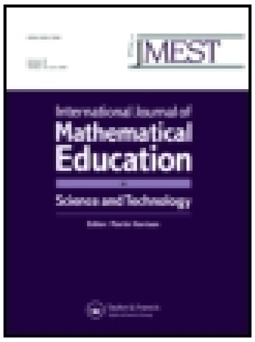


Taylor & Francis
Taylor & Francis Group

International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: http://www.tandfonline.com/loi/tmes20

Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples

Dubravka Glasnovic Gracin

To cite this article: Dubravka Glasnovic Gracin (2018): Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples, International Journal of Mathematical Education in Science and Technology, DOI: 10.1080/0020739X.2018.1431849

To link to this article: https://doi.org/10.1080/0020739X.2018.1431849

Published online: 08 Feb 2018.

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY, 2018
https://doi.org/10.1080/0020739X.2018.1431849

Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples

Dubravka Glasnovic Gracin

Faculty of Teacher Education, University of Zagreb, Zagreb, Croatia

ABSTRACT

Mathematics textbooks play a very important role in mathematics education and textbook tasks are used by students for practice to a large extent. Since the nature of the tasks may influence the way students think it is important that the textbooks provide a balance of a variety of tasks. The analyses of the requirements in textbook tasks contain the usual dimensions of content, cognitive demands, question type and contextual features. The aim of this study is to embed a new fifth dimension into the framework: mathematical activities. This addresses the question of what a student should do in a particular textbook task: to represent, to compute, to interpret or to use argumentation. The analysis encompassed more than 22,000 tasks from the most commonly used Croatian mathematics textbooks in the 6th, 7th and 8th grade. The results show that the textbooks do not provide a full range of task types. There is an emphasis on computation, while argumentation and interpretation activities, reflective thinking and open answer tasks are underrepresented. The study revealed that incorporating mathematical activities into the multidimensional framework of textbook tasks may help to better understand the opportunities to learn which are afforded students by using mathematics textbooks.

ARTICLE HISTORY
Received 7 September 2016

KEYWORDS
Content analysis; mathematics textbook; task types; textbook analysis; mathematical activities; textbook research

SUBJECT CLASSIFICATION CODE
97U20

1. Introduction

Textbooks play an important role in mathematics education [1,2] and are used to a great extent in mathematics classrooms [3–5]. They are regarded as artefacts that translate policy into pedagogy and represent a link between the intended and implemented curriculum; they reflect the potentially implemented curriculum [6]. Textbooks are considered to be the most frequently used resources in lesson preparation in some countries, even more so than the curriculum outlines [7]. Usiskin [8] describes the experience from the United States where the classroom practice is more oriented to what is written in the textbooks than what the intended curriculum says. These findings are in line with Howson's [9] thinking that textbooks are closer to classroom reality than curriculum outlines. Therefore, the results of textbook research have the potential to provide a broader and deeper picture of both curriculum requirements and classroom practices.

CONTACT Dubravka Glasnovic Gracin dubravka.glasnovic@ufzg.hr

The research results indicate that textbooks are used to a great extent by students as a source of tasks, particularly practice exercises [3,4,7]. Generally, 'the tasks are considered as devices for initiating activity' [10,p.238] and they create opportunities for learning mathematics [11]. Therefore, the nature of textbook tasks 'can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged' [12,p.525]. Thus, it is important that the textbooks and other curricular materials used in classrooms provide rich and worthwhile mathematical tasks.

Stenmark [13] suggests some possible features of rich tasks, they should be: essential (fit into the core of the curriculum); authentic (use processes appropriate to the discipline); rich (lead to other problems); engaging (thought-provoking); active (learners construct meaning and deepen understanding); feasible (appropriate for learners); equitable (develop thinking in a variety of styles), and open (have more than one answer or approach). Many of the features mentioned connect rich tasks with high cognitive demands. Cognitive demand refers to the different kinds of thinking required in a task: memorization, procedures without connections to concept, procedures with connections to concept and doing mathematics [14]. *Memorization* involves reproducing rules or definitions, and *procedures without connections* require conducting algorithms which 'have no connections to the concepts or meaning that underlie the procedure being used' [14,p.348]. *Procedures with connections* develop deeper levels of understanding of mathematical concepts and ideas. *Doing mathematics* means requiring complex and non-algorithmic thinking with considerable cognitive effort. Memorization and procedures without connections to concept (e.g. formulas) may be characterized as low-level tasks, while the other two high-level tasks encompass 'comprehension, interpretation, flexible application of knowledge and skills, and assembly of information from several different sources to accomplish work' [15,p.171].

The need for 'rich' tasks means that mathematics textbooks should provide tasks that will engage students and challenge them to reason, as well as influence the quality of instruction and provide opportunities for developing understanding. This does not mean that there should be no low-level tasks in the textbooks or instruction. Vincent and Stacey [16] discuss how it is important 'that students are presented with a balanced curriculum experience. The balance will need to be different for high and low achieving students, but all students need exposure to the full range of problem types' ([16 ,p.103]).

The various features of different types of mathematical tasks mean that their analysis is complex. The purpose of this paper is to analyze the features and diversity of textbook tasks according to a multidimensional tool, which, besides the usual dimensions including cognitive demands and context types, also consists of an activity dimension to find *what* should be done in a particular task.

The analysis encompassed Croatian mathematics textbooks in grades 6, 7 and 8. A brief outline of education and research on textbooks in Croatia is given below. Compulsory education in Croatia lasts for 8 years and is divided into grades 1–4 and 5–8. All pupils in compulsory education follow the same national educational program. All textbooks used in schools are approved by the state board. In Croatia, textbooks are traditionally bought by parents or local communities so students have their own copies of the textbooks the school has selected. The textbooks are brought to every mathematics lesson and used at home for homework. The previous study comprised a survey on the role of mathematics textbooks in grades 5–8. It involved nearly one thousand mathematics teachers, which is about half of the total number of mathematics teachers in grades 5–8 in Croatia. The findings indicate that mathematics textbooks are in use to a great extent, especially in teachers' preparation, in practice exercises for students and in their homework [7]. Other materials are used in classrooms, but not to such a large extent. Also, the results showed that the most important factor in teachers' choosing a textbook was the quality of textbook exercises.

Since textbook exercises greatly influence mathematics teaching, it is reasonable to pose questions about the nature of and demands in these exercises, whether they help to enhance mathematical understanding and to what extent they can be labelled as rich.

2. Literature review

Since mathematical tasks have great potential for challenging and engaging students [11], it is important to consider the different studies and frameworks designed and used for investigating textbook tasks.

2.1. Analyses of textbook exercises and examples

Textbook examples and particularly exercises are the most important source of textbook tasks. Since the worked examples and exercises are used to a large extent by students in the classroom or for homework [3], they surely influence the understanding of mathematical concepts and have the potential to challenge and engage students [11]. Keitel et al. [17] report that teachers find the quality and differentiation of the tasks to be the most important thing about textbooks. A similar result can be found in [7]. Most of the textbooks follow the structure of *rule-example-practice*. Love and Pimm [18] claim that the 'exposition – examples – exercises model' is the most common way of organizing text in mathematics textbooks (p.386). *Exposition* refers to the parts where the author presents the subject matter. *Examples* offer students a model to be implemented in the subsequent exercises. *Exercises* refer to the various tasks that students should do and thus be active readers of the text. Within this triad, the exercises part plays a very important role in mathematics education because mathematics is usually learned through different tasks [11]. Therefore, some textbook research has focused on the analysis of textbook exercises and problems from various interesting aspects [6,16,19–21].

