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SURVEY PAPER

Transformation of the mathematics classroom with the internet

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Abstract
Growing use of the internet in educational contexts has been prominent in recent years. In this survey paper, we describe how the internet is transforming the mathematics classroom and mathematics teacher education. We use as references several reviews of use of the internet in mathematics education settings made in recent years to determine how the field has evolved. We identify three domains in which new approaches are being generated by mathematic educators: principles of design of new settings; social interaction and construction knowledge; and tools and resources. The papers in this issue reflect different perspectives developed in the last decade in these three domains, providing evidence of the advances in theoretical frameworks and support in the generation of new meanings for old constructs such as 'tool', 'resources' or 'learning setting'. We firstly highlight the different ways in which the use of digital technologies generates new ways of thinking about mathematics and the settings in which it is learnt, and how mathematics teacher educators frame the new initiatives of initial training and professional development. In this survey paper, we identify trends for future research regarding theoretical and methodological aspects, and recognise new opportunities requiring further engagement.

Keywords Humans-with-media · Learning environments · Blended learning · Mathematics teaching · Mathematics teacher education · MOOC · Hyper-personalisation · Collaboration · Learning management system

1 Introduction

The central theme for this special issue is the *evolution and transformation of the classroom* with the growing integration of the internet and interactive digital devices into mathematics teaching and mathematics teacher education. At the time of finalising this special issue, the entire globe is in the middle of the COVID-19 pandemic. Because of the extreme relevance of online and blended learning programmes at the current time, an editorial on the timeliness of this issue will be included in this special issue. Since the 1970s technology has changed mathematics education and it will certainly be a major factor in how education in the future differs from education today. Educators realise that we need to rethink the entire model of education and redesign it so that it is more student-centred. This means adopting new technologies, but it also means giving up on certain attitudes about what constitutes educational success. These new technologies also seriously influence the nature of mathematics, e.g., application of procedures is becoming less important and new ways of validation (and practising in general) in mathematics are being developed. However, it is beyond the scope of this special issue to explore this matter in depth.

With the rapid development of new digital technology, specific characteristics of these developments are continuously changing. After the *baby boomers* and the *Generation X* groups, new micro-generations evolved in cycles of about four years, giving birth to new concerns, new motivations and new challenges in all aspects of their lives (Morin 2016). These new micro-generations are as follows:

- The *Echo-Boomers* or first digital natives *Gen Y*, born between 1989 and 1994; with the establishment of the Web, they grew up with the new technology.

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- For the *net generation*, often called *Gen Z*, born between 1994 and 1998, the internet is an essential part of their lives. They are sometimes called the *hyper-connected 'selfie' generation* and are attached to their smart phones as if they have become extensions of their personalities.
- The *Post-Millennial Generation Z* or young mobile generation, born between 2002 and 2006, who did their learning through social networks and mobile technology.
- The youngest group of students, born between 2006 and 2010, is named *Gen Z—Silent Generation*, who have been connected since birth.

Although the micro-generations differ, there are strong common characteristics—they multitask and want information quickly using visuals, sounds, and colour from multiple multimedia sources in a novel or useful way (Dineva et al. 2019). Our current students have grown up in a digital world of computers, internet, and social online media such as Instagram, Facebook, Twitter, Google and other social networks (Jukes et al. 2010). They learn by interacting with other individuals online. Many students have a blog on the Web and a profile on the internet (Curtis 2009). They like to be active and collaborate using the latest technology and visualisation opportunities. Today's students prefer seeking their own information rather than being presented with it (Morin 2016), they prefer on-demand access to knowledge, disseminated over the internet, and absorb knowledge rapidly across different channels. Using networks to share and create new knowledge, they are in frequent contact with their friends.

The change in our students implies that the way of teaching should be completely adapted to meet these challenges and to respond to the new requests (Dineva et al. 2019). In recent decades a more social and connected Web has developed, supporting the idea of network learning. Open network learning environments are digital environments that empower students to conduct social networking, organise social contents and manage social acts by connecting people, resources, and tools by integrating internet tools to design environments that are transparent (Borba et al. 2016; Tu et al. 2012).

In a special issue of ZDM in 2012 (Borba and Llinares 2012), the emergent field regarding online mathematics teacher education showed key topics such as communities and networks of teachers in online environments; sustainability of these communities; knowledge-building practices in technology-mediated work group interactions; and online interactions among teachers. These topics also gave rise to new theoretical developments. The exponential development of interactive digital devices, and the use of the internet in teaching mathematics and online learning environment since the last special issue, support the need to frame some of these initiatives developing in the world (Borba et al. 2013; Borba et al. 2016; Trouche et al. 2013).

The development of digital technology use in mathematics education has been taking place in distinct phases (Borba et al. 2016). In recent years, the development of the internet has introduced a relationship revolution—communication has changed dramatically (Borba et al. 2016; Engelbrecht and Harding 2005a, b; Van de Sande 2011). This development brings us the possibility of two-way communication via the internet, and enhanced opportunities for collaborative learning. It also brings us the personalisation of the internet through personal devices. The role of social media is becoming increasingly important, moving the education process from a 'push' to a student-centred 'pull' approach in which the students become an integral part of many facets of the process (Martinovic et al. 2013). Furthermore, the introduction of digital resources and tools questions the efficacy of current teacher practices and traditional classrooms (Drijvers et al. 2013; Gueudet and Pepin 2020). Social aspects of the internet become more and more relevant and notions such as 'humans-with-media' emphasise that if media are changed, the entire knowledge-acquiring process may change (Borba et al. 2018). Moreover, as humans develop and construct new media, these media seem to transform and 'construct' a new human.

In this development process, the classroom, as we know it, may change entirely from a physical area with defined boundaries to a virtual environment including various components that will probably be determined by the student rather than only by the teacher. Mobile technology, personal learning environments, digital learning objects and other artefacts are 'stretching' the classroom, transforming the classroom, to the extent that it can hardly be recognised as such. Currently it seems clear that digital technology is 'deconstructing' the notion of the classroom. Flipped classrooms change the notion of what is in and outside of the classroom, and also change the roles of students and teachers. There is a profusion of online resources (e.g., widgets, videos), designed with respect to specific mathematical content, which transforms the presentation of content and allows students access to solving mathematical tasks and sharing their mathematical explorations. Our students also have to evaluate the quality of the knowledge disseminated over the internet—they need to be able to select valid resources.

On the other hand, the development of mobile digital technologies, such as forums, wikis, Twitter, Instagram and Facebook is allowing the new generation of different kinds of learning opportunities, supported in new social interaction spaces. These interactive technologies allow students to collaborate with their peers when they use multimedia and the internet, allowing new social ways of knowledge construction (Goos and Geiger 2012; Llinares and Olivero 2008). Finally, different blends are being forged between face-to-face education and online distance education.

