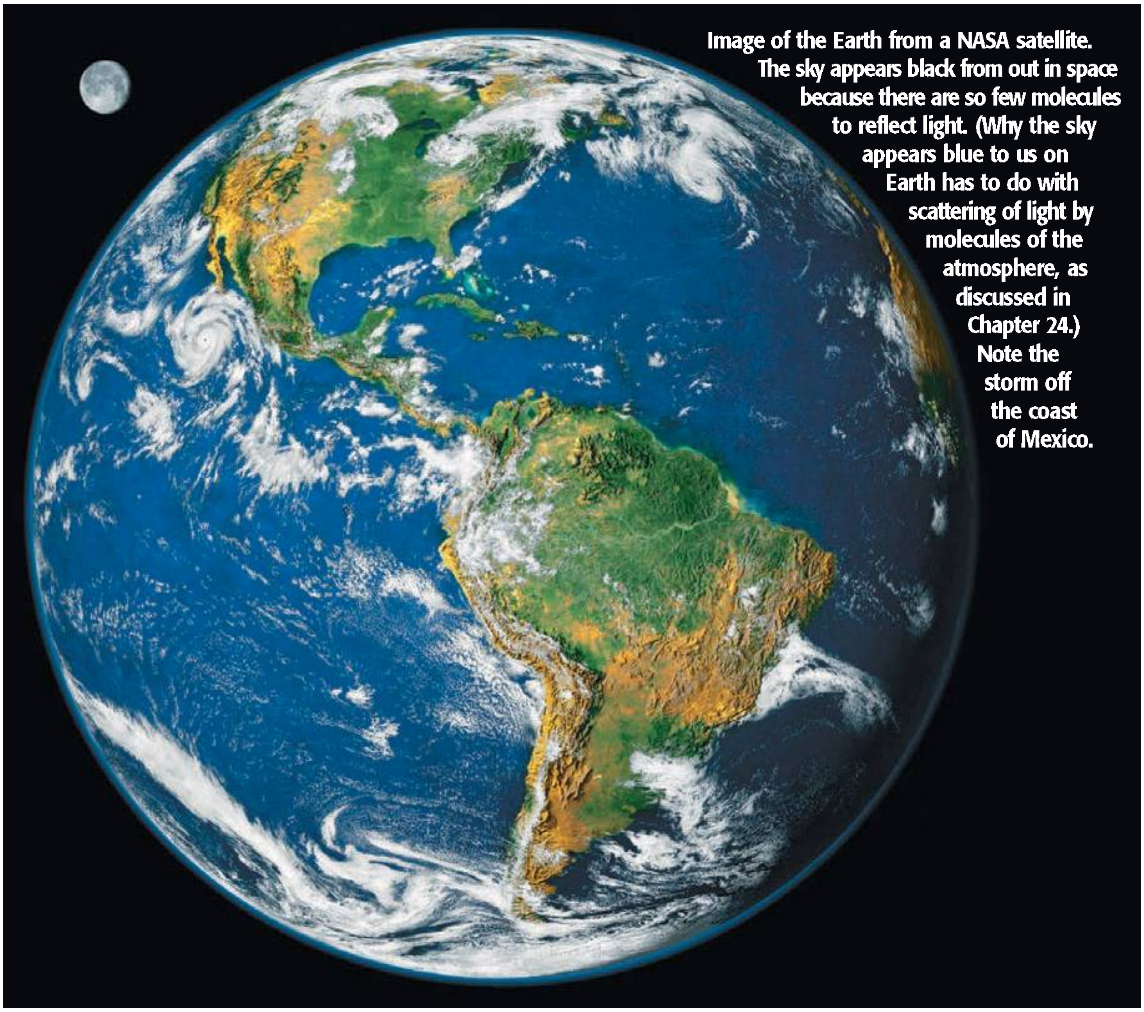

Image of the Earth from a NASA satellite. The sky appears black from out in space because there are so few molecules to reflect light. (Why the sky appears blue to us on Earth has to do with scattering of light by molecules of the atmosphere, as discussed in Chapter 24.) Note the storm off the coast of Mexico.

CHAPTER 1

# Introduction, Measurement, Estimating

**CHAPTER-OPENING QUESTIONS—Guess now!**

**1.** How many $cm^3$ are in $1.0\ m^3$?

**(a)** 10. **(b)** 100. **(c)** 1000. **(d)** 10,000. **(e)** 100,000. **(f)** 1,000,000.

**2.** Suppose you wanted to actually measure the radius of the Earth, at least roughly, rather than taking other people's word for what it is. Which response below describes the best approach?

**(a)** Use an extremely long measuring tape.
**(b)** It is only possible by flying high enough to see the actual curvature of the Earth.
**(c)** Use a standard measuring tape, a step ladder, and a large smooth lake.
**(d)** Use a laser and a mirror on the Moon or on a satellite.
**(e)** Give up; it is impossible using ordinary means.

[*We start each Chapter with a Question—sometimes two. Try to answer right away. Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table. If they are misconceptions, we expect them to be cleared up as you read the Chapter. You will usually get another chance at the Question(s) later in the Chapter when the appropriate material has been covered. These Chapter-Opening Questions will also help you see the power and usefulness of physics.*]

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Physics is the most basic of the sciences. It deals with the behavior and structure of matter. The field of physics is usually divided into *classical physics* which includes motion, fluids, heat, sound, light, electricity, and magnetism; and *modern physics* which includes the topics of relativity, atomic structure, quantum theory, condensed matter, nuclear physics, elementary particles, and cosmology and astrophysics. We will cover all these topics in this book, beginning with motion (or mechanics, as it is often called) and ending with the most recent results in fundamental particles and the cosmos. But before we begin on the physics itself, we take a brief look at how this overall activity called "science," including physics, is actually practiced.

## 1–1 The Nature of Science

The principal aim of all sciences, including physics, is generally considered to be the search for order in our observations of the world around us. Many people think that science is a mechanical process of collecting facts and devising theories. But it is not so simple. Science is a creative activity that in many respects resembles other creative activities of the human mind.

One important aspect of science is **observation** of events, which includes the design and carrying out of experiments. But observation and experiments require imagination, because scientists can never include everything in a description of what they observe. Hence, scientists must make judgments about what is relevant in their observations and experiments.

Consider, for example, how two great minds, Aristotle (384–322 B.C.; Fig. 1–1) and Galileo (1564–1642; Fig. 2–18), interpreted motion along a horizontal surface. Aristotle noted that objects given an initial push along the ground (or on a tabletop) always slow down and stop. Consequently, Aristotle argued, the natural state of an object is to be at rest. Galileo, the first true experimentalist, reexamined horizontal motion in the 1600s. He imagined that if friction could be eliminated, an object given an initial push along a horizontal surface would continue to move indefinitely without stopping. He concluded that for an object to be in motion was just as natural as for it to be at rest. By inventing a new way of thinking about the same data, Galileo founded our modern view of motion (Chapters 2, 3, and 4), and he did so with a leap of the imagination. Galileo made this leap conceptually, without actually eliminating friction.

**FIGURE 1–1** Aristotle is the central figure (dressed in blue) at the top of the stairs (the figure next to him is Plato) in this famous Renaissance portrayal of *The School of Athens*, painted by Raphael around 1510. Also in this painting, considered one of the great masterpieces in art, are Euclid (drawing a circle at the lower right), Ptolemy (extreme right with globe), Pythagoras, Socrates, and Diogenes.

Observation, with careful experimentation and measurement, is one side of the scientific process. The other side is the invention or creation of **theories** to explain and order the observations. Theories are never derived directly from observations. Observations may help inspire a theory, and theories are accepted or rejected based on the results of observation and experiment.

Theories are inspirations that come from the minds of human beings. For example, the idea that matter is made up of atoms (the atomic theory) was not arrived at by direct observation of atoms—we can't see atoms directly. Rather, the idea sprang from creative minds. The theory of relativity, the electromagnetic theory of light, and Newton's law of universal gravitation were likewise the result of human imagination.

The great theories of science may be compared, as creative achievements, with great works of art or literature. But how does science differ from these other creative activities? One important difference is that science requires **testing** of its ideas or theories to see if their predictions are borne out by experiment. But theories are not "proved" by testing. First of all, no measuring instrument is perfect, so exact confirmation is not possible. Furthermore, it is not possible to test a theory for every possible set of circumstances. Hence a theory cannot be absolutely verified. Indeed, the history of science tells us that long-held theories can sometimes be replaced by new ones, particularly when new experimental techniques provide new or contradictory data.

A new theory is accepted by scientists in some cases because its predictions are quantitatively in better agreement with experiment than those of the older theory. But in many cases, a new theory is accepted only if it explains a greater *range* of phenomena than does the older one. Copernicus's Sun-centered theory of the universe (Fig. 1–2b), for example, was originally no more accurate than Ptolemy's Earth-centered theory (Fig. 1–2a) for predicting the motion of heavenly bodies (Sun, Moon, planets). But Copernicus's theory had consequences that Ptolemy's did not, such as predicting the moonlike phases of Venus. A simpler and richer theory, one which unifies and explains a greater variety of phenomena, is more useful and beautiful to a scientist. And this aspect, as well as quantitative agreement, plays a major role in the acceptance of a theory.

**FIGURE 1–2** (a) Ptolemy's geocentric view of the universe. Note at the center the four elements of the ancients: Earth, water, air (clouds around the Earth), and fire; then the circles, with symbols, for the Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, the fixed stars, and the signs of the zodiac. (b) An early representation of Copernicus's heliocentric view of the universe with the Sun at the center. (See Chapter 5.)