Such research requires a well-based framework. Zhu and Fan [19] conducted a comparative study of textbook tasks in the US and China at the lower secondary grade level. They examined whether tasks are routine or non-routine, open or closed-ended, application or non-application, traditional or non-traditional. The results showed that routine, closed and traditional exercises without relevance to real-world situations dominate in both countries. However, 'more application problems, especially authentic ones, were found in the US books' ([19, p.621]).

Another comparative study of Chinese and US textbooks was conducted by Li [20]. Li compared addition and subtraction of integers exercises in several American and Chinese mathematics textbooks in the 7th grade. For this purpose, a three-dimensional (3D) framework was developed. It encompassed mathematical features (single or multiple computation procedure required), contextual features (pure mathematical context or illustrative context/story), and performance requirements (response type and cognitive requirement) of every task examined. The study showed a predominance of simple computation procedures and purely mathematical contexts in both countries. Differences are obtained in the performance requirements of the problems: the results indicate that the US textbooks included more variety in problem requirements (e.g. explanation or solution required, conceptual understanding required).

Another interesting instrument for textbook analysis was established by Dole and Shield [22]. The authors examined the proportional reasoning in the worked examples of two eighth-grade Australian mathematics textbooks. The results showed a predominance of calculation and procedure activities in comparison to tasks which support the conceptual understanding of proportions.

Vincent and Stacey [16] investigated different topics in eighth-grade Australian mathematics textbooks. The aim was to investigate the 'shallow teaching syndrome' shown by the previous study. The authors developed a framework for task analysis including: procedural complexity, type of solving processes, degree of repetition, proportion of application problems, and proportion of problems requiring deductive reasoning. The procedural complexity of a textbook task may be put at one of three levels: low, moderate, or high. The type of solving processes refers to using a procedure, a concept or making connections. Although some textbooks did challenge students to reason and to make connections, the results show the predominance of lower procedural complexity in the examined textbook tasks. Vincent and Stacey point out that a balance of task types is important for all students, which was not the case in the textbooks examined.

Brändström [23] examined the differentiation of tasks (exercises, problems, word problems) in Swedish 7th-grade textbooks. The results show a low level of challenge in textbooks because the emphasis is on tasks with lower cognitive difficulty.

In Ireland, O'Keeffe and O'Donoghue conducted a mathematics textbook analysis study with the intention of highlighting key textbook features which impact on students' learning [24]. Their framework is primarily based on the TIMSS textbook analysis and comprises Content, Structure, Expectation and Language. The examined textbook series are found to be non-innovative in the sense of comprehension and motivation. The analysis also showed that less than one quarter of all exercises in all the textbooks could be classified as non-routine problems. This finding indicates once again the low expectations in the textbook exercises.

The literature review shows that the instrument for analysing the textbook tasks (exercises, examples and other questions) usually includes: answer type, routine or concept orientation, level of complexity and application. Some studies focus on a particular mathematical topic, while others focus on the mathematics textbook tasks of a whole school grade. The developed frameworks contain a complex structure, which is in line with the multi-layered role of textbook tasks in mathematics education [11]. The results of the studies presented in the literature review generally show the predominance of tasks with lower level expectations in mathematics textbooks.

However, the studies presented in this section did not clearly encompass another important dimension of textbook tasks: mathematical activities, i.e. *what* should be done in a particular task (for example, does the textbook task require the activity of computation or another activity, such as drawing a figure or giving a mathematical explanation). The dominance of just one activity may have a negative effect on the students' understanding of mathematical ideas and may limit their views. For example, Dole and Shield [22] examined proportional reasoning in the worked examples of two 8th-grade Australian mathematics textbooks. The results showed that the 'textbook analysis highlighted the range of computational procedures presented to students through their study of ratio and proportion, and also made apparent the minimal use of diagrams, tables and graphs to further proportional reasoning' [22,p.32]. Different mathematical activities presented within the textbook exercises should challenge students and develop understanding. These results indicate that mathematical activities are also worth including in the analysis of textbook tasks.

2.2. Mathematical activities

The idea of involving mathematical activities has its origin in the competency approach of mathematics education [25]. Niss [25] places the focus on 'mathematical activity by asking what it means to be mathematically competent' [25,p.39]. The author with his expert group in Denmark proposed eight mathematical competencies: mathematical thinking, mathematical problem handling, mathematical modelling, mathematical reasoning, mathematical representation, mathematical symbol and formalism, mathematical communication and mathematical aids and tools. Many national curricula involve these competencies under the heading Mathematical Processes which includes problem solving, reasoning and proof, communication, connections and representation (e.g. US process standards in [26]). These competencies relate to the previously mentioned rich tasks because 'it seems natural to focus on the competencies involved in posing and answering different sorts of questions pertinent to mathematics in different settings, contexts and situations' [25,p.39]. This approach with eight mathematical competences was incorporated into the PISA framework for mathematical literacy [27]. The PISA framework influenced many national curricula for mathematics including the German curriculum [28] which includes six mathematical competencies: mathematical argumentation, problem solving, mathematical modelling, mathematical representation, communication, and the handling of symbolic, formal and technical elements of mathematics. Further, the Austrian standards for mathematics [29] contain four 'mathematical activities': representations and modelling, calculation and operation, interpretation, and argumentation and reasoning. Common to every framework is the emphasis on 'what it takes to *do* mathematics' [25,p.42].

Besides finding the context, answer form and cognitive requirements in textbook tasks [16,19,20], this study also includes the idea of finding what students are expected do in a textbook task and which mathematical activity [29] must be performed to solve the task successfully.

2.3. Research questions

The aim of this study is to compose a multidimensional framework including an activity dimension in order to evaluate the requirements in mathematics textbook tasks. Related to that, two research questions are formed for this study: To what extent do the textbooks offer the full range of task types? How can implementation of the activities dimension into the framework help in understanding the demands of the textbook tasks?

3. Theoretical framework

In order to examine the requirements in textbook tasks in this study, a 5D framework was established. It consists of the following aspects: content, mathematical activities, complexity levels, answer forms and contextual features. This type of multidimensional framework required a combination of two theoretical sources: the Austrian educational standards [29]

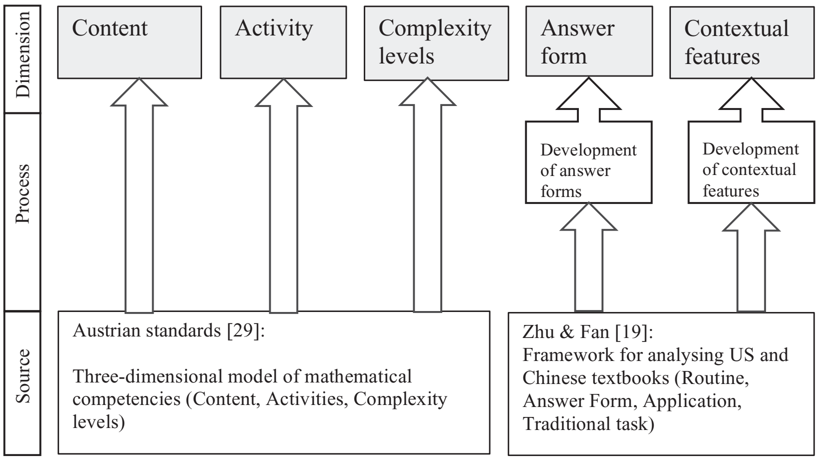


Figure 1. Five-dimensional framework outline (content, activities, complexity levels, answer forms and contextual features).

and the framework by Zhu and Fan [19]. The Austrian educational standards [29] provide the theoretical framework for *content*, *mathematical activities* and *complexity levels* as given in Section 3.1, while the dimensions of *answer forms* and *contextual features* lean on the study presented in Zhu and Fan [19] and are given in Sections 3.2 and 3.3. The aspects from Austrian educational standards are taken as they are given in the original source, while the aspects of answer forms and contextual features are further developed and modified for the purposes of the research presented in this paper. The theoretical framework outline is presented in Figure 1.