In this paper, following mainly Borba et al. (2016) and other surveys and descriptions of the state of the art (Borba et al. 2013; Silverman and Hoyos 2018), we aim to support the building of the domain *blended mathematics education*. By addressing three main strands, the different papers in this special issue provide particular views and describe steps in each of the following areas:

- Principles of design: How mathematics educators enact the principles of design in Massive Open Online Courses (MOOCs) and blended approaches to designing professional development opportunities and mathematics teaching contexts.
- Social interaction and construction of knowledge: How technologies in online contexts support social interaction among participants as a medium to support mathematical knowledge construction and teaching competences.
- Tools and resources: Different meanings associated with the idea of online resources and how their use is conceptualised in different mathematics teaching contexts, given the emergence of new online mathematics resources and ways of teaching.

While the papers in this issue address some of the topics in these domains, this collection is far from exploring all the different paths of digital technology in the mathematics classroom. We conclude the paper by identifying some trends and issues to be developed in the future.

Before we go any further, we should mention that, as we were finalising this paper, it became clear that the COVID-19 crisis was more than ‘a little cold’ as some prominent politicians claimed. The COVID-19 pandemic has ‘paused’ the world, and it has paused many mathematics education activities (e.g.. ICME 14, PME 44). In a certain way it also paused our paper. It seemed odd to write about digital technology, without connecting it to the current situation, which strongly impacts the world-wide use of digital technology in teaching. On the one hand, the worldwide lockdown increases the rate of change of using digital technology in education but at the same time it creates an awareness of the need to feel connected to each other. Rather than rewriting the paper, we added a section at the end in which we ‘blend’ the formal conclusions of the paper with questions and doubts that arose from the ‘agency’ of COVID-19, or that were catalysed by it.

2 How mathematics teacher educators enact the principles of design in blended approaches and MOOCs

Scaling up professional development for mathematics teachers and mathematics teacher educators is a need linked to mathematics teaching improvement. Technology and internet networks are allowing the design of courses for a large number of participants and in ways that were not available in more traditional contexts (Silverman and Hoyos 2018). Different large-scale professional development programmes have been adapted to impact practice (Carney et al. 2019). In particular, two different approaches are been developed to design and scale up professional development: Massive Open Online Courses (MOOCs) and blended approaches. MOOCs are online courses aimed at ample participation and open access via the Web (Avineri et al. 2018), opening up the possibilities for teachers to engage in a variety of learning opportunities.

Scaling up of professional development is concerned with how the programme is implemented across multiple instructors and settings and about the participation effects, such as course participants’ changes in practice. Some initiatives focused on identifying factors that contribute to the sustainability and scaling up of professional development (Goos et al. 2018), recognising that examining their impact is complex, and lacking solid frameworks to explain learning in open online settings (Joksimovic et al. 2018). Digital technologies allow new approaches to design, implement and analyse large scale professional development and determine the influence on teacher practice and student achievement (Bell et al. 2010). For example, the internet allows development of new approaches through which it is possible to support the sustainability and scaling up of professional development focused on the introduction of dynamic technologies in using Web-based toolkits (Clark-Wilson and Hoyles 2019). The growing activities of designing MOOCs and using blended learning approaches are defining new issues related to how design principles can be generated.

2.1 MOOCs

Borba et al. (2016) mentioned MOOCs as a relevant theme, as follows:

> The potential of MOOCs to disrupt the institutional and hierarchical nature of traditional education, offering students opportunities to access courses without prerequisites, without fees (unless they require a record of course completion), and the potential of MOOCs to affect access to and the quality of mathematics education is not well understood. (p. 606)

Right now, the reflection on the design, implementation and sustainability of MOOCs in mathematics education, generates theoretically informed perspectives and how these issues can inform decision making.

The design of MOOCs in different cultural contexts places emphasis on key aspects such as the possibility of sharing materials and ideas, and implicates methodologies of mathematics teaching and the different theoretical frameworks used to support their designs. In this special issue, the two papers focused on MOOCs present different conceptual perspectives to support the principles of design. The common feature in both papers is how the principles of design were theoretically informed and how the conceptual frames influenced the implementation of MOOCs.

Taranto and Arzarello (2020) report on a conceptual framework that guided the design of their MOOC and its use in interpreting the dynamics characterising teacher education. The conceptual framework presents the necessary hybridisation of three different theoretical approaches for mathematics teacher education through MOOCs, namely, the meta-didactical transposition model, the instrumentation/ instrumentalisation process and the network of knowledge of connectivism. Taranto and Arzarello argue that the conceptual framework allows the identification of specific features of the new learning environments defined by the MOOCs—how the interactions between participants change their knowledge and beliefs.

Hollebrands and Lee (2020) adopted a different conceptual approach to report on the effects of design principles of MOOCs. They examined how design principles were enacted in the development of the MOOC and how these influenced the engagement of participants. The principles considered were self-directed learning, learning from multiple voices, job-connected learning and peer-supported learning. From these principles and using the Interconnected Model of Professional Growth (Clarke and Hollingsworth 2002), Hollebrands and Lee considered the ways in which elements of the external domain influenced the personal domain of participants. The conceptual framework used emphasises the teacher change occurring through the process of enacting and reflecting on practice. In particular, they studied how the personal domain of participants in the MOOCs—knowledge, beliefs, and attitudes—interacts, as participants engaged with and reflected upon elements from the external domain, namely, readings, tasks, and frameworks, that define the MOOCs. How design principles are enacted in the design of MOOCs and how they support the learning opportunities for participants became a point for reflection.

A common feature, in the paper of Taranto and Arzarello, and that of Hollebrands and Lee, is their reflection on how the design principles adopted by designers of MOOCs influence participants' personal and social aspects. However, the ideas that support the design principles differ. On the one hand, Hollebrands and Lee use research-based practices used in face-to-face mathematics teacher education and design principles for the online environment to frame their decisions. On the other hand, Taranto and Arzarello hybridise three theoretical perspectives on mathematics teaching and learning to create a new conceptual framework for the purpose of making decisions as designers, researchers and teacher educators.

Other issues are related to how designers consider time constraints to strengthen the impact of the interventions. Three of these needs can be identified. Firstly, an issue is how theoretical perspectives can help us to improve our understanding of the processes of design and implementation of MOOCs and blended initiatives. Secondly, we should identify factors contributing to learning and sustainability in the MOOCs and blended approaches—factors influencing teacher participation in these initiatives. Finally, we have to evaluate different MOOCs and blended learning approaches across the educational settings to provide information about the role played by different contextual and cultural variables.

2.2 Blended approaches

Blended learning courses often tend to replicate traditional teaching methods and are developed for reasons of efficiency by making minor changes to pedagogy with additional resources and supplementary materials (Graham 2006), rather than by employing new views of pedagogy in teaching and learning in a significant manner (Collis and Van der Wende 2002). Because it is quite challenging to develop a rich and effective blended course, widespread adoption of such programmes is proving a challenge (Torrisi-Steele and Drew 2013).