(a) (b)

An important aspect of any theory is how well it can quantitatively predict phenomena, and from this point of view a new theory may often seem to be only a minor advance over the old one. For example, Einstein's theory of relativity gives predictions that differ very little from the older theories of Galileo and Newton in nearly all everyday situations. Its predictions are better mainly in the extreme case of very high speeds close to the speed of light. But quantitative prediction is not the only important outcome of a theory. Our view of the world is affected as well. As a result of Einstein's theory of relativity, for example, our concepts of space and time have been completely altered, and we have come to see mass and energy as a single entity (via the famous equation $E = mc^2$).

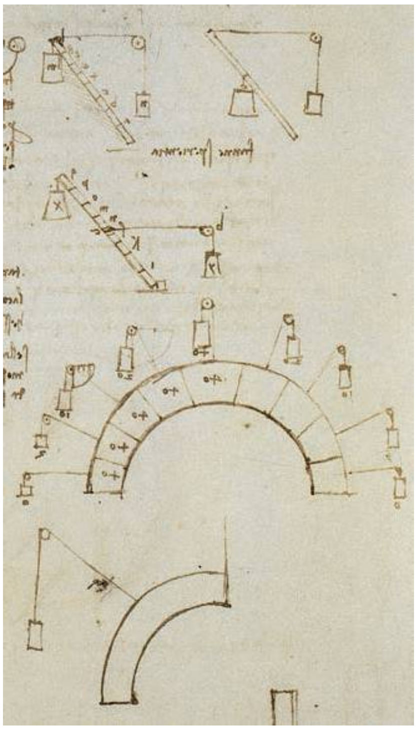


**FIGURE 1–3** Studies on the forces in structures by Leonardo da Vinci (1452–1519).

## 1–2 Physics and its Relation to Other Fields

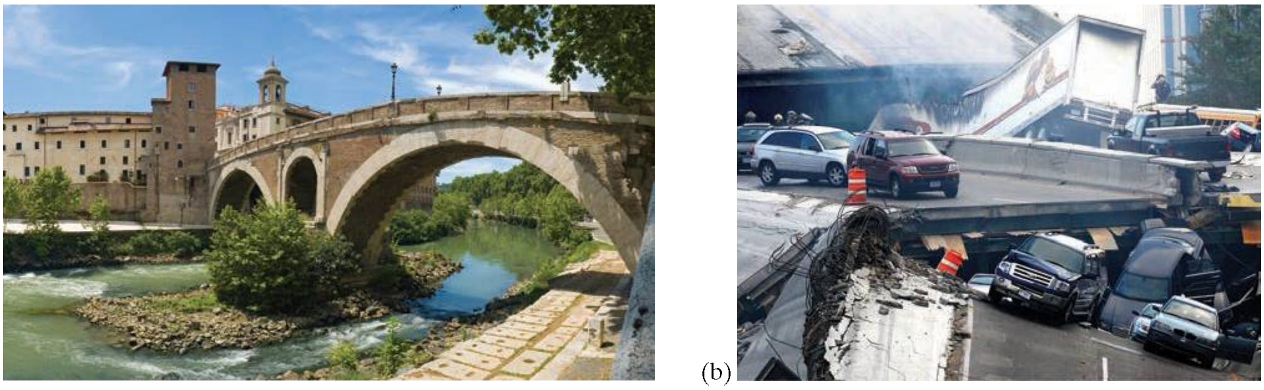
For a long time science was more or less a united whole known as natural philosophy. Not until a century or two ago did the distinctions between physics and chemistry and even the life sciences become prominent. Indeed, the sharp distinction we now see between the arts and the sciences is itself only a few centuries old. It is no wonder then that the development of physics has both influenced and been influenced by other fields. For example, the notebooks (Fig. 1–3) of Leonardo da Vinci, the great Renaissance artist, researcher, and engineer, contain the first references to the forces acting within a structure, a subject we consider as physics today; but then, as now, it has great relevance to architecture and building.

Early work in electricity that led to the discovery of the electric battery and electric current was done by an eighteenth-century physiologist, Luigi Galvani (1737–1798). He noticed the twitching of frogs' legs in response to an electric spark and later that the muscles twitched when in contact with two dissimilar metals (Chapter 18). At first this phenomenon was known as "animal electricity," but it shortly became clear that electric current itself could exist in the absence of an animal.

Physics is used in many fields. A zoologist, for example, may find physics useful in understanding how prairie dogs and other animals can live underground without suffocating. A physical therapist will be more effective if aware of the principles of center of gravity and the action of forces within the human body. A knowledge of the operating principles of optical and electronic equipment is helpful in a variety of fields. Life scientists and architects alike will be interested in the nature of heat loss and gain in human beings and the resulting comfort or discomfort. Architects may have to calculate the dimensions of the pipes in a heating system or the forces involved in a given structure to determine if it will remain standing (Fig. 1–4). They must know physics principles in order to make realistic designs and to communicate effectively with engineering consultants and other specialists.

**FIGURE 1–4** (a) This bridge over the River Tiber in Rome was built 2000 years ago and still stands. (b) The 2007 collapse of a Mississippi River highway bridge built only 40 years before.

(a) (b)



From the aesthetic or psychological point of view, too, architects must be aware of the forces involved in a structure—for example instability, even if only illusory, can be discomforting to those who must live or work in the structure.

The list of ways in which physics relates to other fields is extensive. In the Chapters that follow we will discuss many such applications as we carry out our principal aim of explaining basic physics.

# 1–3 Models, Theories, and Laws

When scientists are trying to understand a particular set of phenomena, they often make use of a **model**. A model, in the scientific sense, is a kind of analogy or mental image of the phenomena in terms of something else we are already familiar with. One example is the wave model of light. We cannot see waves of light as we can water waves. But it is valuable to think of light as made up of waves, because experiments indicate that light behaves in many respects as water waves do.

The purpose of a model is to give us an approximate mental or visual picture—something to hold on to—when we cannot see what actually is happening. Models often give us a deeper understanding: the analogy to a known system (for instance, the water waves above) can suggest new experiments to perform and can provide ideas about what other related phenomena might occur.

You may wonder what the difference is between a theory and a model. Usually a model is relatively simple and provides a structural similarity to the phenomena being studied. A **theory** is broader, more detailed, and can give quantitatively testable predictions, often with great precision. It is important, however, not to confuse a model or a theory with the real system or the phenomena themselves.

Scientists have given the title **law** to certain concise but general statements about how nature behaves (that electric charge is conserved, for example). Often the statement takes the form of a relationship or equation between quantities (such as Newton's second law, $F = ma$).

Statements that we call laws are usually experimentally valid over a wide range of observed phenomena. For less general statements, the term **principle** is often used (such as Archimedes' principle). We use "theory" for a more general picture of the phenomena dealt with.

Scientific laws are different from political laws in that the latter are *prescriptive*: they tell us how we ought to behave. Scientific laws are *descriptive*: they do not say how nature *should* behave, but rather are meant to describe how nature *does* behave. As with theories, laws cannot be tested in the infinite variety of cases possible. So we cannot be sure that any law is absolutely true. We use the term "law" when its validity has been tested over a wide range of cases, and when any limitations and the range of validity are clearly understood.

Scientists normally do their research as if the accepted laws and theories were true. But they are obliged to keep an open mind in case new information should alter the validity of any given law or theory.

# 1–4 Measurement and Uncertainty; Significant Figures

In the quest to understand the world around us, scientists seek to find relationships among physical quantities that can be measured.

## Uncertainty

Reliable measurements are an important part of physics. But no measurement is absolutely precise. There is an uncertainty associated with every measurement.

**FIGURE 1–5** Measuring the width of a board with a centimeter ruler. Accuracy is about $\pm 1$ mm.

Among the most important sources of uncertainty, other than blunders, are the limited accuracy of every measuring instrument and the inability to read an instrument beyond some fraction of the smallest division shown. For example, if you were to use a centimeter ruler to measure the width of a board (Fig. 1–5), the result could be claimed to be precise to about 0.1 cm (1 mm), the smallest division on the ruler, although half of this value might be a valid claim as well. The reason is that it is difficult for the observer to estimate (or "interpolate") between the smallest divisions. Furthermore, the ruler itself may not have been manufactured to an accuracy very much better than this.

When giving the result of a measurement, it is important to state the **estimated uncertainty** in the measurement. For example, the width of a board might be written as $8.8 \pm 0.1$ cm. The $\pm 0.1$ cm ("plus or minus 0.1 cm") represents the estimated uncertainty in the measurement, so that the actual width most likely lies between 8.7 and 8.9 cm. The **percent uncertainty** is the ratio of the uncertainty to the measured value, multiplied by 100. For example, if the measurement is 8.8 cm and the uncertainty about 0.1 cm, the percent uncertainty is

$$\frac{0.1}{8.8} \times 100\% \approx 1\%,$$

where $\approx$ means "is approximately equal to."

Often the uncertainty in a measured value is not specified explicitly. In such cases, the

**uncertainty in a numerical value is assumed to be *one or a few units* in the last digit specified.**

For example, if a length is given as 8.8 cm, the uncertainty is assumed to be about 0.1 cm or 0.2 cm, or possibly even 0.3 cm. It is important in this case that you do not write 8.80 cm, because this implies an uncertainty on the order of 0.01 cm; it assumes that the length is probably between about 8.79 cm and 8.81 cm, when actually you believe it is between about 8.7 and 8.9 cm.

**CONCEPTUAL EXAMPLE 1–1** **Is the diamond yours?** A friend asks to borrow your precious diamond for a day to show her family. You are a bit worried, so you carefully have your diamond weighed on a scale which reads 8.17 grams. The scale's accuracy is claimed to be $\pm 0.05$ gram. The next day you weigh the returned diamond again, getting 8.09 grams. Is this your diamond?