The three main research dimensions are taken from the Austrian standards because during the preparation of this study Croatia was in the process of putting together the new curriculum framework and the new Austrian standards were one of the documents which were considered an important possible influence. The reason for this is that in the past the two countries have shared similar educational traditions and the Austrian standards offered a new concept of mathematical competence which could also be interesting for reflecting on mathematics education in the region.

3.1. Competence model: content, activities and complexity

The Austrian standards provide a 3D model of mathematical competencies: competency is described as a long-term disposable cognitive ability which can be developed by the learner and which enables the learner to practice various activities and skills and to apply them. Further, mathematical competencies refer to mathematical activities, to mathematical content and to the complexity level of the connections required. According to the Austrian standards, mathematical competencies have an activity dimension (*Handlung*), a content dimension (*Inhalt*) and a complexity dimension (*Komplexität*). In this way, a specific mathematical competence could be defined and described through a triple (content, activity, complexity). For example, the triple (functions, interpretation, connections) refers to a mathematical competence where a student is able to interpret graphical representations of functional dependences, in which some facts/relations/representations/activities should be connected. The following three subsections contain descriptions of the three mentioned dimensions.

3.1.1. Content

The content requirements refer to finding out what *mathematical knowledge* a student should possess in order to solve a particular textbook task. The content field of the Austrian standards is divided into: numbers and measures (coded as I1); variables and functional dependences (I2); geometric shapes and solids (I3); and statistic representations and parameters (I4). These content fields follow the current national curriculum for the lower secondary level of mathematical education. Numbers and measurements encompass integers, rational and irrational numbers, arithmetic operations and measurement units. Variables and functional dependences involve terms and formulas, equations and linear equation systems, proportionality and linear and quadratic function. Geometric shapes and solids refer to 2D and 3D shapes, similarity, isometric functions in the plane, Pythagorean theorem, circumference, area, surface area and volume of geometric shapes. In this framework, the statistics content refers to representations of statistical data, probability and random event.

3.1.2. Mathematical activities

The mathematical activities field is divided into: representations and modelling; calculation and operation; interpretation; and argumentation and reasoning. Representation (H1) concerns 'translation' of the given mathematical data into another mathematical representation (for example, transmissions from one statistical representation to another). Modelling involves recognizing relevant mathematical relationships from the given situation and representing the same problem in a mathematical mode (symbolic, graphical, etc.). Calculation (H2) concerns conducting elementary computation operations with concrete or generalized numbers. Operation is the concrete, sensible and efficient conducting of computational or constructional steps. It also refers to transforming measure units, transforming mathematical expressions, solving equations, estimating results, approximations and conducting elementary geometrical constructions. Interpretation (H3) concerns recognizing relations and relevant data given in the mathematical representations (graphical, symbolical and tabular) and their interpretation in the given context. Interpreting includes appropriate reading of the graphical or symbolical mathematical representations and their interpretation in the given context. Argumentation (H4) refers to the description of mathematical aspects that speak pro or contra a particular decision. It requires concrete and appropriate implementation of mathematical relations and characteristics, mathematical rules, as well as the correct usage of mathematical language. Reasoning concerns the sequence of true arguments that lead to a conclusion.

For example, Table 1 gives a geometry task requiring various mathematical activities.

Table 1. Examples of four mathematical activities.

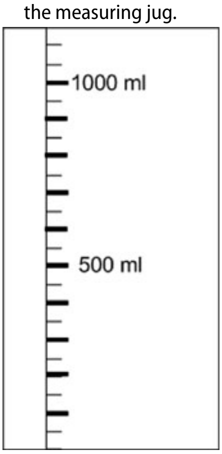
Mathematical activity	Example
H1 Representation	A cylinder-shaped barrel is ½ filled with water. The height of the barrel is 82 cm. Its base has a diameter of 82 cm. Construct its base using a scale 1:100.
H2 Computation and operation	A cylinder-shaped barrel is ½ filled with water. Barrel height is 82 cm. Its base has a diameter of 82 cm. Find the volume of water in the barrel and express it in litres.
H3 Interpretation	A cylinder-shaped barrel is ½ filled with water. The height of the barrel is 82 cm. Its base has a diameter of 82 cm. What is expressed with the formula $(\frac{82}{2})^2 \cdot \pi$?
H4 Argumentation	A cylinder-shaped barrel is ½ filled with water. The height of the barrel is 82 cm. Its base has a diameter of 82 cm. Does the height of water in the barrel influence the surface area of the barrel? Explain your opinion.

Various examples of each activity mentioned can be found in the Austrian standards [29].

3.1.3. Complexity levels

The complexity field is divided into: direct application of basic knowledge and skills; constructing and dealing with connections; reflection or applying reflective knowledge. Namely, the content and activities themselves are not enough to identify the competences required from students in a particular textbook task. Some mathematical tasks could have the same content (for example, the circle), and the same activity (for example, calculation), but they could differ in terms of cognitive complexity. For example, one task could require skills on the reproduction complexity level, while another might require the construction of more complex connections. Applying basic knowledge and skills (K1) encompasses reproduction or direct application of mathematical concepts, rules, procedures and representations. Making connections (K2) and dealing with connections refer to more complex tasks where several concepts or activities are required to be combined in order to solve the problem. It encompasses, for example, making connections between a variety of terms, theorems, methods and representations. Reflective thinking (K3) involves reflecting on mathematical ideas that are not directly readable from the given problem. Reflective knowledge means applying creative knowledge about mathematics. For example, Table 2 gives examples with various complexity levels.

Table 2. Examples of different complexity levels.

K1 Reproduction	Write the decimal 0.35 as a fraction.
K2 Connections	Ana pours 1/8 of a litre of milk into the measuring jug shown below. Mark the height of the milk in the measuring jug. 
K3 Reflection	Sometimes it is more appropriate to express rational numbers as fractions, and sometimes as decimals. Give two examples of when people would use fractions, and two examples of when people would use decimals. Explain your answer.

Forty-eight mathematical competencies are described in this manner because it is the number of possible triples (I, H, K). The Austrian standards provide worked examples of all 48 competencies [29].

3.2. Answer forms

In their study of textbook problem types, Zhu and Fan [19] distinguish open-ended and closed-ended tasks. Open-ended tasks refer to tasks with several or many correct answers, while closed-ended tasks have only one answer. The PISA 2003 Framework [27] distinguishes closed constructed response, open constructed response and multiple-choice. The closed constructed response can be easily judged as either correct or incorrect. Multiple choice problems offer 'a limited number of defined response-options' [27,p.51]. For example, Table 3 gives examples with various answer forms.

Table 3. Examples of different answer forms.