Blended learning provides students with both delivery options, whatever their current stage of development may be (Chaney 2016; Vasileiou 2009). By integrating online learning into the system, blended learning expands the learning environment into the virtual world where traditional limitations are removed. Through the online component, differentiation between student needs becomes easier and combines with the social aspect of the actual classroom to create a strong learning system.

A blended learning system includes the important face-to-face interaction that Vygotsky considered to be vital and thus provides all of the benefits of the social aspects of learning (Ting and Chao 2013). Blended learning recognises the need for peer interaction and practical application in order to bring learning to maturity. Blended learning engages students since it offers them an opportunity to develop their own opinions, consider new ideas in collaboration with other students online, and try out their own ideas in a relatively anonymous environment (Holley and Oliver 2010).

Three papers in this special issue describe blended approaches to professional development for mathematics teachers as a reply to contextual needs, namely, improving primary and middle teachers' mathematics knowledge for teaching, focusing on 'out-of-field' mathematics teachers, and supporting the introduction of technology in mathematics teaching. They describe professional development initiatives from Chile, Ireland, and South Africa. One other paper describes a blended approach for first year engineering mathematics in Australia.

Martinez, Guíñez, Zamora, Bustos and Rodríguez (2020) describe a blended learning professional development programme for primary and middle school teachers aimed at developing mathematical knowledge for teaching. The ways in which face-to-face workshops and virtual activities are integrated illustrates the use of design principles by an instructional model characterised by a learning sequence and a construction model. The construction model is formed by four types of activities: activation, analysis, institutionalisation and practice. The authors underline that the design principles allow participants to have a high level of autonomous self-directed online learning and have learning opportunities to unpack and analyse elementary mathematics from a teaching perspective. Three general principles are used to design and analyse the implementation of this programme: the constructivist view of learning; contextualised problem-based learning, and that the teaching of mathematics requires specific knowledge. These principles are operationalised during the design, for example by means of the design of online activities around contexts operating as a general frame to problematise various aspects of the content. This feature allows generated learning situations in which tasks help to develop different aspects of mathematics knowledge for teaching.

Goos, O'Donoghue, Ní Ríordáin, Faulkner, Hall and O'Meara (2020) focus their initiative on secondary teachers that have to teach mathematics with no formal training or education in the field—*out-of-field teachers*. They analyse the design principles underpinning the development and delivery of a blended learning programme of professional development for out-of-field teachers of secondary school mathematics in Ireland. Three theoretical frameworks informed their analysis of the blended learning design. The first framework examines definitions, dimensions, and rationales for blended learning, the second framework characterises out-of-field teaching as a boundary-crossing event, and the third concerns effective teacher professional development using structural and core features. They integrated these frameworks within a blended learning context, attempting to find out how such an environment contributes to effective professional learning for out-of-field mathematics teachers. They found that engagement is long in duration and intensive among participants, but also learnt that epistemic considerations must be addressed when professional mathematicians engage with school mathematics teachers in curriculum development. For example, the nature of blended learning deserved more attention because a better appreciation could lead to better learning opportunities and outcomes for teachers.

A relatively new development in the professional development of teachers, with a strong foundation in mathematics education, is called *lesson study*. It is based on teachers collaborating to design lessons. This collaboration, however, can be challenging for isolated teachers who cannot communicate face-to-face on a regular basis. Joubert, Callaghan and Engelbrecht (2020) conducted a study in which they presented a blended mode course in the use of technology in teaching to practising teachers from different subject fields, including mathematics. The purpose of this course was to develop teachers' knowledge and skills in the use and integration of mobile technology in their teaching. In their study they investigated how lesson study can be adapted into a blended format to support isolated teachers who cannot meet face-to-face on a regular basis, using an Learning Management System (LMS) to communicate. They identified aspects that should be incorporated into a blended lesson study process to support isolated teachers in teaching with technology. A framework with three dimensions emerged, supported by aspects relating to teachers' collaboration, to the instructional design, and to the iterative improvement process.

Quinn and Aarão (2020) addressed the issue of what the contributions of online and face-to-face experiences should be in an ideal first year engineering mathematics blended learning experience. Developments trialled included online quizzes for changing attitudes and teaching foundational concepts, online lecture options in addition to face-to-face lectures, the adoption of what they called *board tutorials* (a technique that supports peer-to-peer learning and improves engagement), and learning outcomes supported by a problem-solving approach for more complex engineering modelling problems and online interactive problems. The individual student could determine the amount of blending. They found online quizzes to be a useful tool that, coupled with supporting material, can level the playing field in relation to the assumed knowledge for students transitioning to university. In addition, the use of online quizzes of foundational knowledge efficiently replaced a myriad of potential time consuming tutor–student conversations, such as diagnosis, feedback, independent work and the re-assessment, which would be impractical in a face-to-face environment. Quinn and Aarão also used board tutorials, changing the passive tutorials from 'mini-lectures', to experiences that both students and teachers valued highly. Quinn and Aarão argue that successful implementation of blended learning in first year mathematics occurs when one automates as many of the routine conversations between teacher and student as possible, and reserve what tutor–student interaction time you have for high-impact face-to-face learning activities such as board tutorials and for supporting project work.

The papers in this special issue that are mentioned in this section and centre on the design principles of scaling up professional development initiatives, show the needs that should be addressed by research. These needs focus on how to understand teacher change as a consequence of participating in MOOCS and blended learning initiatives.

3 Social networking and construction of knowledge

Social constructivist learning theory, as proposed and developed by Vygotsky, has long been seen to improve student engagement and learning, and many studies support this theory (Grady et al. 2012; Schmidt 2013). In their research on instructional technology, Pepin et al. (2017) found that many studies on the topic are predominantly framed by sociocultural theories underlining the role of discourse in learning. The development of information and communication technologies has undergirded the emergence of new forms of discourse and has the potential to change social relations and the ways through which we come to understand the development of knowledge (Llinares and Olivero 2008; Llinares and Valls 2010; Clay et al. 2012). Interaction in online contexts allows us to consider the links between the processes of meaning construction and of participation underpinning learning. The links between construction of meaning and participation is scaffolded by social artefacts or tools, such as online collaboration, mind mapping or sharing narratives in online forums to discuss relevant aspects in mathematics teaching. Sharing interaction spaces, such as those that facilitate asynchronous[1] online discussion, creates opportunities for participants to reorganise their knowledge in the course of the social interaction. In this sense, the affordance of new media helps participants to communicate knowledge in multimodal ways generating different ways of discourse.

