to carry out a modelling process intersect with twenty-first century skills to a large extent. To elaborate on the second reason, we present the theoretical background of mathematical modelling.

Mathematical modelling has a variety of different definitions (Kaiser and Sriraman 2006). We define mathematical modelling as the solving of a problem from the real world by employing a cyclic process (Niss et al. 2007). Based on Blum and Leiss (2007), we conceptualize the following steps of the modelling process: (1) understanding the real world situation (situation model); (2) making assumptions and simplifying the situation model (real model); (3) mathematizing the real model (construction of a mathematical model); (4) working within the mathematical model (mathematical solution); (5) interpreting the solution; and (6) validating the interpreted solution. The modelling process may be represented by the following diagram (Fig. 1).

An understanding of modelling competencies and skills is closely related to the definition of the modelling process. Different perspectives on modelling processes may therefore imply different views on modelling competencies and skills. Several studies refer to a number of sub-competencies that are part of modelling competencies. For example, Blomhøj and Jensen (2007) differentiate between the parts of the modelling process to be carried out, the mathematics to be used and the context in which students have to work. The empirical study by Maass (2006) produced evidence to suggest the following:

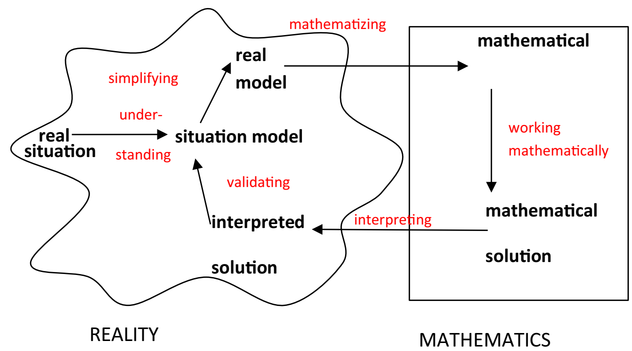

Fig. 1 An idealized scheme of the modelling process (according to Blum and Leiss 2007, p. 225)

- Students need competencies to carry out the single steps of the modelling process.
- There are also further sub-competencies which do not belong to a specific modelling step, but are needed throughout the whole modelling process.

For carrying out the single steps of the modelling process, competencies such as critical thinking, problem solving, teamwork, communication and negotiation skills, analytical skills and creativity—which are all twenty-first century skills—are indispensable (English 2016a).

Additionally, research indicates that more sophisticated levels of self-awareness and explicitness about strategies are associated with greater success in problem-solving (Kapa 2001; Schneider and Artelt 2010). Consequently, instructional interventions have been developed and implemented to enhance metacognition so as to improve modelling competencies (e.g., Goos et al. 2002; Maass 2004; Stillman and Galbraith 1998) and have become an object of systematic study (Vorhölter 2018). Metacognition plays "a critical role in successful problem-solving and modelling" (English 2016b, p. 9). Aligned with metacognitive processes is the notion of *anticipation*—looking forward in the modeling process to take full advantage of a problem solvers' cognitive and available digital and physical resources (Geiger et al. 2018).

Research has also shown that modelling activities have a positive impact on students' competence in applying mathematics to complex situations (see, e.g., Maass 2007), on students' attitudes towards mathematics (e.g., Mischo and Maass 2013) and on mathematical competencies (e.g., English and Watson 2018).

The implementation of modelling in mathematics lessons has the potential to motivate students to develop their talents fully (European Commission 2013; Kaiser 1995), support an appropriate view on mathematics (Kaiser 1995), foster mathematical and scientific literacy (Steen 2001) and develop an in-depth understanding of mathematical content (English 2016b; Gravemeijer 2007). Last but not least, mathematical modelling is also linked with students' development of twenty-first century skills (Ärlebäck and Doerr 2018; English 2016a) and education for democracy (Artigue and Blomhoej 2013).