**RESPONSE** The scale readings are measurements and are not perfect. They do not necessarily give the "true" value of the mass. Each measurement could have been high or low by up to 0.05 gram or so. The actual mass of your diamond lies most likely between 8.12 grams and 8.22 grams. The actual mass of the returned diamond is most likely between 8.04 grams and 8.14 grams. These two ranges overlap, so the data do not give you a strong reason to doubt that the returned diamond is yours.

## Significant Figures

The number of reliably known digits in a number is called the number of **significant figures**. Thus there are four significant figures in the number 23.21 cm and two in the number 0.062 cm (the zeros in the latter are merely place holders that show where the decimal point goes). The number of significant figures may not always be clear. Take, for example, the number 80. Are there one or two significant figures? We need words here: If we say it is *roughly* 80 km between two cities, there is only one significant figure (the 8) since the zero is merely a place holder. If there is no suggestion that the 80 is a rough approximation, then we can often assume (as we will in this book) that it has 2 significant figures: so it is 80 km within an accuracy of about 1 or 2 km. If it is precisely 80 km, to within $\pm 0.1$ or $\pm 0.2$ km, then we write 80.0 km (three significant figures).

When specifying numerical results, you should avoid the temptation to keep more digits in the final answer than is justified: see boldface statement on previous page. For example, to calculate the area of a rectangle 11.3 cm by 6.8 cm, the result of multiplication would be 76.84 $\text{cm}^2$. But this answer can not be accurate to the implied 0.01 $\text{cm}^2$ uncertainty, because (using the outer limits of the assumed uncertainty for each measurement) the result could be between $11.2\text{ cm} \times 6.7\text{ cm} = 75.04\text{ cm}^2$ and $11.4\text{ cm} \times 6.9\text{ cm} = 78.66\text{ cm}^2$. At best, we can quote the answer as 77 $\text{cm}^2$, which implies an uncertainty of about 1 or 2 $\text{cm}^2$. The other two digits (in the number 76.84 $\text{cm}^2$) must be dropped (rounded off) because they are not significant. As a rough general "significant figure" rule we can say that

*the final result of a multiplication or division should have no more digits than the numerical value with the fewest significant figures.*

In our example, 6.8 cm has the least number of significant figures, namely two. Thus the result 76.84 $\text{cm}^2$ needs to be rounded off to 77 $\text{cm}^2$.

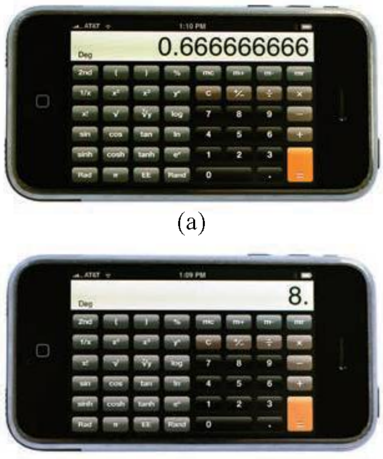
**EXERCISE A** The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (*a*) 14.625 $\text{cm}^2$; (*b*) 14.63 $\text{cm}^2$; (*c*) 14.6 $\text{cm}^2$; (*d*) 15 $\text{cm}^2$.

When adding or subtracting numbers, the final result should contain no more decimal places than the number with the fewest decimal places. For example, the result of subtracting 0.57 from 3.6 is 3.0 (not 3.03). Similarly $36 + 8.2 = 44$, not 44.2.

Be careful not to confuse significant figures with the number of decimal places.

**EXERCISE B** For each of the following numbers, state the number of significant figures and the number of decimal places: (*a*) 1.23; (*b*) 0.123; (*c*) 0.0123.

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not 0.666666666 as calculators give (Fig. 1–6a). Digits should not be quoted in a result unless they are truly significant figures. However, to obtain the most accurate result, you should normally *keep one or more extra significant figures throughout a calculation, and round off only in the final result.* (With a calculator, you can keep all its digits in intermediate results.) Note also that calculators sometimes give too few significant figures. For example, when you multiply $2.5 \times 3.2$, a calculator may give the answer as simply 8. But the answer is accurate to two significant figures, so the proper answer is 8.0. See Fig. 1–6b.




(a)

(b)

**FIGURE 1–6** These two calculations show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result would be 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

**PROBLEM SOLVING**

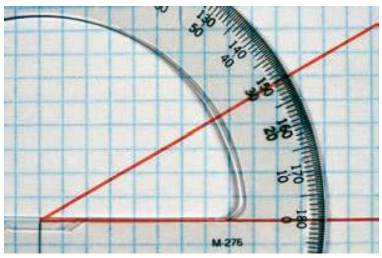
*Report only the proper number of significant figures in the final result. But keep extra digits during the calculation*

**CONCEPTUAL EXAMPLE 1–2** **Significant figures.** Using a protractor (Fig. 1–7), you measure an angle to be 30°. (*a*) How many significant figures should you quote in this measurement? (*b*) Use a calculator to find the cosine of the angle you measured.

**RESPONSE** (*a*) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely 30° (not 30.0°). (*b*) If you enter cos 30° in your calculator, you will get a number like 0.866025403. But the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; you must round your answer to two significant figures.

**NOTE** Trigonometric functions, like cosine, are reviewed in Chapter 3 and Appendix A.

**FIGURE 1–7** Example 1–2. A protractor used to measure an angle.



## Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as $3.69 \times 10^4$, or 0.0021 as $2.1 \times 10^{-3}$. One advantage of scientific notation (reviewed in Appendix A) is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of 10 notation the ambiguity can be avoided: if the number is known to three significant figures, we write $3.69 \times 10^4$, but if it is known to four, we write $3.690 \times 10^4$.

**EXERCISE C** Write each of the following in scientific notation and state the number of significant figures for each: (*a*) 0.0258; (*b*) 42,300; (*c*) 344.50.

## * Percent Uncertainty vs. Significant Figures

The significant figures rule is only approximate, and in some cases may underestimate the accuracy (or uncertainty) of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of $\pm 1$ if no other uncertainty is stated. Both $92 \pm 1$ and $97 \pm 1$ imply an uncertainty of about 1% ($1/92 \approx 0.01 = 1\%$). But the final result to two significant figures is 1.1, with an implied uncertainty of $\pm 0.1$, which is an uncertainty of about 10% ($0.1/1.1 \approx 0.1 \approx 10\%$). It is better in this case to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of $\pm 0.01$ which is $0.01/1.05 \approx 0.01 \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

## Approximations

Much of physics involves approximations, often because we do not have the means to solve a problem precisely. For example, we may choose to ignore air resistance or friction in doing a Problem even though they are present in the real world, and then our calculation is only an approximation. In doing Problems, we should be aware of what approximations we are making, and be aware that the precision of our answer may not be nearly as good as the number of significant figures given in the result.

## Accuracy vs. Precision

There is a technical difference between "precision" and "accuracy." **Precision** in a strict sense refers to the repeatability of the measurement using a given instrument. For example, if you measure the width of a board many times, getting results like 8.81 cm, 8.85 cm, 8.78 cm, 8.82 cm (interpolating between the 0.1 cm marks as best as possible each time), you could say the measurements give a *precision* a bit better than 0.1 cm. **Accuracy** refers to how close a measurement is to the true value. For example, if the ruler shown in Fig. 1–5 was manufactured with a 2% error, the accuracy of its measurement of the board's width (about 8.8 cm) would be about 2% of 8.8 cm or about $\pm 0.2$ cm. Estimated uncertainty is meant to take both accuracy and precision into account.

# 1–5 Units, Standards, and the SI System

The measurement of any quantity is made relative to a particular standard or **unit**, and this unit must be specified along with the numerical value of the quantity. For example, we can measure length in British units such as inches, feet, or miles, or in the metric system in centimeters, meters, or kilometers. To specify that the length of a particular object is 18.6 is insufficient. The unit *must* be given, because 18.6 meters is very different from 18.6 inches or 18.6 millimeters.

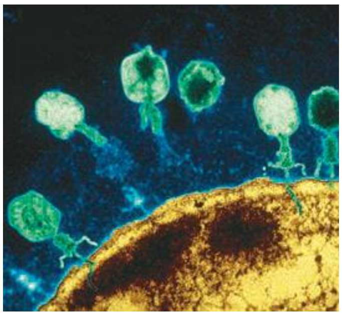
For any unit we use, such as the meter for distance or the second for time, we need to define a **standard** which defines exactly how long one meter or one second is. It is important that standards be chosen that are readily reproducible so that anyone needing to make a very accurate measurement can refer to the standard in the laboratory and communicate with other people.

## Length

The first truly international standard was the **meter** (abbreviated m) established as the standard of **length** by the French Academy of Sciences in the 1790s. The standard meter was originally chosen to be one ten-millionth of the distance from the Earth's equator to either pole,[†] and a platinum rod to represent this length was made. (One meter is, very roughly, the distance from the tip of your nose to the tip of your finger, with arm and hand stretched out horizontally.) In 1889, the meter was defined more precisely as the distance between two finely engraved marks on a particular bar of platinum–iridium alloy. In 1960, to provide even greater precision and reproducibility, the meter was redefined as 1,650,763.73 wavelengths of a particular orange light emitted by the gas krypton-86. In 1983 the meter was again redefined, this time in terms of the speed of light (whose best measured value in terms of the older definition of the meter was 299,792,458 m/s, with an uncertainty of 1 m/s). The new definition reads: "The meter is the length of path traveled by light in vacuum during a time interval of 1/299,792,458 of a second."[‡]

British units of length (inch, foot, mile) are now defined in terms of the meter. The inch (in.) is defined as exactly 2.54 centimeters (cm; 1 cm = 0.01 m). Other conversion factors are given in the Table on the inside of the front cover of this book. Table 1–1 presents some typical lengths, from very small to very large, rounded off to the nearest power of 10. See also Fig. 1–8. [Note that the abbreviation for inches (in.) is the only one with a period, to distinguish it from the word "in".]