Different theoretical perspectives about learning and knowledge have been used to understand the links between interaction in online contexts and the construction of knowledge to conceptualise technology-mediated interaction (Clay et al. 2012; Goos and Geiger 2012; Llinares and Valls 2010). Recently, new perspectives have emerged that consider how newly introduced media reorganise human thinking, favouring connections and group discussion (Borba et al. 2018). These approaches underline that media act and interact in knowing when participants interact collaboratively (in blended or online contexts). It is assumed that the nature of the interface between participants affects the potential for knowledge building (Borba and Llinares 2012). The notion of knowledge construction in collaborative settings assumes that the nature of participation and content of discourse is related to how the process of construction of knowledge is developed. One aspect here is the different forms of discourse that the participants adopt, and how participants create points of focus around a nexus that organises the negotiation of meanings. The media inserted in the specific online context can provide different ways of articulating the discourse for the participants to notice, represent, interpret or use theoretical elements. Two papers in this special issue focus their attention on different ways of articulating the discourse in collaborative contexts, namely, the construction of collaborative mind maps, and sharing and discussing narratives about teaching in an online forum. These two different types of discourse generated in collaborative online contexts focus the participants' attention on meanings, combining participation and reification processes. In these initiatives, several aspects of mathematics knowledge for teaching emerge, and mathematically significant pedagogical opportunities for mathematics teaching that builds on student thinking.

Cendros-Araujo and Gadanidis (2020) report on how the use of collaborative and multimodal technologies supports the construction of knowledge. The specific context is an undergraduate blended course in a primary teacher education programme in Canada, including the use of different tools for online collaborative mind mapping activities. The theoretical frame underlines the notion that new media reorganise human thinking. In this case mind maps are understood as ways of visualising mathematics education knowledge that is collectively constructed, and the different technological tools are used as means to support and organise topics, creating visual connections and inserting other different semiotic representations such as videos or images. The analysis of artefacts created by the prospective teachers (mind maps) and the flux of interaction during the construction process enabled Cendros-Araujo and Gadanidis to report on a grounded theory of knowledge building through mind mapping. They describe how pre-service mathematics teachers construct knowledge when they interact through online collaborative mind mapping and how the different technical characteristics of the tools impact on the reorganisation of knowledge by the different means of discourse. In this case the technological tools in a collaborative context allow new ways of knowing to be shown through a new type of discourse—such as integrating the visual and text, and combining graphical, narrative and symbolic realisations.

Fernandez, Llinares and Rojas (2020) report on the development of prospective secondary mathematics teachers' noticing as a consequence of sharing narratives of their own teaching in an asynchronous forum with other colleagues and university tutors. In this case, narratives of his/her own teaching are the artefacts that allow a prospective mathematics teacher to reify what is noticed on each occasion. The context is an online distance teacher education

[1] 'Synchronous' means 'in synch' (at the same time), while 'asynchronous' means 'out of synch' or at a later time.

programme in Costa Rica aimed at developing the competence of prospective teachers to identify *mathematically significant pedagogical opportunities* (MOSTs) in order to build on student thinking. In this intervention, integration of thinking and doing is evident when prospective teachers reason about the teaching events in order to decide how to act. This paper shows two features of how online social interaction influences the development of teaching competence, such as noticing. The first feature is defined by the role of writing and sharing narratives in an online forum, and the second is defined by the role of feedback as a way of interactive collaboration. In this case, the development of noticing was related to aspects of discourse (communication processes) such as identifying MOSTs, providing more details of students' thinking and providing explicit reasons behind the prospective teachers' actions.

Both papers can be considered to be instances of how an online context, supporting the participants' interaction, helps to rearrange their own activity and, at the same time, as a way in which participants think and share their knowledge using certain technologies. These ideas are supported by the description of stages of knowledge building through mind mapping—introducing a topic, building a concept, and making sense of the whole picture—and by the ways in which the MOSTs are taken advantage of, and by the reasons given for the prospective teachers' actions. These ideas conform to the approach called humans-with-media, developed by Borba and Villarreal (2005). Furthermore, collaborative construction of mind mapping and sharing narratives of their own teaching can be seen as resources for thinking and communicating knowledge when participants are attempting to build new knowledge. The papers underline three relevant features, as follows. Firstly, how the media supporting social interaction can deploy several semiotic possibilities, giving forms to different types of discourse. Secondly, how sharing and co-creating tools generate the context in which participants can compare and share their ideas and justify and evaluate their arguments. Finally, the role played by cognitive scaffolding such as feedback from others and prompts to carry out the activities. These features help us to understand a little more about the relationships between social interaction in online environments and the knowledge construction and development of teaching competence.

4 Resources, tools and new learning environments: changing the relationships between mathematical knowledge, learners and teachers

The development of new digital technologies provides new opportunities to mathematics educators, and new ways of thinking about how the teaching and the design of teaching–learning environments evolve, generating new practices and establishing goals which we did not think about several years ago. Arcavi (2020) pointed out that, in the educational field, tools facilitate the performing of a task, thereby extending the power of human capabilities and amplifying the power of the mind. Although these tools impose constraints on learning activities, they also generate new opportunities for learning. These opportunities for learning are linked to new ways of looking for information, which shape students' mathematical experiences (Van de Sande 2011) and determine how practising teachers can make curricular decisions (Cooper et al. 2019).

In this special issue, several types of tools are analysed and discussed as they concern mathematics teaching and mathematics teacher education, showing new ways of thinking but also defining how we frame our activities as mathematics educators. Tools mediate the actions of mathematics educators, allowing us to design new learning environments, but this new context also determines new ways of thinking. There are several papers that focus on how using different types of tools determines changes in the relationships between mathematical knowledge and learners and teachers. Furthermore, the development of the new digital tools (or new uses of old tools) has generated the necessity of exploring innovative uses of digital technology.

The papers in Sect. 4.1 show three different approaches to digital tools: describing innovative practices; making explicit principles of design, and developing a new teaching stage. Innovative practices include using arts and digital technology to create virtual instruments with music software (Scucuglia) or the production of videos as a resource to produce meaning and to change the dynamics in classrooms, reported on by Oechsler and Borba. Another innovative practice is how students in mathematics courses can benefit by seeking assistance on the internet for help in solving their mathematical problems (Sanchez-Aguilar and Esparza Puga).

A second approach, as explored by the papers in Sect. 4.2, is to consider how new digital tools are framed by conceptual perspectives. In particular these perspectives influence the design of assessment tasks for computer based assessment to assess students' mathematical learning (Yerushalmy and Olsher), or they influence the production of tools such as video as resources to support teachers' professional learning, in which existing resources support teachers' professional learning (Bennison, Goos and Geiger). Finally, they illustrate how designing a new organisation of teaching supported by online access to video, shows that the typical lecture and homework elements of a course can be reversed in a flipped classroom environment (Voigt, Fredriksen and Rasmussen).