A1 Closed-ended	Solve the equation: $2x + 6 = 12$
A2 Open-ended	Write a problem from everyday life which refers to solving the equation $2x + 6 = 12$.
A3 Multiple choice	What is the solution of the equation $2x + 6 = 12$? A. $x = 6$; B. $x = 3$; C. $x = -3$; D. $x = -6$

3.3. Contextual features

Contextual features refer to what extent, and in which ways, real-world experiences are incorporated into the (textbook) tasks. In their research, Zhu and Fan [19] distinguish between application and non-application problems in mathematics textbooks. A non-application problem is unrelated to the real world, while an application problem arises in the context of a real-life situation. Application problems can be fictitious or authentic. The fictitious application problems contain data made up by the textbook author. The authentic problems contain real-life situations or data 'collected by students themselves from their daily lives' [19,p.614]. Sullivan et al. [11] discuss contextualized tasks which are related to students' experiences. Such tasks 'have great potential for challenging and engaging students, and showing how mathematics can help them to make sense of the world' [11 ,p.42]. Table 4 gives examples with various contextual features.

Table 4. Examples of different contextual features.

C1 Intra-mathematical context (non-application tasks)	Write the fraction $\frac{3}{5}$ as a percentage.
C2 Realistic (fictitious) context	A student counted that on Monday from 3 to 4 pm 12 cars, 3 vans, 4 motorbikes and 1 bus passed in front of his school. Show these data using a relative frequency table.
C3 Authentic context	Count the vehicles in front of your school and make a relative frequency table of motorbikes, cars, buses and vans.

4. Method

The examined textbooks were Croatian mathematics textbooks for grades 6, 7 and 8, which correspond to the ages 12, 13 and 14, respectively. Since this study was part of a wider

Table 5. Instrument for textbook analysis.

Dimension	Question	Details and codes
Mathematical content	What content must a student know to perform a particular task?	Numbers and measurements (I1) Algebra, variables and functional dependences (I2) Geometric shapes and solids (I3) Descriptive statistics and probability (I4)
Mathematical activity	What mathematical activities should be performed to carry out the task successfully?	Representations and modelling (H1) Calculation and operation (H2) Interpretation (H3) Argumentation and reasoning (H4)
Complexity level	What is the complexity of knowledge and activities that a student needs in order to perform the task?	Direct application of basic knowledge and skills (K1) Constructing and dealing with connections (K2) Reflection or applying reflective knowledge (K3)
Answer form	What answer form does the task require?	Closed answer (A1) Open answer (A2) Multiple choice (A3)
Context	What is the context in the task?	Intra-mathematical situation (C1) Realistic context (C2) Authentic context (C3)

research [30] connected to the comparison of the PISA requirements and mathematical competencies required in school, grades 6, 7 and 8 were included because of their importance in developing the mathematical competencies important for the upper secondary levels.

Each textbook task was examined in order to identify categories it requires (content, activity, complexity level, answer form and context, as shown in Tables 1–4). These categories are derived from theoretical background and research questions. Within each category, the particular task was given a code according to Table 5. For this purpose, a qualitative approach was needed [31] because the meaning of the text revealed the code (for example, in finding the complexity level). A qualitative approach was accomplished through the qualitative content analysis method [31].

4.1. Sampling

In this study, all examined items are called tasks [10]; the word 'task' covers all situations that require an answer in the textbooks, no matter if the solution is given or not. These are mostly exercises, but also worked examples, revision tasks and other questions. The study encompassed all the tasks from the two most frequently used mathematics textbook series in Croatia for grades 6, 7 and 8. In each grade, the two textbook series examined were used by more than half of the total student population in Croatia of that grade. Altogether, the study encompassed more than 22,000 textbook tasks. For the purposes of the study the two textbook series will be referred to as textbook A and textbook B.

4.2. Instrument for textbook analysis

Based on the theoretical background, the following five classifications of tasks are established and developed in the study (Table 5).

4.2.1. Content

Mathematical content refers to the content from the Austrian standards. Since the content of the Austrian standards does not completely correspond to the Croatian curriculum content the requirements were slightly modified for the purposes of this study. For example, the Austrian curriculum and standards include more topics within descriptive statistics and probability than the current Croatian curriculum. Therefore, this study focused on topics included in the Croatian curriculum and textbooks: graphical data representations, arithmetic mean and probability of a random event.

4.2.2. Mathematical activities

The four main mathematical activities which should be performed in order to carry out a particular task successfully are Representations and Modelling, Calculation and Operation, Interpretation, and Argumentation and Reasoning. They are described in Chapter 3 of this paper. For example, the following task requires calculation activities: $6 + 9 - 12 \times 3$. And the following textbook task requires representation activities: 'Construct two obtuse angles with parallel arms, respectively.' Some tasks required more than one activity or content. In that case, the activity or content which was considered dominant in performing the task was chosen and coded. In cases where two activities are required in equal measure to perform the task, they were both coded and included in the analysis.

4.2.3. Complexity level

The complexity of knowledge and activities that a student needs in order to perform a task can be put on three levels. These are: direct application of basic knowledge and skills (level 1); constructing and dealing with connections (level 2); and reflection or applying reflective knowledge (level 3). They are described in Chapter 3 of this paper and more in-depth in the Austrian standards [29].

4.2.4. Answer form

In this study, three answer forms were distinguished. These are closed answer, open answer and multiple choice. Multiple-choice tasks offer a limited number of already defined responses. Closed answer tasks require one correct (usually short) answer. Open answer tasks require a more free expression of students' ideas about mathematics. Closed answer tasks put more emphasis on the final solution, while open answer tasks are more concerned with the process and the way of solving a particular task.

4.2.5. Context type

Within this research, the term authentic context means that the task situation is taken from authentic reality or from the authentic experience of the student. For example, the textbook task 'Count the vehicles in front of the school and make a relative frequency table of motorbikes, cars, buses and vans' deals with an authentic situation. Realistic context encompasses situations with simulated reality. An intra-mathematical situation refers to mathematical tasks without context. Such tasks contain the relevant data represented in mathematical notation (symbolic, graphical, etc.) and use specific mathematical terms.

These classifications make up the instrument for textbook analysis (Table 5).

4.3. Task examples and exemplary analysis

This section presents two tasks from the analyzed mathematics textbooks. They are followed by the analysis according to the 5D framework.

Example 1: 'Write the number 3/100 as decimal number and as percentage.' This task refers to the content of numbers (Table 5). This requires representation activities because the given fraction should be represented with two other representations of a rational number. The task is put on the lowest complexity level (direct application of basic knowledge and skills). The required answer is closed and the task is intra-mathematical.

Example 2: 'Find the circumference of the isosceles triangle ABC with the leg $b = 73$ cm and the base height $h = 55$ cm.' This task refers to the content of geometry and requires computation activities. This requires using the Pythagorean Theorem and connecting it to the knowledge of the isosceles triangle and its circumference; therefore, it is on the complexity level of connections. The required answer is closed and the task is intra-mathematical (Table 5).

4.4. Procedure

The analysis of all the tasks in the selected textbooks was conducted using the framework described above. Each of the 22,168 tasks was examined according to the 5D instrument and then coded into the corresponding category. Since this study was part of a wider doctoral research [30], the accuracy and reliability of coding was ensured by checking samples of the tasks with the thesis mentors who were creators of and experts on the Austrian standards. To ensure the consistency of the framework application across such a huge number of tasks, the analysis was conducted in steps over 9 months. Each step was followed by the checking of samples. In addition, discussions with the mentors were especially valuable in coding some of the more problematic and equivocal tasks.