**FIGURE 1–8** Some lengths: (a) viruses (about $10^{-7}$ m long) attacking a cell; (b) Mt. Everest's height is on the order of $10^{4}$ m (8850 m above sea level, to be precise).



(a)

(b)

## Time

The standard unit of **time** is the **second** (s). For many years, the second was defined as 1/86,400 of a mean solar day $(24\,\text{h/day} \times 60\,\text{min/h} \times 60\,\text{s/min} = 86{,}400\,\text{s/day})$. The standard second is now defined more precisely in terms of the frequency of radiation emitted by cesium atoms when they pass between two particular states. [Specifically, one second is defined as the time required for 9,192,631,770 oscillations of this radiation.] There are, by definition, 60 s in one minute (min) and 60 minutes in one hour (h). Table 1–2 presents a range of measured time intervals, rounded off to the nearest power of 10.

[†]Modern measurements of the Earth's circumference reveal that the intended length is off by about one-fiftieth of 1%. Not bad!

[‡]The new definition of the meter has the effect of giving the speed of light the exact value of 299,792,458 m/s.

**TABLE 1–1 Some Typical Lengths or Distances (order of magnitude)**

| Length (or Distance) | Meters (approximate) |
|---|---|
| Neutron or proton (diameter) | $10^{-15}$ m |
| Atom (diameter) | $10^{-10}$ m |
| Virus [see Fig. 1–8a] | $10^{-7}$ m |
| Sheet of paper (thickness) | $10^{-4}$ m |
| Finger width | $10^{-2}$ m |
| Football field length | $10^{2}$ m |
| Height of Mt. Everest [see Fig. 1–8b] | $10^{4}$ m |
| Earth diameter | $10^{7}$ m |
| Earth to Sun | $10^{11}$ m |
| Earth to nearest star | $10^{16}$ m |
| Earth to nearest galaxy | $10^{22}$ m |
| Earth to farthest galaxy visible | $10^{26}$ m |

**TABLE 1–2 Some Typical Time Intervals (order of magnitude)**

| Time Interval | Seconds (approximate) |
|---|---|
| Lifetime of very unstable subatomic particle | $10^{-23}$ s |
| Lifetime of radioactive elements | $10^{-22}$ s to $10^{28}$ s |
| Lifetime of muon | $10^{-6}$ s |
| Time between human heartbeats | $10^{0}$ s (= 1 s) |
| One day | $10^{5}$ s |
| One year | $3 \times 10^{7}$ s |
| Human life span | $2 \times 10^{9}$ s |
| Length of recorded history | $10^{11}$ s |
| Humans on Earth | $10^{13}$ s |
| Age of Earth | $10^{17}$ s |
| Age of Universe | $4 \times 10^{17}$ s |

**TABLE 1–3 Some Masses**

| Object | Kilograms (approximate) |
|---|---|
| Electron | $10^{-30}$ kg |
| Proton, neutron | $10^{-27}$ kg |
| DNA molecule | $10^{-17}$ kg |
| Bacterium | $10^{-15}$ kg |
| Mosquito | $10^{-5}$ kg |
| Plum | $10^{-1}$ kg |
| Human | $10^{2}$ kg |
| Ship | $10^{8}$ kg |
| Earth | $6 \times 10^{24}$ kg |
| Sun | $2 \times 10^{30}$ kg |
| Galaxy | $10^{41}$ kg |

## Mass

The standard unit of **mass** is the **kilogram** (kg). The standard mass is a particular platinum–iridium cylinder, kept at the International Bureau of Weights and Measures near Paris, France, whose mass is defined as exactly 1 kg. A range of masses is presented in Table 1–3. [For practical purposes, 1 kg weighs about 2.2 pounds on Earth.]

When dealing with atoms and molecules, we usually use the **unified atomic mass unit** (u or amu). In terms of the kilogram,

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}.$$

Precise values of this and other useful numbers are given on page A-72.

The definitions of other standard units for other quantities will be given as we encounter them in later Chapters.

## Unit Prefixes

In the metric system, the larger and smaller units are defined in multiples of 10 from the standard unit, and this makes calculation particularly easy. Thus 1 kilometer (km) is 1000 m, 1 centimeter is $\frac{1}{100}$ m, 1 millimeter (mm) is $\frac{1}{1000}$ m or $\frac{1}{10}$ cm, and so on. The prefixes "centi-," "kilo-," and others are listed in Table 1–4 and can be applied not only to units of length but to units of volume, mass, or any other unit. For example, a centiliter (cL) is $\frac{1}{100}$ liter (L), and a kilogram (kg) is 1000 grams (g). An 8.2-megapixel camera has a detector with 8,200,000 pixels (individual "picture elements").

In common usage, $1\ \mu\text{m}\ (= 10^{-6}\text{ m})$ is called 1 **micron**.

## Systems of Units

**PROBLEM SOLVING**
*Always use a consistent set of units*

When dealing with the laws and equations of physics it is very important to use a consistent set of units. Several systems of units have been in use over the years. Today the most important is the **Système International** (French for International System), which is abbreviated SI. In SI units, the standard of length is the meter, the standard for time is the second, and the standard for mass is the kilogram. This system used to be called the MKS (meter-kilogram-second) system.

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title. The **British engineering system** (although more used in the U.S. than Britain) has as its standards the foot for length, the pound for force, and the second for time.

We use SI units almost exclusively in this book.

**TABLE 1–4 Metric (SI) Prefixes**

| Prefix | Abbreviation | Value |
|---|---|---|
| yotta | Y | $10^{24}$ |
| zetta | Z | $10^{21}$ |
| exa | E | $10^{18}$ |
| peta | P | $10^{15}$ |
| tera | T | $10^{12}$ |
| giga | G | $10^{9}$ |
| mega | M | $10^{6}$ |
| kilo | k | $10^{3}$ |
| hecto | h | $10^{2}$ |
| deka | da | $10^{1}$ |
| deci | d | $10^{-1}$ |
| centi | c | $10^{-2}$ |
| milli | m | $10^{-3}$ |
| micro† | $\mu$ | $10^{-6}$ |
| nano | n | $10^{-9}$ |
| pico | p | $10^{-12}$ |
| femto | f | $10^{-15}$ |
| atto | a | $10^{-18}$ |
| zepto | z | $10^{-21}$ |
| yocto | y | $10^{-24}$ |

† $\mu$ is the Greek letter "mu."

## * Base vs. Derived Quantities

Physical quantities can be divided into two categories: *base quantities* and *derived quantities*. The corresponding units for these quantities are called *base units* and *derived units*. A **base quantity** must be defined in terms of a standard. Scientists, in the interest of simplicity, want the smallest number of base quantities possible consistent with a full description of the physical world. This number turns out to be seven, and those used in the SI are given in Table 1–5.

**TABLE 1–5 SI Base Quantities and Units**

| Quantity | Unit | Unit Abbreviation |
|---|---|---|
| Length | meter | m |
| Time | second | s |
| Mass | kilogram | kg |
| Electric current | ampere | A |
| Temperature | kelvin | K |
| Amount of substance | mole | mol |
| Luminous intensity | candela | cd |

All other quantities can be defined in terms of these seven base quantities,† and hence are referred to as **derived quantities**. An example of a derived quantity is speed, which is defined as distance divided by the time it takes to travel that distance. A Table on page A-73 lists many derived quantities and their units in terms of base units. To define any quantity, whether base or derived, we can specify a rule or procedure, and this is called an **operational definition**.

# 1–6 Converting Units

Any quantity we measure, such as a length, a speed, or an electric current, consists of a number *and* a unit. Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure that a shelf is 21.5 inches wide, and we want to express this in centimeters. We must use a **conversion factor**, which in this case is, *by definition*, exactly

$$1\text{ in.} = 2.54\text{ cm}$$

or, written another way,

$$1 = 2.54\text{ cm/in.}$$

Since multiplying by the number one does not change anything, the width of our shelf, in cm, is

$$21.5\text{ inches} = (21.5\ \cancel{\text{in.}}) \times \left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 54.6\text{ cm}.$$

Note how the units (inches in this case) cancelled out (thin red lines). A Table containing many unit conversions is found on page A-73. Let's consider some Examples.

**EXAMPLE 1–3 The 8000-m peaks.** There are only 14 peaks whose summits are over 8000 m above sea level. They are the tallest peaks in the world (Fig. 1–9 and Table 1–6) and are referred to as "eight-thousanders." What is the elevation, in feet, of an elevation of 8000 m?

**APPROACH** We need to convert meters to feet, and we can start with the conversion factor 1 in. = 2.54 cm, which is exact. That is, 1 in. = 2.5400 cm to any number of significant figures, because it is *defined* to be.

**SOLUTION** One foot is 12 in., so we can write

$$1\text{ ft} = (12\ \cancel{\text{in.}})\left(2.54\,\frac{\text{cm}}{\cancel{\text{in.}}}\right) = 30.48\text{ cm} = 0.3048\text{ m},$$

which is exact. Note how the units cancel (colored slashes). We can rewrite this equation to find the number of feet in 1 meter:

$$1\text{ m} = \frac{1\text{ ft}}{0.3048} = 3.28084\text{ ft}.$$

(We could carry the result to 6 significant figures because 0.3048 is exact, $0.304800\cdots$.) We multiply this equation by 8000.0 (to have five significant figures):

$$8000.0\text{ m} = (8000.0\ \cancel{\text{m}})\left(3.28084\,\frac{\text{ft}}{\cancel{\text{m}}}\right) = 26{,}247\text{ ft}.$$

An elevation of 8000 m is 26,247 ft above sea level.