4.1 Tools, new ways of thinking and new practices

The new ways of using different digital tools portray a range of different teaching scenarios. The papers in this special issue consider the different meanings associated with the design and implementation of resources, tools and learning contexts, and changes in mathematics learning perspectives and the digital tools. The meanings associated with the notion of design and implementation of resources, tools and learning contexts, change mathematics learning perspectives; and the digital tools presented in different papers in this special issue show new ways of interaction between students, teachers and mathematical knowledge, defining new practices. Traditional tools, such as videos and assessment tasks, are used differently and define new practices. Furthermore, the accessibility of huge sources of information provided by the internet, generates new needs, such as how to determine criteria in looking for help to solve problems, or explicit conceptual frameworks to develop online resources to support teacher professional learning or to assess students' mathematical learning. There is also an increased need for students to think critically about the wealth of new opportunities associated with all the various resources with which they can interact, and how they can be empowered to develop this critical attitude towards evaluating new resources.

Scucuglia (2020) reports on the integrated use of the arts and digital technology in mathematics education creating pedagogic scenarios. In this case, the creation of a virtual instrument with music software is used for developing new ways of mathematics teaching. This approach generates new teaching scenarios that provide aesthetic mathematical experiences. The theoretical frame takes into account that mathematical knowledge is not produced by humans alone but by humans-with-media, in this case, analysing the pre-service teachers' mathematical experience when they are engaged in musical production. This new learning scenario underlines the link between music and mathematics education. The focus on the link between mathematics and music using digital tools opens possibilities for considering the hypothetical potential of thinking-with-media and music in terms of representation, patterning and algorithms, as aspects of computational thinking.

Oechsler and Borba (2020) investigated how the creation of videos with mathematical content, by the students themselves, may contribute to the process of changing the classroom, and how this activity can become a teaching and learning tool. They ground their discussion in social semiotics, a theory that considers the context of production and the negotiations between actors, to analyse how the production of videos in the classroom can help in the communication of mathematical knowledge and in the change of the dynamics in the classroom. They found that video production provided a classroom dynamic in which students could become protagonists of the teaching and learning process, with teachers mediating this activity. They argue that video production is a different way to express mathematics, and it is particularly well-suited to expressing what students have understood. Using videos, a new kind of mathematics can emerge in the classroom, integrating its traditional symbolic language with other modes, such as language, gesture, image and music. In production of the videos, students showed their understanding of the content, and through this activity, the students themselves became aware of their difficulties and sought ways to overcome them. In this sense, video production assists in encouraging students' discussion and reflection about content and its exposition to produce meaning and promotes a change in the dynamics of the classroom, breaking the barrier between the classroom and the outside world. This activity is seen as a new, emergent facet of education, in which the student searches for content outside the classroom.

Sánchez Aguilar and Puga (2020) used monitoring software to observe how a group of students use the internet to solve a mathematical task, whose mathematical underpinnings they are not completely familiar with, to produce a characterisation of the help-seeking behaviours that students display. They made use of self-reports, complemented with the analysis of students' solving of mathematics tasks, supported by the use of monitoring software. They found that students manifest instrumental help-seeking behaviours mostly associated with the procedural items of the task. A general pattern of behaviour, manifested by the students who participated in the study, was dominated by the use of search engines and keywords to identify sources of mathematical help. They observed how the internet and its resources were fundamental for some students to seek for help, and that, with this help, students were able to successfully solve a mathematical task on a topic partially unknown to them, which could be interpreted as a positive development of a self-regulated learning strategy. However, some students only used appropriate keywords to perform the help-seeking process—so students could find ways to solve mathematical tasks in which there was no need to exert a priori reasoning about the structure of the task or the nature of the mathematical situation at stake, before deciding on specific algorithmic procedures.

A common issue, considered by all three papers in this section, is the use of digital tools to emphasise a purposeful, jointly undertaken activity (creating virtual instruments with music software to underline mathematical thinking, videos with mathematical content, and using the internet to look for help). In these cases, the use of different types of tools shows how to create spaces for multiple voices in the learning environments, underlining the technological mediation played by the digital tools. These new uses of tools define new practices in mathematics education.

4.2 Developing new tools

The design of digital tools has generated the necessity to make the conceptual framing underpinning their development more explicit. The development of new tools strengthens the growth of this field, linking the quality of available resources and learning. This is the focus in papers in this issue by Bennison, Goos and Geiger, and by Yerushalmy and Olscher. Bennison and colleagues report on a research-informed instructional design approach to developing an online resource to support teacher professional learning. This approach provides a framework to determine the utility of existing resources and identify new needs, and also a conceptual framework to inform the development of new sets of videos. The specific focus is how video resources targeting specific teacher learning needs can be designed and made available via open access online.

Yerushalmy and Olscher report on a study concerning a special kind of task for assessing students' reasoning skills when establishing the validity of geometry statements about the similarity of triangles. By creating examples, students have to verify claims that argue for conjunction or disjunction of given relations. They focus on characterising the properties of the conjunction/disjunction design for automatically assessing conceptions related to examples generated by the learner with interactive diagrams. Their analysis shows that the Seeing the Entire Picture (STEP) environment, which supports interactive example eliciting tasks, and the design principles of conjunction and disjunction of geometric relations, enabled them to assess the students' exploration of the logic of universal claims, characterise successful and partial answers, and differentiate between students. By analysing the student-generated example spaces, they explored the opportunities of the environment and the specific task design pattern to automatically provide feedback and assess students' mathematical skills based on logical relations between examples and universal statements.

The papers of Bennison, Goos and Geiger (2020), and Yerushalmy and Olsher (2020) show how mathematics educators should take into account the conceptual frame underpinning the design of new digital tools (to provide opportunities for teacher professional learning and to asses students' mathematical learning). Explicit principles of design to develop tools (and determine ways of using them), is a growing field in mathematics teacher education (Van Es et al. 2020) and teaching mathematics (Leung and Baccaglini-Frank 2017).

4.3 New ways of teaching: flipped classrooms

The idea of a *flipped classroom,* where students watch earlier prepared lectures outside the classroom, has recently become quite popular in blended learning (Schmidt 2013).

In this pedagogical model, students completed preparatory activities (e.g., readings and reflections) before class and then participated in collaborative activities in class. (Crouch and Mazur 2001).

The flipped classroom approach in mathematics education is related to increasing in-class time for task/practice, the possibility of integrating new knowledge with existing beliefs and real-time feedback. But challenges are also reported, such as students' unfamiliarity with flipped learning, and significant start-up effort on the part of instructors (Lo et al. 2017).