The next step was to analyze the coding results using quantitative methods, which encompassed finding the relative frequencies of codes within a specific mathematical topic. For this purpose, the SPSS program was used.

5. Results

The results are presented in five main sections, which follow the five dimensions of the framework.

5.1. Content

All school textbooks in Croatia are approved by an expert group appointed by the Minister of Education, with the purpose of ensuring that the textbook content matches the curricular requirements. Although this section is about content findings, they cannot be fully separated from other aspects of the framework (Tables 6–13).

5.1.1. Numbers and measurements

Learning about numbers in grade 6 in Croatia encompasses the topics: Integers, Fractions, and Set Q [32]. Percentages are taught in grade 7, and the topic Irrational Numbers in grade 8. Table 6 presents the proportions of task features in textbook A, and Table 7 refers to textbook B.

Every line in Tables 6 and 7 refers to a topic which is presented as a chapter in the textbooks. For example, textbook A encompassed 943 tasks in the topic Integers in grade 6.

Table 6. Requirements in 'Numbers and Measurements' chapters of textbook A.

G	Topic (no. of tasks)	H1	H2 in %	H3	H4	K1	K2	K3 in %	CA	IM
6	Integers (943)	4	95	3	0	89	11	0	100	96
6	Fractions (1091)	7	96	1	0	70	30	0	100	94
6	Set **Q** (699)	9	90	3	0	66	33	0	99	99
7	Percentages (61)	44	49	7	0	87	13	0	100	92
8	Square, square root (929)	16	84	1	1	72	28	0	100	99
8	Set **R** (360)	17	71	35	7	55	45	0	100	100

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

Table 7. Requirements in 'Numbers and Measurements' chapters of textbook B.

G	Topic (no. of tasks)	H1	H2 in %	H3	H4	K1	K2	K3 in %	CA	IM
6	Integers (957)	4	96	4	0	91	9	0	100	97
6	Fractions (1079)	4	98	1	0	54	46	0	100	90
6	Set **Q** (848)	5	95	1	0	68	32	0	100	99
7	Percentages (388)	36	64	0	0	73	27	0	100	91
8	Square, square root (1300)	18	85	2	0	78	22	0	100	99
8	Set **R** (592)	19	59	28	0	75	25	0	100	98

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

Table 6 shows that 4% of these 943 tasks required students to represent integers, usually on the number line. 95% of all items required calculation activities with integers, 3% of items required interpreting a given picture or mathematical formula, while none of the 943 items required argumentation or reasoning activities. Similar results are obtained in textbook series B (Table 7). The activities aspect does not add up to 100% because some of the tasks required more than one activity. Further, the results presented in Tables 6 and 7 show that about 90% of all integer tasks require low cognitive demands (K1) and that almost all tasks include intra-mathematical situations (IM) with integers (96% in textbook A and 97% in textbook B).

Focusing on the mathematical activities in the topics of numbers and measurements (Tables 6 and 7) the results show that textbook requirements emphasize operation activities and calculation techniques in arithmetic education. The findings point to a lack of presentation, interpretation and above all argumentation activities about numbers. Only the topic 'Percentages' significantly requires representation activities in the tasks (44% in textbook A and 36% in textbook B, Tables 6 and 7). These tasks refer to translation from one representation of rational number to another. The interpretation activities in the topic 'Set **R**' are represented with 35% in textbook A and 28% in textbook B. These activities are to do with observing decimal notation of rational and irrational numbers and comparing them.

In the topic 'Numbers' the level of the activities is reproductive or simpler connections, higher cognitive activities such as reflective thinking are not encouraged in the topics Numbers and Measurements. Also, all the textbook tasks in this topic are of the closed answer type. Intra-mathematical tasks dominate in textbooks, even within topics which have a strong connection to everyday life, such as fractions and percentages. The results in relation to the topic 'Numbers' indicate that the emphasis is on symbolic tasks and on following rules for operations.

5.1.2. Algebra, Variables and Functional Dependences

The content on Algebra, Variables and Functional Dependences in grades 6–8 comprises the topics of equations, direct and indirect proportionality, algebraic terms and linear function [32]. Tables 8 and 9 present the proportions of task features in textbooks A and B, respectively.

Table 8. Requirements in 'Functions and Algebra' chapters of textbook A.

G	Topic (no. of tasks)	H1	H2 in %	H3	H4	K1	K2	K3 in %	CA	IM
6	Equations (236)	16	100	0	0	21	79	0	100	89
7	Equation systems (208)	20	100	0	0	53	47	0	100	88
8	Equat. $x^2 = a$ (54)	0	100	0	0	56	44	0	100	100
7	Proportionality (283)	6	93	1	0	57	41	2	99	22
8	Terms (556)	0	98	3	1	75	25	0	100	100
7	Linear func. (273)	33	67	32	0	72	28	0	100	99

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

Table 9. Requirements in 'Functions and Algebra' chapters of textbook B.

G	Topic (no. of tasks)	H1	H2 in %	H3	H4	K1	K2	K3 in %	CA	IM
6	Equations (373)	10	100	0	0	25	75	0	100	90
7	Equation syst. (519)	12	99	0	0	53	47	0	100	91
8	Equat. $x^2 = a$ (103)	0	100	0	0	54	46	0	100	100
7	Proportionality (482)	6	91	4	1	85	14	1	99	28
8	Terms (661)	0	97	3	2	80	20	0	98	100
7	Linear func. (1026)	36	48	40	0	73	27	0	99	94

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

The results indicate high proportions of calculation and operation activities in the examined tasks, on the symbolic level and requiring closed answers (Tables 8 and 9). Interpretative, argumentative and reflective skills are not required from students in the topics Algebra and Functional Dependences.

The topic 'Equations' encompasses linear equations with a single variable, the system of two linear equations with two variables and a simple quadratic equation $x^2 = a$. The textbook tasks in these areas focus on intra-mathematical calculation tasks, without using authentic tasks or the idea of equivalent equations. Linear equations and equation systems, apart from 100% of operation and calculation activities, also require representations in 10%–20% of tasks. This means that in the textual tasks students should express the given problem in a symbolic way with an equation or equation system.

The chapters on linear function require representations in about one third of tasks, calculations in two thirds of tasks in textbook A and in about half of the tasks in textbook B, and interpretation activities in about one third of tasks. The required activities in linear function comprise of drawing a graph of a linear function given in a symbolic form (representation activities), calculating function value for given argument x (calculation), or reading the values from a given graph (interpretation).

Functions and algebra frequently use symbolic tasks, except for the topic of direct and indirect proportionality. This topic places more emphasis on exercises with realistic or authentic contexts. In textbook A, 78% of 283 examined tasks had realistic context (Table 8). Similarly, in textbook B, 72% of 482 examined proportionality tasks had realistic or authentic context (Table 9). These results indicate that the textbook authors extensively used the context-oriented tasks in this topic. Although the topic of proportionality may be included in arithmetic (Numbers), its approach in Croatian textbooks and curriculum is more algebraic, meaning it is included in the content of functional dependences in the results.

5.1.3. Geometric shapes and solids

The geometry content in grades 6–8 encompasses the topics Triangle and Quadrilateral in grade 6, Polygons, Circle and Similarity of Triangles in grade 7, and Pythagorean Theorem, Plane Isometries, Relationships of the lines in space, and Geometric Solids in grade 8 [32]. Tables 10 and 11 present the proportions of task features in textbooks A and B, respectively.