The definition of modelling given above and the link with twenty-first century competencies, emphasize the role of problems taken from the real world. By their very nature these problems are often interdisciplinary and thereby support the advancement of mathematics education within the STEM field. Problems from the real world often also include issues and decisions that have ethical, moral, social or cultural dimensions and thus are suitable vehicles for promoting the type of discussion that is central to the development of responsible citizenship.

3.3 Responsible citizenship education

The third approach we have identified for promoting the role of mathematics within interdisciplinary STEM instruction is through connection to education for responsible citizenship. The need to link individual subjects and responsible citizenship education is a common recommendation in policy documents (e.g., Eurydice 2016) and there have been a range of educational initiatives that aim to foster young people's social, civic and intercultural understandings and capabilities. One example is the European project MaSDiV, which links citizenship education to mathematics and science teaching (https://icse.eu/international-projects/masdiv/). These understandings and capabilities are vital if young people, as they move into adulthood, are to be equipped to make the moral and ethical decisions and judgements needed to ensure a sustainable, equitable and peaceful transition to a world of rapid technological, social and economic change.

While arguments promoting the STEM agenda are mostly connected to personal financial security and national economic prosperity, proponents of twenty-first century skills recognize the need to support the development of socially sensitive and responsible citizens. For example, it is argued in the framework for twenty-first century learning (Partnership for 21st Century Skills 2002) that global awareness, civic literacy and environmental literacy are interdisciplinary themes that should be woven into key subjects. From this perspective, the skills necessary to negotiate life in the twenty-first century are inclusive of the need to be ethical, accountable and socially responsible.

While, to date, a focus on the role of responsible citizens and consequently issues such as ethics, equity and social justice, has not been prominent in promotion of STEM education or in the marketing of STEM careers (Mildenhall et al. 2019), there is increasing advocacy internationally, particularly within science education, for the need to explore the ethical issues that almost always accompany innovation and invention. As UNESCO (2005) points out:

> Many of the most important ethical predicaments the world community is facing today arise in connection with science, in scientific research and in the development and applications of new technologies… (p. 3).

This commentary makes it clear that scientific development is linked to ethical dilemmas such as those in which national economic or military priorities are balanced against social issues such as environmental sustainability, improving the lives of the oppressed and equity within and across societies (e.g., Steele et al. 2012; Steele 2016). Such dilemmas have motivated the development of teaching and learning programs within the natural sciences in which controversial contexts are deliberately selected in order to catalyse discussion involving ethical, moral, social or cultural aspects of the solutions to scientific problems, that is, socio-scientific issues (Maass et al. 2019).

There is also considerable history, within mathematics education, of advocacy for the use of mathematics to highlight and clarify issues related to responsible citizenship. For example, the ethno-mathematics and critical mathematics movements of the late 1990s (e.g., D'Ambrosio 1999; Skovsmose and Nielsen 1996) heralded a line of research enquiry that positioned mathematics as an essential tool for effective socially-conscious decision-making. In this view, the capacity to use mathematics when critically evaluating the arguments and propositions of authority is vital for the development of an ethical, moral and socially just citizenry. This perspective was extended by D'Ambrosio (1999), in the case of Brazil, to the importance of a mathematically critical citizenary to the defence of democracy.

Others, such as Zevenbergen (1995), have also emphasized the importance of social critique as a fundamental underpinning to responsible citizenship. In particular, she drew on Habermas' (1972) tripartite theory of knowledge to identify a role for mathematics in critical or emancipatory capabilities. Extending this line of argument, Ernest (2002) suggested that mathematics is a tool for social empowerment—an essential for making socially just decisions and judgements, that is, acting as a socially critical citizen:

> The empowered learner will not only be able to pose and solve mathematical questions (mathematical empowerment), but also will be able to understand and begin to answer important questions relating to a broad range of social uses and abuses of mathematics (social empowerment). Many of the issues involved will not seem primarily to be about mathematics, just as keeping up to date about current affairs from reading broadsheet newspapers is not primarily about literacy. Once mathematics becomes a 'thinking tool' for viewing the world critically, it will be contributing to both the political and social empowerment of the learner, and hopefully to the promotion of social justice and a better life for all. (p. 6).