**NOTE** We could have done the unit conversions all in one line:

$$8000.0\text{ m} = (8000.0\ \cancel{\text{m}})\left(\frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}}\right)\left(\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\right)\left(\frac{1\text{ ft}}{12\ \cancel{\text{in.}}}\right) = 26{,}247\text{ ft}.$$

The key is to multiply conversion factors, each equal to one (= 1.0000), and to make sure which units cancel.

†Some exceptions are for angle (radians—see Chapter 8), solid angle (steradian), and sound level (bel or decibel, Chapter 12). No general agreement has been reached as to whether these are base or derived quantities.

**FIGURE 1–9** The world's second highest peak, K2, whose summit is considered the most difficult of the "8000-ers." K2 is seen here from the south (Pakistan). Example 1–3.

**PHYSICS APPLIED**
*The world's tallest peaks*

**TABLE 1–6 The 8000-m Peaks**

| Peak | Height (m) |
|---|---|
| Mt. Everest | 8850 |
| K2 | 8611 |
| Kangchenjunga | 8586 |
| Lhotse | 8516 |
| Makalu | 8462 |
| Cho Oyu | 8201 |
| Dhaulagiri | 8167 |
| Manaslu | 8156 |
| Nanga Parbat | 8125 |
| Annapurna | 8091 |
| Gasherbrum I | 8068 |
| Broad Peak | 8047 |
| Gasherbrum II | 8035 |
| Shisha Pangma | 8013 |

**EXAMPLE 1–4** **Apartment area.** You have seen a nice apartment whose floor area is 880 square feet ($ft^2$). What is its area in square meters?

**APPROACH** We use the same conversion factor, 1 in. = 2.54 cm, but this time we have to use it twice.

**SOLUTION** Because $1\text{ in.} = 2.54\text{ cm} = 0.0254\text{ m}$, then

$$1\text{ ft}^2 = (12\text{ in.})^2(0.0254\text{ m/in.})^2 = 0.0929\text{ m}^2.$$

So

$$880\text{ ft}^2 = (880\text{ ft}^2)(0.0929\text{ m}^2/\text{ft}^2) \approx 82\text{ m}^2.$$

**NOTE** As a rule of thumb, an area given in $ft^2$ is roughly 10 times the number of square meters (more precisely, about 10.8×).

**EXAMPLE 1–5** **Speeds.** Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (*a*) in meters per second (m/s) and (*b*) in kilometers per hour (km/h)?

**APPROACH** We again use the conversion factor $1\text{ in.} = 2.54\text{ cm}$, and we recall that there are 5280 ft in a mile and 12 inches in a foot; also, one hour contains $(60\text{ min/h}) \times (60\text{ s/min}) = 3600\text{ s/h}$.

**SOLUTION** (*a*) We can write 1 mile as

$$\begin{aligned} 1\text{ mi} &= (5280\,\cancel{\text{ft}})\left(12\,\frac{\cancel{\text{in.}}}{\cancel{\text{ft}}}\right)\left(2.54\,\frac{\cancel{\text{cm}}}{\cancel{\text{in.}}}\right)\left(\frac{1\text{ m}}{100\,\cancel{\text{cm}}}\right) \\ &= 1609\text{ m}. \end{aligned}$$

We also know that 1 hour contains 3600 s, so

$$\begin{aligned} 55\,\frac{\text{mi}}{\text{h}} &= \left(55\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(1609\,\frac{\text{m}}{\cancel{\text{mi}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\text{ s}}\right) \\ &= 25\,\frac{\text{m}}{\text{s}}, \end{aligned}$$

where we rounded off to two significant figures.

(*b*) Now we use $1\text{ mi} = 1609\text{ m} = 1.609\text{ km}$; then

$$\begin{aligned} 55\,\frac{\text{mi}}{\text{h}} &= \left(55\,\frac{\cancel{\text{mi}}}{\text{h}}\right)\left(1.609\,\frac{\text{km}}{\cancel{\text{mi}}}\right) \\ &= 88\,\frac{\text{km}}{\text{h}}. \end{aligned}$$

**PROBLEM SOLVING**
*Conversion factors* = 1

**NOTE** Each conversion factor is equal to one. You can look up most conversion factors in the Table inside the front cover.

**EXERCISE D** Return to the first Chapter-Opening Question, page 1, and answer it again now. Try to explain why you may have answered differently the first time.

**EXERCISE E** Would a driver traveling at 15 m/s in a 35 mi/h zone be exceeding the speed limit? Why or why not?

**PROBLEM SOLVING**
*Unit conversion is wrong if units do not cancel*

When changing units, you can avoid making an error in the use of conversion factors by checking that units cancel out properly. For example, in our conversion of 1 mi to 1609 m in Example 1–5(*a*), if we had incorrectly used the factor $\left(\frac{100\text{ cm}}{1\text{ m}}\right)$ instead of $\left(\frac{1\text{ m}}{100\text{ cm}}\right)$, the centimeter units would not have cancelled out; we would not have ended up with meters.

# 1–7 Order of Magnitude: Rapid Estimating

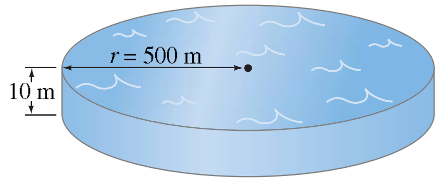
We are sometimes interested only in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check a calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

A rough estimate can be made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again keeping only one significant figure. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

**PROBLEM SOLVING**
*How to make a rough estimate*

Let's do some Examples.

(a)




(b)

**FIGURE 1–10** Example 1–6. (a) How much water is in this lake? (Photo is one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\,\mathrm{kg/m^3}$, so this lake has a mass of about $(10^3\,\mathrm{kg/m^3})(10^7\,\mathrm{m^3}) \approx 10^{10}\,\mathrm{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lb, slightly larger than a British ton, 2000 lb.)]

**EXAMPLE 1–6 ESTIMATE** **Volume of a lake.** Estimate how much water there is in a particular lake, Fig. 1–10a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

**APPROACH** No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 1–10b).

**PHYSICS APPLIED**
*Estimating the volume (or mass) of a lake; see also Fig. 1–10*

**SOLUTION** The volume $V$ of a cylinder is the product of its height $h$ times the area of its base: $V = h\pi r^2$, where $r$ is the radius of the circular base.† The radius $r$ is $\frac{1}{2}\,\mathrm{km} = 500\,\mathrm{m}$, so the volume is approximately

$$V = h\pi r^2 \approx (10\,\mathrm{m}) \times (3) \times (5 \times 10^2\,\mathrm{m})^2 \approx 8 \times 10^6\,\mathrm{m^3} \approx 10^7\,\mathrm{m^3},$$

where $\pi$ was rounded off to 3. So the volume is on the order of $10^7\,\mathrm{m^3}$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $(10^7\,\mathrm{m^3})$ is probably better to quote than the $8 \times 10^6\,\mathrm{m^3}$ figure.

**NOTE** To express our result in U.S. gallons, we see in the Table on the inside front cover that $1\text{ liter} = 10^{-3}\,\mathrm{m^3} \approx \frac{1}{4}\text{ gallon}$. Hence, the lake contains $(8 \times 10^6\,\mathrm{m^3})(1\text{ gallon}/4 \times 10^{-3}\,\mathrm{m^3}) \approx 2 \times 10^9\text{ gallons}$ of water.

†Formulas like this for volume, area, etc., are found inside the back cover of this book.

**EXAMPLE 1–7 ESTIMATE** **Thickness of a sheet of paper.** Estimate the thickness of a page of this book.

**APPROACH** At first you might think that a special measuring device, a micrometer (Fig. 1–11), is needed to measure the thickness of one page since an ordinary ruler can not be read so finely. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of this book are equal in thickness.

**SOLUTION** We can use a ruler to measure hundreds of pages at once. If you measure the thickness of the first 500 pages of this book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 numbered pages, counted front and back, is 250 separate pieces of paper. So one sheet must have a thickness of about

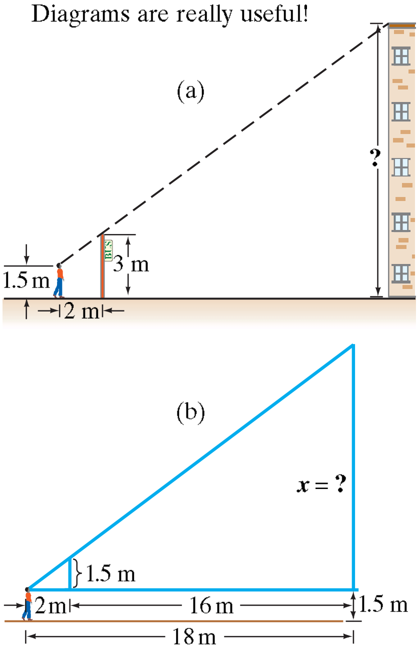
$$\frac{1.5\,\text{cm}}{250\,\text{sheets}} \approx 6 \times 10^{-3}\,\text{cm} = 6 \times 10^{-2}\,\text{mm},$$

or less than a tenth of a millimeter (0.1 mm).

**FIGURE 1–11** Example 1–7. Micrometer used for measuring small thicknesses.

It cannot be emphasized enough how important it is to draw a diagram when solving a physics Problem, as the next Example shows.

**FIGURE 1–12** Example 1–8. Diagrams are really useful!




**EXAMPLE 1–8 ESTIMATE** **Height by triangulation.** Estimate the height of the building shown in Fig. 1–12, by "triangulation," with the help of a bus-stop pole and a friend.