Voigt, Fredriksen and Rasmussen (2020) conducted a study on flipped classrooms. In their study, they addressed the efficacy of using a flipped classroom approach on student outcomes. They accounted for the classroom activities and learning theories used to design the curriculum, uniting the at-home video and in-class curricular components of the flipped classroom via design heuristics that empowered students to think critically about mathematical problems individually before engaging with the task in a collective environment. They illustrate how elements of the instructional design theory of Realistic Mathematics Education (Freudenthal 1991) and Culturally Responsive Pedagogy (Ladson-Billings 1994) influenced the written and hidden curriculum and how those considerations were then experienced by calculus students at a Norwegian university, as part of the enacted components of the curriculum. By linking the content presented in the video lectures with the experiences of students inside the classroom, they highlighted how design theories can be leveraged to create a richer flipped classroom model and provide an opportunity to analyse critically how flipped classrooms can be designed in a way that values the diversity of student experience and moves beyond a transferable mode of learning.

5 Implications for future directions

From the outset there has been uncertainty about how effective online teaching may be (Cavanaugh et al. 2004; Chaney 2016), but because of the accessibility of computer technology in classrooms, the popularity of using digital tools has grown rapidly throughout the educational systems of the world. As a result, using digital tools has attracted the attention of researchers who embark on the process of empirical investigation needed for thorough analysis (Chaney 2016).

In 2016, Borba et al. (2016) identified five trends of development in e-learning in mathematics education that need to be addressed:

1. The relationship between students and mathematics created by student access to mobile technologies disrupts the traditional flow of mathematics knowledge from

teacher to student, and that is not well understood from a research perspective.
2. The role of MOOCs, disrupting the institutional and hierarchical nature of traditional education is not well understood.
3. The availability of online mathematics learning resources means that many students now turn to these resources before they consult a teacher or a textbook, raising questions about how the resources are facilitated to foster conceptual understanding.
4. Current technologies, such as social media, provide extensive collaborative and social networking affordances. This raises questions about the design and use of LMSs and personal learning environments.
5. Teachers are still uncertain about the amount and nature of blending in blended learning courses and how to employ a flipped classroom model to make the classroom a place for extension and elaboration rather than direct instruction.

Although some of the issues in these trends are being addressed, the questions are still very much open. From the surveyed literature, it is becoming increasingly clear that more empirical evidence is needed to determine the effectiveness of online or blended instruction in classrooms at all levels of mathematics education and how the use of digital tools determines new practices.

The nature of mathematics is also changing. With computing devices that can do the procedural mathematics faster and more accurately than humans, there is a shift from seeing mathematics as an application of procedures to an emphasis on creative problem solving (Devlin 2011).

Along with the trends mentioned above, we identify four domains that help us to see changes caused by the use of the internet: the changing classrooms; new ways of thinking and human-with-media; collaboration in online contexts, and hyper-personalisation of learning.

5.1 The changing classroom

Singh (2018) related a story of a mathematics teacher who used to give out “insanely hard” mathematics problems. He encouraged students to get help from any teacher in or outside the school, but excluding himself. His main objective was not really to get students to arrive at the correct answer or solution—he wanted to initiate mathematics conversations outside the classroom. He wanted people talking about mathematics. His ideas are central to the new approach of expanding the classroom outside the boundaries that we are used to.

The physical classroom, as we know it, is changing. Menninger (2011) encouraged educators to seriously consider and discuss the changes needed in the classroom. He wanted educators to regard teaching as a work of art, in that.

> …a total work of art serves not only as an all encompassing intellectual, emotional, and spiritual experience, but also as a means of conceptualizing the act of teaching itself; instruction should inspire while it informs, just as art informs as it inspires. (p. 97)

Many current initiatives take a technology push approach in which learning content is pushed onto a group of students in a closed environment in a one-size-fits-all, centralised, static, top-down, and knowledge-push as in models of traditional learning. Researchers who feel a shift towards a more open student-pull model for learning are needed—a shift towards a more personalised, social, open, dynamic, and knowledge-pull model (Borba et al. 2016). Chatti et al. (2010) suggested the 3P learning model, consisting of three core components, namely, personalisation, participation, and knowledge-pull, as a new approach for addressing the growing complexity and constant change in knowledge that is required for the new generation.

The hyper-connected students of today live in a world of instant interpersonal communication and unlimited access to information and educational resources (Christen 2009). To an extent, this networked world, and the powerful learning tools it offers, has started to penetrate the typical classroom. Schools should take full advantage of the twenty-first century learning technologies. We are entering an educational transformation that aligns the learning with the learners themselves and the employment that awaits them after they leave school.

We may think that few people still believe that ‘the role of the teacher is to transmit information’. However, many teachers-with-blackboard, or teachers-with-power-point are still considering themselves as *information transmitters*. Almost parallel to this issue, at the end of last century there was a heated debate about distance versus face-to-face education. As many researchers predicted, it now looks as if a blended, changing classroom is taking shape. Whether one calls it face-to-face, blended or distance, it almost always is a blended experience, in which on the one hand, students who are in a traditional classroom use internet and mobile technology constantly, and in which on the other hand, almost every distance education course has some face-to-face components.

As argued by Souto and Borba (2018), the internet has become the main source of transmitting information, changing traditional classrooms. The role of teachers, and of the community (face-to-face and virtual) is to build knowledge and propose new problems that have not already been solved on the internet. Together with these approaches, we still have (and it seems necessary) to have traditional instruction in many instances. Different ways of communicating mathematical ideas transform the production of knowledge, and it seems likely that the classroom (as we currently know it) will not fit in future education systems. The papers in this issue are instances of how the meaning of the classroom is changing in mathematics teaching as well as mathematics teacher education.

5.2 New ways of thinking and humans-with-media

In Vygotsky's theory, social interaction with others plays the primary role in the construction of learning—interaction with adults/teachers but also with peers. This interplay enables the student to move to higher levels of understanding and achievement (Blatchford et al. 2003).

The traditional online learning environment was first viewed with some skepticism and expected to be less effective at developing higher cognitive thinking processes than traditional classroom learning (Chaney 2016; Cicconi 2014). However, with the currently available features, blended learning incorporates the social aspect more fully through the presence of many other students, teachers and online resources. Chaney (2016) suggested that in blended learning the combination of a human teacher with online resources provides an effective way for students to construct learning socially (Chaney 2016). Researchers suggested that online blended learning fits well with Vygotsky's concept of a zone of proximal development despite challenges that arise (Cicconi 2014; Deulen 2013).

Borba et al. (2018) claimed that technological advances have changed societies. In particular, although not very rapidly, educational processes are being transformed (Almeida 2015). As students incorporate the internet into the classroom, digital technologies invade the teaching process (Borba 2009).

There is a growing relationship between humans and media, as originally proposed more than twenty years ago. Some authors have claimed that artefacts shape the human mind, but Borba and Villarreal (2005) documented that things happen the other way around as well: humans shape technology beyond the design of tools and of digital tools. Besides, technology is seen as having agency. Digital technology is saturated with humanity in its design and in its conception, and humans are impregnated by technology, and in particular digital technology.