The notion of mathematics as a tool for empowerment has become even more relevant in a world where data and information are so freely accessible. Steen (2001) foreshadowed the necessity for citizens to have the tools to make critical judgements and decisions in their personal lives in what he termed a "data drenched" world. This view has been reinforced by Jablonka (2015) and Frankenstein (2001), who describe how mathematics is used to manipulate or shape opinions in order to pursuade the public about social or political issues. Frankenstein (2001) further points out that it is those at the margins of society who are most susceptible to being disadvantaged because they often lack the capabilities necessary to detect bias in arguments promoted within the mass media or see how they could be taken advantage of during financial transactions.

The issue of equitable and just dealings in financial matters is also a theme taken up by Sawatzki (2013) in a study of 14 teachers and 300 Years 5 and 6 students in Australia. In this study, she examined students' responses to tasks designed as "financial dilemmas"—open-ended problems that featured both social and mathematical dimensions. These responses indicated that students' socio-economic background influenced their engagement with tasks. For example, in response to one task that required students from a low socio-economic area to develop and cost a plan for a group of friends to go to the movies, they expressed surprise at the prices of tickets and food. As a consequence, they proposed a lower cost alternative to the excursions by way of hiring a DVD and purchasing snacks from the supermarket.

There is, therefore, considerable advocacy for mathematics teaching and learning to play a role in the formation of responsible citizens through attention to critical reasoning and social critique during schooling. *But what is the role of mathematics education in supporting responsible citizenship education within the STEM area?*

Two approaches that have the potential to address this question are numeracy (also known as mathematical literacy in some international contexts) and socio-scientific issues. The numeracy approach focuses on the need to equip young people and adults with the knowledge and capabilities required to accommodate the mathematical demands of private and public life and to participate in society as informed, reflective and contributing citizens.. Addressing socio-scientific issues is an approach drawn from science and science education that seeks to raise awareness and find solutions to global issues such as sustainability and climate change.

3.3.1 Numeracy

Geiger et al. (2015b) have argued that the demands of the new, sophisticated problems encountered in the twenty-first century, many of which are related to the social and economical issues associated with innovations in STEM, require far more than the mastery of basic mathematical skills and procedures. To meet these demands, they have described a rich model of numeracy (also known as mathematical literacy in some international contexts) for the twenty-first century (Geiger et al. 2015b), in which mathematics is the foundation for evidence-based decision-making and judgement forming capabilities that are essential for participatory citizenship. In their view:

> Numeracy is a concept used to identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life, and to participate in society as informed, reflective, and contributing citizens. (p. 531)

The twenty-first century numeracy model (Geiger et al. 2015b) positions real world contexts, mathematical knowledge, dispositions towards the use of mathematics and competence with physical, representational and digital tools, as foundational dimensions for being numerate. Overarching and intertwined through these dimensions is an evaluative component—a critical orientation. This component brings the dimensions of the model to bear, in a coordinated fashion, on real world problems for the purpose of making evidence-based decisions and judgements. In the context of social engagement or societal issues, this means a numerate person has the capability to promulgate their own arguments or to challenge the positions or claims of others. A representation of the model appears in Fig. 2.