Table 10. Requirements in 'Geometry' chapters of textbook A.

G	Topic (no. of tasks)	H1	H2	H3	H4	K1	K2	K3	CA	IM
			in %					in %		
6	Triangle (568)	38	59	29	0	49	50	1	98	98
6	Quadrilateral (304)	45	74	23	1	50	50	0	99	98
7	Polygons (264)	22	88	8	0	55	45	0	99	100
7	Circle (306)	27	71	10	5	45	55	0	94	92
7	Similarity(137)	37	73	37	7	36	64	0	93	88
8	Pythagorean theorem (723)	7	95	13	2	37	63	0	98	98
8	Isometries (586)	72	18	45	3	50	50	0	96	100
8	Lines, space (467)	22	14	80	4	76	24	0	95	99
8	Geom. solids (726)	3	92	33	1	34	65	1	97	94

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

Table 11. Requirements in 'Geometry' chapters of textbook B.

G	Topic (no. of tasks)	H1	H2	H3	H4	K1	K2	K3	CA	IM
6	Triangle (677)	30	68	41	3	50	49	1	97	97
6	Quadrilateral (352)	22	75	32	3	65	35	0	97	96
7	Polygons (570)	23	73	18	2	70	30	0	98	100
7	Circle (753)	25	62	22	0	72	28	0	100	96
7	Similarity (320)	33	50	50	15	45	55	0	83	91
8	Pythagorean theorem (797)	8	92	13	0	41	59	0	100	91
8	Isometries(798)	69	7	49	0	46	54	0	100	89
8	Lines, space (513)	41	21	62	0	80	20	0	100	100
8	Geom. solids (1048)	4	86	32	0	59	41	0	100	89

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

The results show that geometry tasks mostly focus on reproduction, simple connections, closed answers and intra-mathematical requirements. In terms of activities, the results show an emphasis on routines in calculating or constructing activities. Presentation activities refer to sketching or constructing a geometric picture, while interpretation activities consist of reading data from a given or constructed picture. Also, the results showed a deficit of argumentation activities and open answers. In terms of the complexity level, there is a clear lack of reflective thinking requirements.

The textbook chapters on Pythagorean Theorem show that the task requirements put strong emphasis on operation and calculation activities (Table 10 and 11). The Pythagorean Theorem is introduced in the 8th grade after students have been taught square roots [32]. Textbook A offered 723 tasks, and textbook B 797 tasks within this topic. More than 90% of them required calculation and only 13% required interpretation of a given picture. This means that the tasks mostly cover two values in the textual form, and students are supposed to find (compute) the length of the third side of a right-angled triangle. Although the importance of the Pythagorean Theorem has always been in its implementation to problems from everyday life, the results show the Pythagorean Theorem presented as a rule applicable in the pure intra-mathematical objects (98% in textbook A and 91% in textbook B).

The topic Isometric Mappings in the Plane puts emphasis on representation activities: conducting translation, rotation or reflection of a given geometric object, including the composition of these mappings. The topic Relationship of lines in space consists of many tasks with a given picture where students have to recognize (interpret) the relationship between two marked geometric objects (e.g. a plane and a line which are parallel). Most of the tasks in this chapter have low cognitive demands.

As in other content topics, argumentation activities, open answers and reflection are not required in the geometry chapters. Still, the topic Similar Triangles in grade 7 encompasses 15% tasks of textbook B which require argumentation activities. These tasks offer a picture of two triangles whose corresponding sides are in the same ratio or whose angles are equal. Students are supposed to prove that the two given triangles are similar using the appropriate similarity postulates.

In comparison to the results of Numbers and Algebra, the Geometry tasks require more representation and interpretation activities, but computation and operation activities remain the most frequent activities in the chapters on geometric shapes and solids in mathematics textbooks.

5.1.4. Descriptive statistics and probability

According to the Croatian curriculum [32], statistics and probability are taught only in the 7th grade and to a very limited extent. Statistics encompasses graphical data representations, frequencies and arithmetic mean, while probability refers to the probability of a random event. The results of textbook requirements within these topics are given in Tables 12 and 13.

Table 12. Requirements in 'Descriptive Statistics and Probability' chapters of textbook A.

G	Topic (no. of tasks)	H1	H2	H3	H4	K1	K2	K3	CA	IM
			in %					in %		
7	Statistics (175)	36	65	33	0	51	47	2	99	2
7	Probability (112)	26	75	9	0	73	27	0	100	6

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

Table 13. Requirements in 'Descriptive Statistics and Probability' chapters of textbook B.

G	Topic (no. of tasks)	H1	H2	H3	H4	K1	K2	K3	CA	IM
			in %					in %		
7	Statistics (86)	30	65	30	0	66	34	0	100	0
7	Probability (117)	1	99	5	1	83	16	1	99	0

Legend: G: grade; H1: representations, H2: calculation, H3: interpretation, H4: argumentation; K1: reproduction, K2: connections, K3: reflection; CA: closed answer required; IM: intra-mathematical context.

About two thirds of tasks within the chapters on statistics in both of the textbooks require calculation activities. This can be explained by the fact that the main requirement of the topic Arithmetic mean is to calculate the average of given numerical data. One third of tasks refers to representation of data, and one third to interpretation of a given data table or graph. The situation is similar for the statistics chapters in textbooks A and B, but in the topic of probability there are significant differences between the two textbooks. Textbook B requires only calculations, while textbook A has representation activities in one quarter of its tasks. Both textbooks favour low cognitive levels and closed answer forms in the tasks. Again, argumentation and reasoning skills, open answer forms and reflective thinking are not represented to a great extent in mathematics textbooks in statistics and probability education in Croatia. However, there are more realistic context exercises than intra-mathematical ones in the descriptive statistics and probability topics.

5.2. Activities

In general, the results show that textbook tasks require mainly calculation activities with numbers and terms, as well as operations such as conducting geometric constructions (Tables 6–13). They are predominant in all the topics analyzed except for Space Relationships and Isometric Mappings. The chapters on equations require calculation activities in 100% of the tasks in all the textbooks examined. The traditional geometry topics such as plane shapes or solids predominantly require operational or computational skills of calculating circumferences, areas, volumes or surface areas. This implies that geometry is actually quickly switched to dealing with numbers or algebraic terms.

The results presented in Tables 6–13 show that argumentation and reasoning activities are not present in Croatian mathematics textbooks at all. Only the chapters on triangle similarity contain exercises requiring argumentation skills (in 15% of tasks in one textbook, and in 7% of tasks in the other). They refer to explaining (proving) the similarity of two triangles using the similarity theorems.

5.3. Complexity levels

Since there are no complexity requirements laid out in the Croatian program for mathematics [32], the extent of competency required in textbook tasks depends mainly on the authors. The analysis shows that the textbook tasks are on the reproductive or simpler connections level (Tables 6–13). There are no reflective thinking activities in the textbook tasks.

5.4. Answer forms

The textbook analysis results indicate the predominance of closed answer tasks in all mathematics topics. In most of the topics, the proportion of required closed answers is more than 97%. The lack of argumentation activities shown in the previous section is closely connected with this issue. Also, multiple choice questions are not common in Croatian mathematics textbooks.

5.5. Context

The results indicate the usage of intra-mathematical tasks to a huge extent (more than 88% in almost all topics). Only the research results for descriptive statistics, probability and proportionality show a higher proportion of realistic contexts.