Souto and Borba (2018) showed examples of how the third generation of activity theory may be transformed by the notion of humans-with-media as agents of production of knowledge. From the theoretical perspective of human capital and activity theory (Souto and Borba 2018), artefacts, community, and subjects are separate vertices of the multiple triangle diagram that illustrates activity in a social approach. Souto and Borba (2018) show examples of the internet being community and subject. The notion of humans-with-media, which has as one of three pillars the first generation of activity theory, with the notion of reorganisation of thinking (Tikhomirov 1981), may now transform the current, and more thorough third generation of activity theory.

5.3 Collaboration in online contexts

Collaboration in learning is becoming increasingly possible and popular in new teaching and learning contexts from the internet, and takes place in different formats. The concept of *personal learning environments* (PLE) was introduced through the work of, e.g., Attwell (2007), Chatti et al. (2010) and Wild et al. (2010). PLEs are systems that enable students to take control of their own learning, setting their own learning objectives, and managing their own learning content to achieve these learning objectives (Borba et al. 2016). A PLE can consist of subsystems, such as a desktop application or some web-based service integrating formal and informal learning, using social networks, and could include collaboration possibilities, such as small groups, to connect a range of resources and systems in an individual space.

PLEs differ from *learning management systems* (LMS) in that the LMS is course-wide (or institution-wide), while a PLE is individual. When students do not have control over what is taught but do have control over what is learnt (Tu et al. 2012), they create a PLE, a collection of all tools they use for learning, thereby enabling a student-controlled integration of myriad learning tools and services into a personalised space (Bidarra and Araújo 2013). The idea of a *personal learning network* (PLN) is related to the concept of a PLE. PLNs extend the PLE framework to include an informal learning network of people with whom to connect for the specific purpose of learning (Borba et al. 2016). In a PLN there is an understanding among participants that they are connecting for the purpose of active learning (Lalonde 2012). Although these environments are not commonly used in mathematics education yet, they are well-known concepts in other disciplines, such as computer science. It is envisaged that, in the foreseeable future, an increasing number of students and institutions will embark on this route of collaborating online to support their learning.

The issues regarding collaboration in online contexts in the papers of this issue underline some of the latter features, including, in particular, how the media supporting social interaction deploy several forms of semiotic possibilities. Also, they underline how features of participants in interactions, such as justifying their positions and evaluating their arguments, are intermingled with the cognitive scaffolding that is more difficult to identify in traditional teaching settings.

5.4 Hyper-personalisation of learning

Students no longer recite their lessons in chorus as they did years ago, but we are still far from a really *personalised* educational system (Paludan 2006). Despite our awareness of a disparity between students, the normal practice is to lump learners together by date of birth. Today, unique, personal characteristics such as creativity, a sense of humour, and special competencies are recognised as important in education (Paludan 2006). Learning can be enhanced when the instructional process accommodates the various learning styles of students (Lin et al. 2017).

New teaching contexts, with the use of the internet, offer every student a personalised approach to learning where they control their own pacing and where they can see themselves as successful students (Staker 2011). In fact, some authors are of the opinion that students should be allowed to choose their own learning pathways (Chaney 2017).

> [They] are able to select learning formats to fit their changing needs…It is not the role of the teacher to prescribe the nature of the blend. (George-Walker and Keeffe 2010, p. 12)

Not everyone agrees with this understanding of the role of the teacher, but the internet provides more options from which students may choose (Chaney 2016). It holds the potential of individualising the learning process to provide for the individual needs of each student (Vasileiou 2009), with participants taking ownership and responsibility of their learning processes and of the tools that they use (Verpoorten et al. 2009).

The idea of *hyper-personalisation* has become quite popular in internet marketing. *Adaptive hypermedia* aims to enhance the functionality of hyperlink-based systems by making the user interaction process personalisable (Brusilovsky et al. 1998a). Adaptive hypermedia is an alternative to the traditional 'one-size-fits-all' approach in the development of hypermedia systems, in that they build a model of the goals, preferences and knowledge of each individual user and this model is used throughout the interaction with the user in order to adapt to the needs of that particular user (Brusilovsky 1996; Kurilovas 2016). In a learning situation, a student in an adaptive educational hypermedia system could be given a presentation that is adapted specifically to his or her knowledge of the topic (Hothi et al. 2000) and the most relevant links to proceed further will be suggested (Brusilovsky et al.1998b; Kavcic 2004).

So the adaptive hypermedia will use knowledge provided by (or captured about) specific students to tailor the information and the links presented to each student (Ohene-Djan and Fernandes 2000; Schuck 2016). Using this knowledge, the system can then support learners in navigating to information units, suggesting relevant links to follow and providing additional information (Ohene-Djan 2002).

Mohan (2013) predicted that the future of all student education—from kindergarten to post-graduate level—will be hyper-personalised. Each student will focus on having their own teacher, their own curriculum and their own books or other resources. In such an environment, the teacher, using adaptive hypermedia, will increasingly become a person who understands the unique needs of each student. Not all students in a class will be at the same level. Some might surge ahead in mathematics while others in literature or art. It will help students to excel in something, rather than be ordinary at everything (Mohan 2013).

Although the idea of personalised learning sounds wonderful, there are some concerns, including the fact that learning cannot be broken down and measured as small bits and pieces that lend themselves to the kind of assessment and record-keeping that the software can handle (Greene 2019); there is also the danger of isolation (France 2018), and the issue of certification of certain skills (Paludan 2006).

5.5 Panic-gogy

The arrival of the COVID-19 pandemic in 2020 forced universities and schools to move to online teaching instead of the traditional face-to-face approach, and this is likely to continue for the indefinite future (Han 2020). This situation makes online and blended learning an essential topic in the teaching of mathematics and other disciplines. If nothing else, the worldwide lockdown has given us a glimpse into the future—a future of remote workspaces, online interactions and digital service delivery. More and more people may be working or studying from home using online educational and meeting platforms, replacing existing processes with digital equivalents, ramping up measurement instruments on websites, interaction on social media, and marketing activities. E-learning is becoming popular worldwide. As a result of the pandemic, blended and online learning has developed from *important* to *essential.*

Teachers all over the world have a tongue-in-cheek name for what everybody is forced to do now: *Panic-gogy*—for panic + pedagogy (Kamanetz 2020).

Panic-gogy means understanding students' practical resources and problems, including availability of devices and the internet, family responsibilities, students sent home who need to find a new place to live, and financial constraints. But it also means how teachers are going to move into this environment with their teaching approaches.

Many teachers do not have the same experience of online instruction as they have of face-to-face teaching and all of a sudden there are many 'experts' giving advice on how an online approach should be employed. Teachers encounter new problems and feel somewhat isolated and uncomfortable in the environment. Teachers are uncertain about the level of students' commitment to learning.

One does get the feeling that, as a result of the COVID-19 crisis, in many instances, institutions move over to using technology without really making use of the existing available research on the topic. Granted, COVID-19 has forced institutions into using technology without the luxury of time to consider research or best practice—they are running crisis management to survive. Moreover, some instititutions use this opportunity to profit from the situation, saving on their face-to-face activities.