These critical capabilities are important attributes all citizens must appropriate in order to participate in the type of societal-forming decisions with which we are increasingly confronted. For example, Barwell (2013) argues that because mathematics is involved at every level of understanding climate change, including the description, prediction and communication of its consequences, there is urgent need for critical approaches to mathematics education that prepare current and future citizens to participate in a dialogue about how this phenomenon can best be managed. Consistent with this view, Jablonka (2003) points to the socio-political aspect of mathematics usage by stakeholders within societies who seek to shape opinions and views of citizens. Thus, it is important that all citizens posses the critical capability to engage with dialogue based on quantitative evidence in which arguments are developed that support or challenge the proposed policy directions of authority (D'Ambrosio 2003; Geiger et al. 2015a; Skovsmose 1994). The need for citizens to engage in informed and critical discussion about

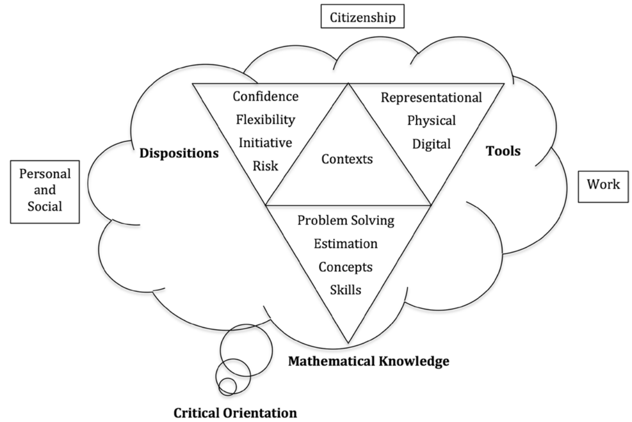

Fig. 2 A model of numeracy for the twenty-first century (Goos et al. 2014, p. 84)

the consequences of political decisions related to the natural environment and associated economic and societal impact highlight the vital role of mathematics within STEM related societal challenges.

The role of mathematics in empowering individuals to make critically reasoned decisions and judgements is being increasingly recognized as an essential capacity in being numerate (e.g., Ernest 2002), as evidenced by, for example, recent definitions of numeracy/mathematical literacy in the Programme of International Student Assessment (PISA) and the Programme for International Assessment of Adult Competencies (PIAAC) (Tout et al. 2017). This recognition provides a connection to scientific literacy and the role of socio-scientific issues in science teaching and learning.

3.3.2 Socio-Scientific Issues

The often controversial nature of real world contexts that are relevant to society, and the necessity of engaging in dialogue, discussion and debate, including moral, ethical or social reasoning, is very much emphasized within approaches to teaching and learning about socio-scientific issues (Zeidler and Nichols 2009). This concept stems from science education but has been also linked to mathematical modelling (Maass et al. 2019).

Within socio-scientific issues, people often have to deal with incomplete information because of conflicting or incomplete scientific evidence and incomplete reporting. This aspect often involves a cost-benefit analysis in which risk interacts with ethical reasoning (Ratcliffe and Grace 2003). Consequently, such contexts especially serve the purpose of educating for scientific citizenship (Owen et al. 2009).

Research in science education has shown that socio-scientific issues can be used as contexts for learning scientific content (Applebaum et al. 2006; Walker 2003; Zohar and

Nemet 2002) and for understanding the nature of science and for citizenship education (Herman et al. 2018; Radakovic 2015; Sadler et al. 2007). An example of a socio-scientific issue would be the question of whether wind power plants should be set up in a particular area. Whilst there may be sufficient scientific evidence that the new venture will produce renewable energy, other issues such as the endangerment of birds, drop shadows or infra sound, lead to debates that influence the decisions on approval and/or the conditions under which a project might be permitted to go ahead. Naturally, discussing this issue not only involves science but also mathematics. This is also the case for many other socio-scientific issues, as they are of an interdisciplinary nature.

The connection between socio-scientific issues and mathematics prompted Maass et al. (2019) to suggest an extension to the modelling cycle that supports decision-making in relation to potentially controversial real world applications, by taking into account moral, ethical or social dimensions. This approach thus links socio-scientific issues to mathematical modelling and responsible citizenship education. Consequently, looking at (interdisciplinary) socio-scientific issues from the modelling perspective can help to advance the role of mathematics in the STEM area.

In summary, both numeracy education and the investigation of socio-scientific issues via a modelling approach are avenues for responsible citizenship education. Accordingly, approaching responsible citizenship education through one of these educational perspectives can contribute to advancing the role of mathematics in the STEM field.