**APPROACH** By standing your friend next to the pole, you estimate the height of the pole to be 3 m. You next step away from the pole until the top of the pole is in line with the top of the building, Fig. 1–12a. You are 5 ft 6 in. tall, so your eyes are about 1.5 m above the ground. Your friend is taller, and when she stretches out her arms, one hand touches you, and the other touches the pole, so you estimate that distance as 2 m (Fig. 1–12a). You then pace off the distance from the pole to the base of the building with big, 1-m-long steps, and you get a total of 16 steps or 16 m.

**SOLUTION** Now you draw, to scale, the diagram shown in Fig. 1–12b using these measurements. You can measure, right on the diagram, the last side of the triangle to be about $x = 13\,\text{m}$. Alternatively, you can use similar triangles to obtain the height $x$:

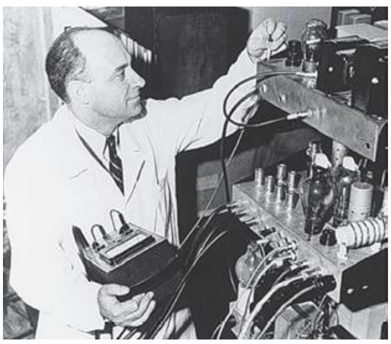
$$\frac{1.5\,\text{m}}{2\,\text{m}} = \frac{x}{18\,\text{m}},$$

so

$$x \approx 13\tfrac{1}{2}\,\text{m}.$$

Finally you add in your eye height of 1.5 m above the ground to get your final result: the building is about 15 m tall.

**FIGURE 1–13** Enrico Fermi. Fermi contributed significantly to both theoretical and experimental physics, a feat almost unique in modern times.



Another approach, this one made famous by Enrico Fermi (1901–1954, Fig. 1–13), was to show his students how to estimate the number of piano tuners in a city, say, Chicago or San Francisco. To get a rough order-of-magnitude estimate of the number of piano tuners today in San Francisco, a city of about 800,000 inhabitants, we can proceed by estimating the number of functioning pianos, how often each piano is tuned, and how many pianos each tuner can tune. To estimate the number of pianos in San Francisco, we note that certainly not everyone has a piano. A guess of 1 family in 3 having a piano would correspond to 1 piano per 12 persons, assuming an average family of 4 persons.

As an order of magnitude, let's say 1 piano per 10 people. This is certainly more reasonable than 1 per 100 people, or 1 per every person, so let's proceed with the estimate that 1 person in 10 has a piano, or about 80,000 pianos in San Francisco. Now a piano tuner needs an hour or two to tune a piano. So let's estimate that a tuner can tune 4 or 5 pianos a day. A piano ought to be tuned every 6 months or a year—let's say once each year. A piano tuner tuning 4 pianos a day, 5 days a week, 50 weeks a year can tune about 1000 pianos a year. So San Francisco, with its (very) roughly 80,000 pianos, needs about 80 piano tuners. This is, of course, only a rough estimate.† It tells us that there must be many more than 10 piano tuners, and surely not as many as 1000.

**PROBLEM SOLVING**
*Estimating how many piano tuners there are in a city*

## A Harder Example—But Powerful

**EXAMPLE 1–9 ESTIMATE** **Estimating the radius of Earth.** Believe it or not, you can estimate the radius of the Earth without having to go into space (see the photograph on page 1). If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as $d \approx 6.1$ km. Use Fig. 1–14 with $h = 3.0$ m to estimate the radius $R$ of the Earth.

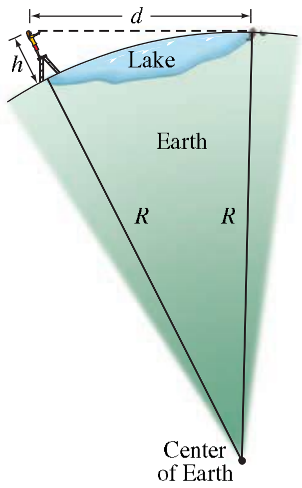


**FIGURE 1–14** Example 1–9, but not to scale. You can just barely see rocks at water level on the opposite shore of a lake 6.1 km wide if you stand on a stepladder.

**APPROACH** We use simple geometry, including the theorem of Pythagoras,

$$c^2 = a^2 + b^2,$$

where $c$ is the length of the hypotenuse of any right triangle, and $a$ and $b$ are the lengths of the other two sides.

**SOLUTION** For the right triangle of Fig. 1–14, the two sides are the radius of the Earth $R$ and the distance $d = 6.1\text{ km} = 6100\text{ m}$. The hypotenuse is approximately the length $R + h$, where $h = 3.0$ m. By the Pythagorean theorem,

$$\begin{aligned} R^2 + d^2 &\approx (R+h)^2 \\ &\approx R^2 + 2hR + h^2. \end{aligned}$$

We solve algebraically for $R$, after cancelling $R^2$ on both sides:

$$\begin{aligned} R \approx \frac{d^2 - h^2}{2h} &= \frac{(6100\text{ m})^2 - (3.0\text{ m})^2}{6.0\text{ m}} \\ &= 6.2 \times 10^6\text{ m} \\ &= 6200\text{ km}. \end{aligned}$$

**NOTE** Precise measurements give 6380 km. But look at your achievement! With a few simple rough measurements and simple geometry, you made a good estimate of the Earth's radius. You did not need to go out in space, nor did you need a very long measuring tape.

**EXERCISE F** Return to the second Chapter-Opening Question, page 1, and answer it again now. Try to explain why you may have answered differently the first time.

†A check of the San Francisco Yellow Pages (done after this calculation) reveals about 60 listings. Each of these listings may employ more than one tuner, but on the other hand, each may also do repairs as well as tuning. In any case, our estimate is reasonable.

## *1–8 Dimensions and Dimensional Analysis

When we speak of the **dimensions** of a quantity, we are referring to the type of base units or base quantities that make it up. The dimensions of area, for example, are always length squared, abbreviated $[L^2]$, using square brackets; the units can be square meters, square feet, $\text{cm}^2$, and so on. Velocity, on the other hand, can be measured in units of km/h, m/s, or mi/h, but the dimensions are always a length $[L]$ divided by a time $[T]$: that is, $[L/T]$.

The formula for a quantity may be different in different cases, but the dimensions remain the same. For example, the area of a triangle of base $b$ and height $h$ is $A = \frac{1}{2}bh$, whereas the area of a circle of radius $r$ is $A = \pi r^2$. The formulas are different in the two cases, but the dimensions of area are always $[L^2]$.

Dimensions can be used as a help in working out relationships, a procedure referred to as **dimensional analysis**. One useful technique is the use of dimensions to check if a relationship is *incorrect*. Note that we add or subtract quantities only if they have the same dimensions (we don't add centimeters and hours); and the quantities on each side of an equals sign must have the same dimensions. (In numerical calculations, the units must also be the same on both sides of an equation.)

For example, suppose you derived the equation $v = v_0 + \frac{1}{2}at^2$, where $v$ is the speed of an object after a time $t$, $v_0$ is the object's initial speed, and the object undergoes an acceleration $a$. Let's do a dimensional check to see if this equation could be correct or is surely incorrect. Note that numerical factors, like the $\frac{1}{2}$ here, do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of speed are $[L/T]$ and (as we shall see in Chapter 2) the dimensions of acceleration are $[L/T^2]$:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2]$$
$$\stackrel{?}{=} \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect: on the right side, we have the sum of quantities whose dimensions are not the same. Thus we conclude that an error was made in the derivation of the original equation.

A dimensional check can only tell you when a relationship is wrong. It can't tell you if it is completely right. For example, a dimensionless numerical factor (such as $\frac{1}{2}$ or $2\pi$) could be missing.

Dimensional analysis can also be used as a quick check on an equation you are not sure about. For example, consider a simple pendulum of length $\ell$. Suppose that you can't remember whether the equation for the period $T$ (the time to make one back-and-forth swing) is $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$, where $g$ is the acceleration due to gravity and, like all accelerations, has dimensions $[L/T^2]$. (Do not worry about these formulas—the correct one will be derived in Chapter 11; what we are concerned about here is a person's recalling whether it contains $\ell/g$ or $g/\ell$.) A dimensional check shows that the former $(\ell/g)$ is correct:

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T],$$

whereas the latter $(g/\ell)$ is not:

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}.$$

The constant $2\pi$ has no dimensions and so can't be checked using dimensions.

*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor, and they are marked with an asterisk (*). See the Preface for more details.

# Summary

[*The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary* cannot *serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.*]

Physics, like other sciences, is a creative endeavor. It is not simply a collection of facts. Important **theories** are created with the idea of explaining **observations**. To be accepted, theories are "tested" by comparing their predictions with the results of actual experiments. Note that, in general, a theory cannot be "proved" in an absolute sense.

Scientists often devise models of physical phenomena. A **model** is a kind of picture or analogy that helps to describe the phenomena in terms of something we already know. A **theory**, often developed from a model, is usually deeper and more complex than a simple model.

A scientific **law** is a concise statement, often expressed in the form of an equation, which quantitatively describes a wide range of phenomena.

**Measurements** play a crucial role in physics, but can never be perfectly precise. It is important to specify the **uncertainty** of a measurement either by stating it directly using the ± notation, and/or by keeping only the correct number of **significant figures**.

Physical quantities are always specified relative to a particular standard or **unit**, and the unit used should always be stated. The commonly accepted set of units today is the **Système International** (SI), in which the standard units of length, mass, and time are the **meter**, **kilogram**, and **second**.

When converting units, check all **conversion factors** for correct cancellation of units.

Making rough, **order-of-magnitude estimates** is a very useful technique in science as well as in everyday life.

[*The **dimensions** of a quantity refer to the combination of base quantities that comprise it. Velocity, for example, has dimensions of [length/time] or $[L/T]$. Working with only the dimensions of the various quantities in a given relationship (this technique is called **dimensional analysis**) makes it possible to check a relationship for correct form.]