6. Discussion and conclusions

6.1. Task design

The research results in general show that there is no balance between different task types. The textbook tasks are found to be computational, intra-mathematical, with low-level cognitive demands and are closed. The authentic context, open-answer tasks and reflective thinking skills are not required at all in the researched textbooks. According to the analysis conducted in [30], these results are not in accordance with the Croatian national mathematics requirements [32] which contain the following educational achievements: to apply mathematics in everyday life, to determine which method is most efficient in reaching the solution, to think about the solution and to discuss it [32,p.230]. These achievements point directly to authentic tasks and reflective thinking, but rich tasks as described in [13] are barely present in the examined textbooks.

The lower cognitive expectations are generally in line with other studies of textbook tasks, as shown in Section 2.1 [19,20,22]. For example, the results on requirements about integers correspond to the findings of Li [20] in Chinese and US textbooks. As in the Chinese textbooks, none (0%) of all examined integer tasks required argumentation or explanations. Also, the emphasis on computational procedures in the chapters on proportionality is in line with the results by Dole and Shield [22] on proportional reasoning in Australian textbooks. The focus on lower cognitive levels matches similar results obtained in studies reported in [16,23,24]. In most textbook topics, intra-mathematical tasks are predominant. Realistic context predominates in only three topics: descriptive statistics, probability and proportionality. These results could in part explain the poor Croatian results on the PISA assessment [33], as the PISA mathematical literacy puts emphasis on solving problems in various situations in order 'to identify and understand the role that mathematics plays in the world' [27,p.24].

The emphasis on performing a large amount of tasks can also be seen in the results of the study presented in this paper. The tables in Section 5 also include the number of textbook tasks per particular topic. The analysis revealed a huge number of tasks within each textbook (about 4000). If we were to do them all within 140 school lessons per year, it would total an average of about 30 textbook tasks per one lesson unit. This finding corresponds to the predominance of simple tasks in textbooks because simple tasks require less time to solve. It is also interesting to note that the tasks in the study do not become more complex as the students move up from grades 6 to 8.

The findings highlight the difficulties in designing tasks, particularly tasks given in such extensively used curriculum resources as textbooks. The issues concerning task design are currently being recognized as an interesting and important area within the research on mathematics education [11] because the textbook requirements influence implemented curriculum and students' opportunities to learn [34,35]. A better implementation of rich tasks [11,13] would encourage the use of authentic context, open tasks or tasks which develop thinking in a variety of styles. This may influence the students' image of mathematics because the tasks help students to create opportunities which can broaden or limit their views of mathematics [11,12]. The relationship of *textbook task features, enacted mathematics curriculum* and *student's image about mathematics* is an idea for further research.

6.2. Discussion on mathematical activities

Incorporating an activities dimension into the analysis of textbook tasks contributed to a better understanding of the learning opportunities afforded to students through the use of mathematics textbooks. The applied framework showed the dominance of just one activity, while Niss's competency approach [25] refers to a wide range of mathematical competences. Related to that, the results presented in this paper may suggest that, according to the textbook requirements, a mathematically competent student only needs to be effective in computation. For example, this study revealed an interesting result about geometry education in Croatia. Most of the geometry topics put emphasis on calculation or operation activities (Tables 10 and 11), which means that the geometry tasks predominantly require the ability to deal with numbers, formulas and terms instead of mastering geometric concepts. This leads to the question of what picture of geometry our students get; what is geometry to them? A set of computation procedures? The aims of geometry education would seem to be much broader than that [36,37]. Therefore, different mathematical activities presented within the textbook tasks would challenge students and help them in developing their understanding.

These results imply the importance of incorporating the mathematical activities dimension into national mathematical curricula, with the aim of encouraging textbook authors and teachers to implement tasks with presentation and modelling, interpretation, argumentation and reasoning activities into mathematics education. The current Croatian curriculum for mathematics [32] does not contain such outlines for competences or activities [30].

6.3. Future research and international implications

The results of the study presented in this paper shows the dominance of one activity (computation), one answer form (closed answer) and one context type (intra-mathematical) in tasks. Also, the previous survey [7] showed that the teachers are satisfied with these textbooks as such. These findings suggest a traditional view on mathematics and teaching mathematics in Croatia from both teachers and textbook authors, which is in line with the results of an empirical research about teachers' perceptions of mathematics education [38] which showed 'that the traditional approach and traditional teaching methods prevail' [38,p.96] in Croatian mathematics classrooms. In general, the findings of the study presented in this paper together with the above-mentioned previous studies [7,38] indicate that mathematics education in Croatia emphasizes the view of 'mathematics as a tool' rather than as a 'medium of communication' [39]. These conclusions have raised new issues for further research and raise a question about the students' perspective: How do the dominant textbook task features influence students' image of mathematics and their mathematical competencies? Another issue relates to the question: What is happening in classroom practice regarding the use of textbook tasks – are they being used directly as they are, or are teachers modifying them or combining them with tasks from other resources? Further to this, another issue for future research relates to the challenge of modifying current textbook tasks in order to create more rich tasks. The thorough analysis of textbook tasks revealed many interesting exercises and examples. Although they are characterized as intra-mathematical, computational or closed, many of these tasks are seen as interesting because they have the potential to be easily modified to create rich tasks. For example, some of them could be transformed from closed answer to open answer tasks with the addition of questions such as 'Why? Can you explain it?' In others, the predominating computation requirement could be modified into another activity. These modifications present ideas for further studies, including stating design principles, developing a variety of rich tasks and intervention in the classroom. Related to this idea, the results indicate that it would be worthwhile gaining further insight into teachers' beliefs and expectations of the textbook content, because if the aim is to change classroom practice, then teachers' attitudes is a very important issue.

With regard to the international context, the research presented in this paper highlights the significance and complexity of textbook task design and analysis in general. The multidimensional framework proved to be a powerful analytical tool for identifying students' opportunities to learn. The provision of a balance of different task types in textbooks and other curriculum materials, including providing more rich tasks, may be of interest to curriculum developers and researchers all over the world. The idea of embedding the dimension of mathematical activities, a domain which refers to *what* a student needs to do in a particular task, can be implemented in textbook analyzes worldwide, with the aim of better understanding national or international curriculum requirements and to gain insights into the opportunities to learn which students have in different countries. In this way, the results of textbook task analyses may help explain the differences in the performance of different countries in international student assessments, such as PISA or TIMSS.

The literature review showed that the textbook tasks in many countries predominantly require low cognitive demands. Correspondingly, it would be interesting to find out whether similar mathematical activities predominate in textbooks from different countries, and to what extent these correlate with the results from large-scale studies. For example, the findings on the high proportion of calculation and operation activities in geometry chapters raise a question about the required activities in geometry in textbooks in other countries. In this way, the results obtained in this study may contribute to the global discussion about the issues on contemporary geometry education and tendencies in mathematics education.

Acknowledgments

The author would like to thank Prof Dr Edith Schneider and Prof Dr Werner Peschek from the Institut für Didaktik der Mathematik – Austrian Educational Competence Centre, Alpen-Adria Universität Klagenfurt for all their helpful suggestions and discussions during the coding and analysing process.

Disclosure statement

No potential conflict of interest was reported by the author.