4 What needs to be done in the future?

In Sect. 3 we discussed three possible areas in which to advance the role of mathematics in interdisciplinary STEM teaching: (1) twenty-first century skills; (2) mathematical modelling; and (3) responsible citizenship education. In this section we consider future directions for research in order to advance the field, in relation to the following questions:

1. What issues are emerging in relation to the role of mathematics and twenty-first century skills?
2. What issues are emerging in relation to connecting mathematical modelling to STEM education?
3. What issues are emerging in relation to connecting mathematics to responsible citizenship?

There is ample evidence that mathematics education researchers are beginning to address questions of curriculum and pedagogy in relation to development of school and university students'twenty-first century skills through attention to how pre-service and practising teachers can be prepared to incorporate twenty-first century skills into their planning, task design and selection of resources. However, there seems to have been limited research into the question of assessment of twenty-first century skills in the mathematics domain of STEM. This is surprising, given the powerful impact of the results of international assessment programmes such as PISA on the design of mathematics curricula for the preparation of mathematics teachers.

Ideas about competency developed by Niss (2018) shaped all the PISA assessment frameworks for mathematical literacy between 2000 and 2012, by "underpinning and developing the notion(s) of mathematical literacy'' (p. 75). While Niss's view of competency was firmly grounded in the discipline of mathematics (Niss and Højgaard 2019) and thus represents only a subset of a broader capability set, it does point to the the importance of the role of mathematics in twenty-first century skills.

Conversely, the need to consider assessment of twenty-first century skills is also highlighted within PISA. The draft mathematics framework for PISA 2021 (OECD 2018) organises the domain in relation to four aspects: mathematical reasoning and processes of mathematical modelling; mathematics content knowledge; the relationship between mathematical literacy and "so-called twenty-first Century skills" (p. 5); and the contexts in which students face mathematical challenges. It is argued within this framework that mathematical reasoning "contributes to the development of a select set of twenty-first century skills"(p. 9). For example, the PISA 2012 framework claims that effective instruction in mathematics, and especially mathematical reasoning, can equip students with the kind of critical thinking skills necessary for detecting fraudulent arguments and contradictions in the information that one encounters in everyday life.

The following eight twenty-first century skills were recommended for inclusion in the PISA 2021 mathematics framework on the basis of their relevance to the discipline of mathematics:

- Critical thinking;
- Creativity;
- Research and inquiry;
- Self-direction, initiative, persistence;
- Information use;
- Systems thinking;
- Communication;
- Reflection.

The PISA 2021 mathematics assessment will include items that sample all the major categories of content, mathematical reasoning and problem solving processes, context, and—for the first time—twenty-first century skills. It is proposed within this framework that the PISA background questionnaires exploring student and school context will also provide insight into how mathematics can contribute to the development of twenty-first century skills (Organisation for Economic Cooperation and Development (OECD) 2018). This result might come into effect through items related to teaching practices, learning time, and curriculum models, through data gathered on the enacted curriculum in mathematics classrooms.

The PISA 2021 assessment of mathematical literacy thus offers new opportunities for large scale investigation of how mathematics instruction might be contributing to development of students' twenty-first century skills that are particularly relevant to mathematics. Such a major international initiative has potential to stimulate renewed research interest in assessment of twenty-first century skills, not only via standardised tests but also through teachers' informal collection of assessment evidence in their own classrooms. Because of the potential connection to mathematics instruction, PISA 2021 might also stimulate research and developments in the area of textbook and curriculum design.