# Questions

**1.** What are the merits and drawbacks of using a person's foot as a standard? Consider both (*a*) a particular person's foot, and (*b*) any person's foot. Keep in mind that it is advantageous that fundamental standards be accessible (easy to compare to), invariable (do not change), indestructible, and reproducible.

**2.** What is wrong with this road sign:

Memphis 7 mi (11.263 km)?

**3.** Why is it incorrect to think that the more digits you include in your answer, the more accurate it is?

**4.** For an answer to be complete, the units need to be specified. Why?

**5.** You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get the diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.

**6.** Express the sine of 30.0° with the correct number of significant figures.

**7.** List assumptions useful to estimate the number of car mechanics in (*a*) San Francisco, (*b*) your hometown, and then make the estimates.

# MisConceptual Questions

[List all answers that are valid.]

**1.** A student weighs herself on a digital bathroom scale as 117.4 lb. If all the digits displayed reflect the true precision of the scale, then probably her weight is
(*a*) within 1% of 117.4 lb.
(*b*) exactly 117.4 lb.
(*c*) somewhere between 117.38 and 117.42 lb.
(*d*) roughly between 117.2 and 117.6 lb.

**2.** Four students use different instruments to measure the length of the same pen. Which measurement implies the greatest precision?
(*a*) 160.0 mm. (*b*) 16.0 cm. (*c*) 0.160 m. (*d*) 0.00016 km.
(*e*) Need more information.

**3.** The number 0.0078 has how many significant figures?
(*a*) 1. (*b*) 2. (*c*) 3. (*d*) 4.

**4.** How many significant figures does $1.362 + 25.2$ have?
(*a*) 2. (*b*) 3. (*c*) 4. (*d*) 5.

**5.** Accuracy represents
(*a*) repeatability of a measurement, using a given instrument.
(*b*) how close a measurement is to the true value.
(*c*) an ideal number of measurements to make.
(*d*) how poorly an instrument is operating.

**6.** To convert from $ft^2$ to $yd^2$, you should
(*a*) multiply by 3.
(*b*) multiply by 1/3.
(*c*) multiply by 9.
(*d*) multiply by 1/9.
(*e*) multiply by 6.
(*f*) multiply by 1/6.

**7.** Which is *not* true about an order-of-magnitude estimation?
(*a*) It gives you a rough idea of the answer.
(*b*) It can be done by keeping only one significant figure.
(*c*) It can be used to check if an exact calculation is reasonable.
(*d*) It may require making some reasonable assumptions in order to calculate the answer.
(*e*) It will always be accurate to at least two significant figures.

***8.** $[L^2]$ represents the dimensions for which of the following?
(*a*) $cm^2$.
(*b*) square feet.
(*c*) $m^2$.
(*d*) All of the above.

For assigned homework and other learning materials, go to the MasteringPhysics website.

# Problems

[*The Problems at the end of each Chapter are ranked* I, II, *or* III *according to estimated difficulty, with* (I) *Problems being easiest. Level* III *are meant as challenges for the best students. The Problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—Problems often depend on earlier material. Next is a set of "General Problems" not arranged by Section and not ranked. Finally, there are "Search and Learn" Problems that require rereading parts of the Chapter.*]

### 1–4 Measurement, Uncertainty, Significant Figures

(Note: *In Problems, assume a number like* 6.4 *is accurate to* $\pm 0.1$; *and* 950 *is* $\pm 10$ *unless* 950 *is said to be "precisely" or "very nearly"* 950, *in which case assume* $950 \pm 1$.)

**1.** (I) How many significant figures do each of the following numbers have: (*a*) 214, (*b*) 81.60, (*c*) 7.03, (*d*) 0.03, (*e*) 0.0086, (*f*) 3236, and (*g*) 8700?

**2.** (I) Write the following numbers in powers of 10 notation: (*a*) 1.156, (*b*) 21.8, (*c*) 0.0068, (*d*) 328.65, (*e*) 0.219, and (*f*) 444.

**3.** (I) Write out the following numbers in full with the correct number of zeros: (*a*) $8.69 \times 10^4$, (*b*) $9.1 \times 10^3$, (*c*) $8.8 \times 10^{-1}$, (*d*) $4.76 \times 10^2$, and (*e*) $3.62 \times 10^{-5}$.

**4.** (II) The age of the universe is thought to be about 14 billion years. Assuming two significant figures, write this in powers of 10 in (*a*) years, (*b*) seconds.

**5.** (II) What is the percent uncertainty in the measurement $5.48 \pm 0.25\,\text{m}$?

**6.** (II) Time intervals measured with a stopwatch typically have an uncertainty of about 0.2 s, due to human reaction time at the start and stop moments. What is the percent uncertainty of a hand-timed measurement of (*a*) 5.5 s, (*b*) 55 s, (*c*) 5.5 min?

**7.** (II) Add $(9.2 \times 10^3\,\text{s}) + (8.3 \times 10^4\,\text{s}) + (0.008 \times 10^6\,\text{s})$.

**8.** (II) Multiply $3.079 \times 10^2\,\text{m}$ by $0.068 \times 10^{-1}\,\text{m}$, taking into account significant figures.

**9.** (II) What, approximately, is the percent uncertainty for a measurement given as $1.57\,\text{m}^2$?

**10.** (III) What, roughly, is the percent uncertainty in the volume of a spherical beach ball of radius $r = 0.84 \pm 0.04\,\text{m}$?

**11.** (III) What is the area, and its approximate uncertainty, of a circle of radius $3.1 \times 10^4\,\text{cm}$?

### 1–5 and 1–6 Units, Standards, SI, Converting Units

**12.** (I) Write the following as full (decimal) numbers without prefixes on the units: (*a*) 286.6 mm, (*b*) 85 $\mu$V, (*c*) 760 mg, (*d*) 62.1 ps, (*e*) 22.5 nm, (*f*) 2.50 gigavolts.

**13.** (I) Express the following using the prefixes of Table 1–4: (*a*) $1 \times 10^6$ volts, (*b*) $2 \times 10^{-6}$ meters, (*c*) $6 \times 10^3$ days, (*d*) $18 \times 10^2$ bucks, and (*e*) $7 \times 10^{-7}$ seconds.

**14.** (I) One hectare is defined as $1.000 \times 10^4\,\text{m}^2$. One acre is $4.356 \times 10^4\,\text{ft}^2$. How many acres are in one hectare?

**15.** (II) The Sun, on average, is 93 million miles from Earth. How many meters is this? Express (*a*) using powers of 10, and (*b*) using a metric prefix (km).

**16.** (II) Express the following sum with the correct number of significant figures: $1.80\,\text{m} + 142.5\,\text{cm} + 5.34 \times 10^5\,\mu\text{m}$.

**17.** (II) Determine the conversion factor between (*a*) km/h and mi/h, (*b*) m/s and ft/s, and (*c*) km/h and m/s.

**18.** (II) A **light-year** is the distance light travels in one year (at speed $= 2.998 \times 10^8\,\text{m/s}$). (*a*) How many meters are there in 1.00 light-year? (*b*) An astronomical unit (AU) is the average distance from the Sun to Earth, $1.50 \times 10^8\,\text{km}$. How many AU are there in 1.00 light-year?

**19.** (II) How much longer (percentage) is a one-mile race than a 1500-m race ("the metric mile")?

**20.** (II) American football uses a field that is 100.0 yd long, whereas a soccer field is 100.0 m long. Which field is longer, and by how much (give yards, meters, and percent)?

**21.** (II) (*a*) How many seconds are there in 1.00 year? (*b*) How many nanoseconds are there in 1.00 year? (*c*) How many years are there in 1.00 second?

**22.** (II) Use Table 1–3 to estimate the total number of protons or neutrons in (*a*) a bacterium, (*b*) a DNA molecule, (*c*) the human body, (*d*) our Galaxy.

**23.** (III) A standard baseball has a circumference of approximately 23 cm. If a baseball had the same mass per unit volume (see Tables in Section 1–5) as a neutron or a proton, about what would its mass be?

### 1–7 Order-of-Magnitude Estimating

(Note: *Remember that for rough estimates, only round numbers are needed both as input to calculations and as final results.*)

**24.** (I) Estimate the order of magnitude (power of 10) of: (*a*) 2800, (*b*) $86.30 \times 10^3$, (*c*) 0.0076, and (*d*) $15.0 \times 10^8$.

**25.** (II) Estimate how many books can be shelved in a college library with $3500\,\text{m}^2$ of floor space. Assume 8 shelves high, having books on both sides, with corridors 1.5 m wide. Assume books are about the size of this one, on average.

**26.** (II) Estimate how many hours it would take to run (at 10 km/h) across the U.S. from New York to California.

**27.** (II) Estimate the number of liters of water a human drinks in a lifetime.

**28.** (II) Estimate how long it would take one person to mow a football field using an ordinary home lawn mower (Fig. 1–15). (State your assumption, such as the mower moves with a 1-km/h speed, and has a 0.5-m width.)


**FIGURE 1–15** Problem 28.

**29.** (II) Estimate the number of gallons of gasoline consumed by the total of all automobile drivers in the U.S., per year.

**30.** (II) Estimate the number of dentists (*a*) in San Francisco and (*b*) in your town or city.

**31.** (III) I agree to hire you for 30 days. You can decide between two methods of payment: either (1) \$1000 a day, or (2) one penny on the first day, two pennies on the second day and continue to double your daily pay each day up to day 30. Use quick estimation to make your decision, and justify it.

**32.** (III) Many sailboats are docked at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. 1–16, where $h = 1.5\,\text{m}$, estimate the radius $R$ of the Earth.