ORCID

Dubravka Glasnovic Gracin http://orcid.org/0000-0001-5195-4873

References

[1] Fan L. Textbook research as scientific research: towards a common ground on issues and methods of research on mathematics textbooks. *ZDM* Math Educ. 2013;45(5):765–777.

[2] Fan L, Zhu Y, Miao Z. Textbook research in mathematics education: development status and directions. *ZDM* Math Educ. 2013;45(5):633–646.

[3] Hopf D. Mathematikunterricht: Eine empirische Untersuchung zur Didaktik und Unterrichtsmethode in der 7. Klasse des Gymnasiums [Mathematics education: an empirical study in the 7th grade of grammar school]. Stuttgart: Klett-Cotta; 1980. German.

[4] Pepin B, Haggarty L. Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *ZDM* Math Educ. 2001;33(5):158–175.

[5] Robitaille DF, Garden RA, editors. The IEA study of mathematics II: context and outcomes of school mathematics. Oxford: Pergamon Press; 1989.

[6] Valverde GA, Bianchi LJ, Wolfe RG, et al. According to the book: using TIMSS to investigate the translation of policy into practice through the world of textbooks. Dordrecht: Kluwer Academic Publishers; 2002.

[7] Glasnovic Gracin D, Domovic V. Upotreba matematičkih udžbenika u nastavi viših razreda osnovne škole [Using maths textbooks in lower secondary education]. Odgoj Znan. 2009;11:297–317. Croatian.

[8] Howson G. Mathematics textbooks: a comparative study of grade 8 texts. TIMSS monograph no. 3. Vancouver (BC): Pacific Educational Press; 1995.

[9] Usiskin Z. Studying textbooks in an information age – a United States perspective. *ZDM* Math Educ. 2013;45(5):713–723.

[10] Mason J, Johnston-Wilder S. Fundamental constructs in mathematics education. London: Routledge; 2004.

[11] Sullivan P, Clarke DM, Clarke BA. Teaching with tasks for effective mathematics learning. New York (NY): Springer; 2013.

[12] Henningsen M, Stein M. Mathematical tasks and student's cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. J Res Math Educ. 1997;28:524–549.

[13] Stenmark JK, editor. Mathematics assessment: myths, models, good questions, and practical suggestions. Reston (VA): National Council of Teachers of Mathematics; 1991.

[14] Smith MP, Stein MK. Selecting and creating mathematical tasks: from research to practice. Math Middle Sch. 1998;3(5):344–350.

[15] Doyle W. Work in mathematics classes: the content of student thinking during instruction. Educ Psychol. 1998;23(2):167–180.

[16] Vincent J, Stacey K. Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. Math Educ Res J. 2008;20(1):82–107.

[17] Keitel C, Otte M, Seeger F. Text, Wissen, Tätigkeit: Das Schulbuch im Mathematikunterricht [Text, knowledge, action: the textbook in mathematics education]. Königstein/Ts: Scriptor; 1980. German.

[18] Love E, Pimm D. ´This is so´: a text on texts. In: Bishop AJ, Clements K, Keitel C, et al. , editors. International handbook of mathematics education. Dordrecht: Kluwer Academic Publishers; 1996. Vol. 1, p. 371–409.

[19] Zhu Y, Fan L. Focus on the representation of problem types in intended curriculum: a comparison of selected mathematics textbooks from Mainland China and the United States. Int J Sci Math Educ. 2006;4(4):609–626.

[20] Li Y. A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. J Res Math Educ. 2000;31(2):234–241.

[21] Huntley MA, Terrel MS. One-step and multi-step linear equations: a content analysis of five textbook series. *ZDM* Math Educ. 2014;46(5):751–766.

[22] Dole S, Shield MJ. The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning. Res Math Educ. 2008;10(1):19–35.
[23] Brändström A. Differentiated tasks in mathematics textbooks: an analysis of the levels of difficulty [master's thesis]. Luleå: Luleå University of Technology; 2005.
[24] O'Keeffe L, O'Donoghue J. The use of evidence based research on mathematics textbooks to increase student conceptual understanding. Int J Cross-Discip Subj Educ. 2011;2(1):304–311.
[25] Niss M. Mathematical competencies and PISA. In: Stacey K, Turner R, editors. Assessing mathematical literacy: The PISA experience. New York (NY): Springer; 2015. p. 35–55.
[26] National Council of Teachers of Mathematics. Principles and standards for school mathematics. Reston (VA): The National Council of Teachers of Mathematics; 2000.
[27] Organisation for Economic Co-operation and Development. The PISA 2003 assessment framework – mathematics, reading, science and problem solving knowledge and skills [Internet]. Paris: OECD; 2003 [cited 2016 Sep 7]. 194 p. Available from: http://www.pisa.oecd.org/dataoecd/46/14/33694881.pdf
[28] Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland. Beschlüsse der Kultusministerkonferenz: Bildungsstandards im Fach Mathematik für den Mittleren Schulabschluss [Conclusions of Standing Conference of the Ministers of Education and Cultural Affairs: educational standards for middle grade mathematics]. München: Wolters Kluwer Deutschland; 2003. German.
[29] Institut für Didaktik der Mathematik. Standards für die mathematischen Fähigkeiten österreichischer Schülerinnen und Schüler am Ende der 8. Schulstufe [Standards for the mathematical competencies of Austrian students at the end of the 8th grade]. Klagenfurt: Alpen-Adria-Universität; 2007. German.
[30] Glasnovic Gracin D. Requirements in mathematics textbooks and PISA assessment [dissertation]. Klagenfurt: Alpen-Adria University of Klagenfurt; 2011.
[31] Mayring P. Qualitative content analysis. Forum QualSoc Res [Internet]. 2000 June [cited 2016 Sep 7];1(2):[28 paragraphs]. Available from: http://www.qualitative-research.net/index.php/fqs/article/view/1089/2385
[32] Ministry of Science, Education and Sports. *Nastavni plan i program za osnovnu školu* [Educational plan and programme for elementary school]. Zagreb: Ministry of Science, Education and Sports; 2006. Croatian.
[33] Organisation for Economic Co-operation and Development. PISA 2012 results in focus: What 15-year-olds know and what they can do with what they know [Internet]. Paris: OECD; 2014 [cited 2016 Sep 7]. 44. p. Available from: http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf
[34] Remillard JT, Harris B, Agodini R. The influence of curriculum material design on opportunities for student learning. *ZDM* Math Educ. 2014;46(5):735–749.
[35] Thompson DR, Senk SL. The same geometry textbook does not mean the same classroom enactment. *ZDM* Math Educ. 2014;46(5):781–795.
[36] Weigand HG. Ziele des Geometrieunterrichts [Objectives of geometry education]. In: Weigand HG, Filler A, Reinhard H, et al, editors. Didaktik der Geometrie für die Sekundarstufe I. Berlin-Heidelberg: Springer; 2014. p. 13–34.
[37] Sinclaire N, Bruce CD. New opportunities in geometry education at the primary school. *ZDM* Math Educ. 2015;47:319–329.
[38] Baranović B, Štibrić M. Math teachers' perceptions of mathematics education in elementary and secondary schools in Croatia: Results of an empirical research [abstract]. In: Pavleković M, editor. Proceedings of the Second International Scientific Colloquium ´Mathematics and Children´ – How to Teach and Learn Mathematics; 2009 Apr 24; Osijek. Zagreb: Element; 2009. p. 96.
[39] Heymann HW. Allgemeinbildung und Mathematik [General education and mathematics]. Weinheim: BeltzVerlag; 1996. German.