The interdisciplinary nature of mathematical modelling provides clear opportunities to connect mathematics to the goal of STEM education—learning how to solve problems in the real world through the application of knowledge from science, technology, engineering and mathematics. There are a number of ways in which mathematical modelling can be introduced as the means of addressing real world problems, for example, Fermi-Problems are discussed by Ärlebäck and Albarracín (2019) as a means to connect modelling to twenty-first century skills and STEM education. Looking more broadly, Maass and Engeln (2019) consider connections to the world of work as a specific context for mathematical modelling. Others, such as Stillman et al. (2013), point to the potential for mathematical modelling for bringing contexts into the classroom that require the application of a socio-critical perspective—which necessitate discussion about ethical and socially responsible conduct, a central tenet of twenty-first century skills and STEM education.

Despite these and other arguments (e.g., English 2016a) for mathematical modelling to act as a vehicle for promoting the role of mathematics in the STEM field, there appears to be limited research with a focus on this opportunity (cf. e.g. Kaiser et al. 2011; Schukajlow et al. 2018; Kaiser et al. 2011; Stillman et al. 2017). Thus, there remain many avenues for investigation; for example, how can a focus on mathematics be maintained when exploring an interdisciplinary problem based in STEM through mathematical modelling? What do teaching approaches that shed light on all STEM aspects look like? What professional learning is needed to support teachers' design and implementation of effective interdisciplinary STEM tasks that build connentions between all constituent disciplines? Consequently, there remain immense possibilities for research into meaningful connections between STEM education and mathematical modelling.

We have also considered in this issue the important role of mathematics in developing responsible citizenship and the connection of this responsiblity to STEM education. Papers by Maass et al. (2019) and Nicol et al. (2019) point to the need for these issues to be addressed in initial teacher education and ongoing professional learning. However, there remain additional challenges to building connections between mathematics and this aspect of STEM education. Forgasz et al. (2015), for example, point out that mathematics teachers appear to be reluctant to take on the role of contributing to citizenship education:

> There are few situations in real life that are devoid of ethical and moral dilemmas and concerns. We are concerned, however, that mathematics teachers generally steer clear of the controversies inherent in many of the contexts they select for students to engage in problem solving, believing that this is the purview of other disciplines in the school curriculum, or are better dealt with at home. With respect to ethical considerations, this level of avoidance may be exacerbated. (p. 148).

According to Forgasz et al., this reluctance exists despite positive encouragement from national curriculum documents within their educational context. This example highlights a need to support teachers of mathematics to engage with real world issues they see as potentially controversial or divisive, but which may assist the preparation of their students for responsible citizenship. The first step in providing this support is to challenge the belief that mathematics is a value neutral discipline that is at odds with the view that every language of a culture, inclusive of mathematics, has a social and ethical dimension that must be accommodated (cf. e.g., Grigutsch et al. 1998). This aspect is most clearly played out when mathematics is utilized in decision making processes that will impact on the happiness and wellbeing of others, or in the gravest situations, the health of our planet—a position that speaks to inclusiveness.

Additionally, teachers appear to find it difficult to design tasks that promote students' critical orientation. Geiger et al. (2015a), for example, pointed to the challenges experienced by teachers in a 3 year long research and development project aimed at supporting teachers to design tasks that required students to make decisions and judgements. Thus, while there may be support at policy and curriculum levels for instructional approaches that promote students' critical capabilities (see for example, Curriculum of Baden-Württemberg, Ministerium für Jugend et al. 2016), significant professional learning support will be needed if teachers are to be effective in adapting or designing tasks and pedagogies in which mathematics takes on a socio-critical role. This support should include evidence-based teaching materials for different educational levels in conjunction with assessment strategies aimed at determining changes in students' understanding of responsible citizenship, as well as their mathematical competence.

Additionally, the question of how much scaffolding should be provided when students engage with tasks directed at promoting responsible citizenship deserves further attention. As we have outlined in Sect. 3.2, metacognition plays an important role in the development of modelling competencies. What metacognitive scaffolding is needed when mentoring students to think critically, ethically and socially responsibly, is an area within mathematics/STEM education that might provide a fruitful direction for further research. As English and King (2015) point out, such support needs to be balanced in terms of establishing an understanding of core concepts and allowing students to apply this learning in ways they choose during problem solution (English 2016a, p. 9).