Most of us agree that teaching should be student-centred: "Teaching should not be based on what knowledge the professors can impart, but instead on what students need" (Han 2020). But when one suddenly has to transfer to an entirely new teaching environment, many teachers tend to just convert their traditional courses to an online platform. We have labelled that a "domestication of a new media": one does not take advantage of the agency of a new medium and simply uses practices from teaching with the old medium (Borba and Villarreal 2005).

People are ambivalent about the 'move online' for various reasons—teachers are underprepared, some wonder whether we are doing the right thing, and in some instances the move to using technology is likely to evoke political responses, serving a range of conflicting agendas (Czerniewicz 2020). It is not easy to design well for effective, meaningful learning in this environment, and hurried, incomplete and rushed efforts to 'teach online' can give blended and online learning a bad name, associated with managing student protests rather than for pedagogical innovation (Czerniewicz 2020). Robin DeRosa (in Kamanetz 2020) pointed out that creating a good online course can take years of development and collaboration, involving people with different skills. She claimed

> I think the first thing is, we are not building online courses or converting your face to face courses to online learning. Really, what we're doing is we are trying to extend a sense of care to our students and trying to build a community that's going to be able to work together to get through the learning challenges that we have. …. so if people think that in three to five days they're going to rejigger their course and build some super amazing online platform, that's probably unlikely to happen. (DeRosa in Kamanetz 2020).

In many countries, there are also digital divides and social inequalities that have to be taken into account. There are many students (and teachers) who do not have access to technology and connectivity.

In the COVID-19 context where social distancing is encouraged, it will be essential to pay even more attention to the human connection and to find ways to ensure that human interaction is continued.

This special issue shows parts of this transformation that are happening in regular education, pre-service and in-service teacher education.

6 Conclusions

It is generally accepted that we do not need a formal 'curriculum' to explicitly teach a young child to speak. An environment permeated by orality seems a friendly one for a small child to try a word, to repeat, and to interiorise and create or construct new sentences.

The classic philosopher Pierre Levy (Levy 1993) discussed the different levels of *orality*. In current terminology, we can connect his ideas to the *multimodal discourse* (Bezemer et al. 2016) that is generated by digital videos, LMSs, and most discourses that are products of collectives of humans-with-media, including computers. Borba and Villarreal (2005) claimed that expressions made by collectives that include computers, may be part of an orality at a next level.

In this phase of technological development, communication that involves different computer platforms, e.g. LMS and social media, is a combination of icons, videos, regular writing, orality, images, graphs and video-clips that seems to bring a new consequence to mathematics education.

Traditionally, education used to be very much teacher driven. However, over the last decades, the focus has been changing from a situation of students passively absorbing information from an educator who is teaching by writing on the blackboard—sometimes referred to as *pushing* knowledge—to a more student driven approach, where students take control of the learning process—referred to as a *pull* process (Bassendowski and Petrucka 2013). This approach encourages students to select and transform information, discover principles, make hypotheses and decisions beyond the given information (Jung and Latchem 2009). Students can become involved in the design and development as well as the delivery of curricula (Lightner et al. 2007). An environment that supports the development of communities and collaborative discussion opportunities can assist students to comprehend and synthesise information, as independent and critical thinkers (Jansen et al. 2011).

In the traditional *push* approach the idea is that only the educator has legitimate knowledge and this knowledge is being transmitted one-way to students. A *pull* model involves the interest and commitment of students to create communities of trust, knowledge sharing, cooperation, and collaboration (Bassendowski and Petrucka 2013) as some papers in this special issue show. Pull approaches are characterised by constructivist and connectivist models—some of which are created by students, as in socially shared environments (Willems 2009). In these models students demand, request, and even create the particular products or information that they need. Students become active participants in their learning by working alongside the educators in both traditional and online settings (Ahn and Class 2011; Willems 2009).

A message throughout our entire paper is that the official 'curriculum' is currently playing a role that is overemphasised. Our modern students want a bigger say in how they are taught and what they are taught. Comparable to a child learning to speak, they want to decide on what mathematics they learn and how, in a pull approach, rather than a curriculum that is pushed onto them by the educational system. We may be at the beginning of a transformation of the classroom. Will mathematics change in the same way that it changed with the appearance of 'paper and pencil'? However, although the papers in this issue push the field forward, there are still many issues to address.

A wide array of media and technology is available to create new hybrid forms of teaching. The integration of technology enables educators to create learning experiences that actively and meaningfully pull students into course content. This technology may form thinking collectives (Levy 1993) with teachers that can break the walls of the regular 'cubic' classroom that is associated with lecturing.

Questions arise considering the new social interactions, the design of new teaching settings and about the new ways of thinking in the use of digital tools. Some of these questions are as follows:

- What technology should students use to support their own learning as well as collaboratively the learning from other students?
- How do we provide ways for students to evaluate and reflect critically about the learning and the resources with which they interact?
- How can mathematics educators develop research-based principles of design regarding new teaching contexts that digital tools provide?
- How can social media tools be combined with the best practices in teaching and contribute effectively to student engagement?

Papers in general, and survey papers in particular, age rapidly when the theme is digital technology (and mathematics education). New trends in mathematics education, and as we emphasise in this paper, changes in the (mathematics) classroom itself and the speed of digital technology may accelerate this 'deterioration' of the paper. With the COVID-19 crisis this may be an even bigger problem. Once this paper is published, or when it reaches a given reader, the COVID-19 problem may have been resolved with an appropriate vaccine and/or treatment for those infected, or we may still be at the stage we are now, with many airports, borders and frontiers closed, or we may be in a 'cloudy scenario', with some activities being resumed, but with everyone using masks and avoiding shaking hands and bodily contact.

The question is, what has this to do with mathematics education and digital technology? Besides the impact on conferences and on the transforming of mathematics classrooms, we may have to ask broader questions:

- Digital technology intensified travelling and our way of living, so it is also partly responsible for the present crisis. Is it possible that the use of digital technology can generate a similar crisis in mathematics education?
- Conversely, if the crisis lasts for a long period, would digital technologies be able to provide alternative ways to implement mathematics education?
- There is not much research on online mathematics education for young children, but if the crisis lasts for a long time, are we going to implement it without sufficient research? If the current crisis is over soon, are we going to develop research on mathematics education for a possible 'COVID-2X' crisis?
- In this paper, among others, we have anthropomorphised media, talking about agency. The notion of humans-with-media as the collective that produces knowledge, may synthesise it, as we discussed in this paper. The COVID-19 virus (SARS-CoV-2) is a non-living being: can we talk about the impact (agency) of COVID-19 on mathematics education and on the world?

These questions are too broad for this paper, and along with other questions will be the theme of other papers. We hope to be there to write and to read them!

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