Marginson et al. (2013) and others (e.g., English 2016a; Fitzallen 2015) have pointed to the under-representation of mathematics in activities and practices used to promote STEM education. As Shaughnessy (2013) commented, the "M" in interdisciplinary STEM teaching must be made more "transparent and explicit." This seems to be particularly important as we cannot assume that all students will acknowledge the mathematics that is inherent in a particular problem (p. 324). Thus, more research is required into ways that may help students make the connections between mathematics and STEM more transparent and meaningful across disciplines. This research should include how such connections can be made at different grade levels and what support teachers may require to make the necessary interdisciplinary connections explicit. To do so will require research-based teaching resources, new pedagogical approaches and professional learning opportunities that contribute to mathematics teaching within interdisciplinary STEM units.

In this paper, we began by asking questions about the role of mathematics within interdisciplinary STEM education and how we can further advance the role of mathematics. A survey of the literature indicated that the emphasis on different subjects within the STEM field is not balanced and the role of mathematics, in particular, is typically understated. To address this issue, we discussed three areas, attention to which may advance the role of mathematics in the field of STEM education: (1) twenty-first century skills; (2) mathematical modelling; and (3) responsible citizenship education. Research in each of these areas in relation to STEM appears to provide important starting points in effectively advancing the role of mathematics in the field of STEM education.

5 Outlook on the journal issue

Papers in this volume provide different perspectives on connections between the field of STEM education and mathematics across various levels of the educational system.

The paper by van der Wal et al. (2019) Teaching strategies to foster Techno-mathematical Literacies in an innovative mathematics course for future engineers, specifically addresses the development of a particular twenty-first century skill, namely techno-mathematical literacy, in future engineers, thus it engages with the level of (tertiary) students. Duijzer et al. (2019) in their paper Supporting primary school students' reasoning about motion graphs through physical experiences, turn to another twenty-first century skill, that of reasoning. They also address students, in this case however, primary school students in general education. Primary school students and twenty-first century skills are also the focus of Miller's (2019) paper, supporting primary school students' reasoning about motion graphs through physical experiences. In her paper she investigates the connection between coding instruction and students' mathematical thinking in relation to identifying mathematical patterns and structures. Geiger's (2019) paper, *Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms*, describes an approach to the design and implementation of tasks that addresses the broad capabilities identified in discussion related to twenty-first century skills and the role of STEM education—offering insight into an approach for making mathematics explicit within STEM teaching and learning.

Bergsten and Frejd's (2019) paper *Preparing pre-service mathematics teachers for STEM education: An analysis of lesson proposals*, turns to the level of pre-service teachers and analyses their lesson proposals for innovative STEM activities in secondary mathematics classrooms drawing on a categorisation of twenty-first century skills. Beswick and Fraser (2019) turn to in-service teachers. In their paper, *Developing mathematics teachers' twenty-first century competence for teaching in STEM contexts*, they describe the process whereby the capabilities of expert teachers of mathematics and science are unpacked and made available to novice teachers.

Two of the papers focus in particular on modelling. The paper by Maass and Engeln, *Professional development on connections to the world of work in mathematics and science education*, presents a study in which the effects of a professional development course on connections to the world of work have been researched. Ärlebäck and Albarracín turn to Fermi-Problems and carry out a meta-analysis of other research papers on *The use and potential of Fermi problems in the STEM disciplines to support the development of twenty-first century competencies*.

Last but not least, the focus of two further papers is on education for responsible citizenship and on the developing related teaching competencies. In *Promoting active citizenship in mathematics teaching*, Maass et al. present a concept for a professional development course designed to support teachers in implementing citizenship education. Nicol et al.

investigate *Learning to Teach the M in/for STEM for Social Justice* for in-service teachers, pre-service teachers and teacher educators.

The whole collection of papers in this volume will contribute to a better understanding of how the role of mathematics can be advanced in the STEM area and will move the related discussion forward.

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