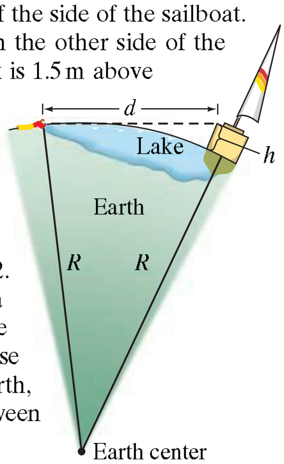



**FIGURE 1–16** Problem 32. You see a sailboat across a lake (not to scale). $R$ is the radius of the Earth. Because of the curvature of the Earth, the water "bulges out" between you and the boat.

### *1–8 Dimensions

*__33.__ (I) What are the dimensions of density, which is mass per volume?

*__34.__ (II) The speed $v$ of an object is given by the equation $v = At^3 - Bt$, where $t$ refers to time. (*a*) What are the dimensions of $A$ and $B$? (*b*) What are the SI units for the constants $A$ and $B$?

*__35.__ (III) The smallest meaningful measure of length is called the **Planck length**, and is defined in terms of three fundamental constants in nature: the speed of light $c = 3.00 \times 10^8\,\text{m/s}$, the gravitational constant $G = 6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2$, and Planck's constant $h = 6.63 \times 10^{-34}\,\text{kg}\cdot\text{m}^2/\text{s}$. The Planck length $\ell_P$ is given by the following combination of these three constants:

$$\ell_P = \sqrt{\frac{Gh}{c^3}}.$$

Show that the dimensions of $\ell_P$ are length $[L]$, and find the order of magnitude of $\ell_P$. [Recent theories (Chapters 32 and 33) suggest that the smallest particles (quarks, leptons) are "strings" with lengths on the order of the Planck length, $10^{-35}\,\text{m}$. These theories also suggest that the "Big Bang," with which the universe is believed to have begun, started from an initial size on the order of the Planck length.]

## General Problems

**36.** **Global positioning satellites (GPS)** can be used to determine your position with great accuracy. If one of the satellites is 20,000 km from you, and you want to know your position to $\pm 2\,\text{m}$, what percent uncertainty in the distance is required? How many significant figures are needed in the distance?

**37.** **Computer chips** (Fig. 1–17) are etched on circular silicon wafers of thickness 0.300 mm that are sliced from a solid cylindrical silicon crystal of length 25 cm. If each wafer can hold 400 chips, what is the maximum number of chips that can be produced from one entire cylinder?

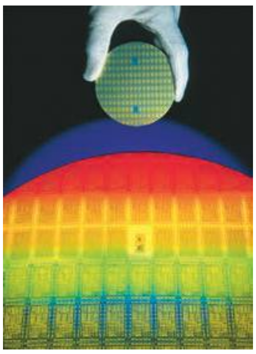


**FIGURE 1–17** Problem 37. The wafer held by the hand is shown below, enlarged and illuminated by colored light. Visible are rows of integrated circuits (chips).

**38.** A typical adult human lung contains about 300 million tiny cavities called alveoli. Estimate the average diameter of a single alveolus.

**39.** If you used only a keyboard to enter data, how many years would it take to fill up the hard drive in a computer that can store 1.0 terabytes $(1.0 \times 10^{12}\text{ bytes})$ of data? Assume 40-hour work weeks, and that you can type 180 characters per minute, and that one byte is one keyboard character.

**40.** An average family of four uses roughly 1200 L (about 300 gallons) of water per day $(1\,\text{L} = 1000\,\text{cm}^3)$. How much depth would a lake lose per year if it covered an area of $50\,\text{km}^2$ with uniform depth and supplied a local town with a population of 40,000 people? Consider only population uses, and neglect evaporation, rain, creeks and rivers.

**41.** Estimate the number of jelly beans in the jar of Fig. 1–18.


**FIGURE 1–18** Problem 41. Estimate the number of jelly beans in the jar.

**42.** How big is a ton? That is, what is the volume of something that weighs a ton? To be specific, estimate the diameter of a 1-ton rock, but first make a wild guess: will it be 1 ft across, 3 ft, or the size of a car? [*Hint*: Rock has mass per volume about 3 times that of water, which is 1 kg per liter $(10^3\,\text{cm}^3)$ or 62 lb per cubic foot.]

**43.** A certain compact disc (CD) contains 783.216 megabytes of digital information. Each byte consists of exactly 8 bits. When played, a CD player reads the CD's information at a constant rate of 1.4 megabits per second. How many minutes does it take the player to read the entire CD?

**44.** Hold a pencil in front of your eye at a position where its blunt end just blocks out the Moon (Fig. 1–19). Make appropriate measurements to estimate the diameter of the Moon, given that the Earth–Moon distance is $3.8 \times 10^5$ km.

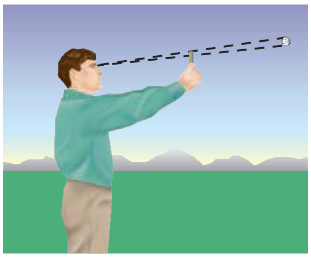


**FIGURE 1–19** Problem 44. How big is the Moon?

**45.** A storm dumps 1.0 cm of rain on a city 6 km wide and 8 km long in a 2-h period. How many metric tons $(1\text{ metric ton} = 10^3\,\text{kg})$ of water fell on the city? ($1\,\text{cm}^3$ of water has a mass of $1\,\text{g} = 10^{-3}\,\text{kg}$.) How many gallons of water was this?

**46.** Estimate how many days it would take to walk around the Earth, assuming 12 h walking per day at 4 km/h.

**47.** A watch manufacturer claims that its watches gain or lose no more than 8 seconds in a year. How accurate are these watches, expressed as a percentage?

**48.** An angstrom (symbol Å) is a unit of length, defined as $10^{-10}$ m, which is on the order of the diameter of an atom. (*a*) How many nanometers are in 1.0 angstrom? (*b*) How many femtometers or fermis (the common unit of length in nuclear physics) are in 1.0 angstrom? (*c*) How many angstroms are in 1.0 m? (*d*) How many angstroms are in 1.0 light-year (see Problem 18)?

**49.** Jim stands beside a wide river and wonders how wide it is. He spots a large rock on the bank directly across from him. He then walks upstream 65 strides and judges that the angle between him and the rock, which he can still see, is now at an angle of 30° downstream (Fig. 1–20). Jim measures his stride to be about 0.8 m long. Estimate the width of the river.

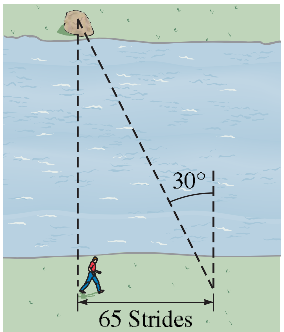



**FIGURE 1–20** Problem 49.

**50.** Determine the percent uncertainty in $\theta$, and in $\sin\theta$, when (*a*) $\theta = 15.0° \pm 0.5°$, (*b*) $\theta = 75.0° \pm 0.5°$.

**51.** If you walked north along one of Earth's lines of longitude until you had changed latitude by 1 minute of arc (there are 60 minutes per degree), how far would you have walked (in miles)? This distance is a **nautical mile**.

**52.** Make a rough estimate of the volume of your body (in $\text{m}^3$).

**53.** One mole of atoms consists of $6.02 \times 10^{23}$ individual atoms. If a mole of atoms were spread uniformly over the Earth's surface, how many atoms would there be per square meter?

**54.** The density of an object is defined as its mass divided by its volume. Suppose a rock's mass and volume are measured to be 6 g and $2.8325\,\text{cm}^3$. To the correct number of significant figures, determine the rock's density (mass/volume).

**55.** Recent findings in astrophysics suggest that the observable universe can be modeled as a sphere of radius $R = 13.7 \times 10^9$ light-years $= 13.0 \times 10^{25}$ m with an average total mass density of about $1 \times 10^{-26}\,\text{kg/m}^3$. Only about 4% of total mass is due to "ordinary" matter (such as protons, neutrons, and electrons). Estimate how much ordinary matter (in kg) there is in the observable universe. (For the light-year, see Problem 18.)

# Search and Learn

**1.** Galileo is to Aristotle as Copernicus is to Ptolemy. See Section 1–1 and explain this analogy.

**2.** Using the French Academy of Sciences' original definition of the meter, determine Earth's circumference and radius in those meters.

**3.** To the correct number of significant figures, use the information inside the front cover of this book to determine the ratio of (*a*) the surface area of Earth compared to the surface area of the Moon; (*b*) the volume of Earth compared to the volume of the Moon.

## ANSWERS TO EXERCISES

**A:** (*d*).

**B:** All three have three significant figures; the number of decimal places is (*a*) 2, (*b*) 3, (*c*) 4.

**C:** (*a*) $2.58 \times 10^{-2}$, 3; (*b*) $4.23 \times 10^4$, 3 (probably); (*c*) $3.4450 \times 10^2$, 5.

**D:** (*f*).

**E:** No: 15 m/s ≈ 34 mi/h.

**F:** (*c*).


The space shuttle has released a parachute to reduce its speed quickly. The directions of the shuttle's velocity and acceleration are shown by the green ($\vec{v}$) and gold ($\vec{a}$) arrows.

Motion is described using the concepts of velocity and acceleration. In the case shown here, the velocity $\vec{v}$ is to the right, in the direction of motion. The acceleration $\vec{a}$ is in the opposite direction from the velocity $\vec{v}$, which means the object is slowing down.

We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

CHAPTER 2

# Describing Motion: Kinematics in One Dimension

**CHAPTER-OPENING QUESTION—Guess now!**

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.*]

Two small heavy balls have the same diameter but one weighs twice as much as the other. The balls are dropped from a second-story balcony at the exact same time. The time to reach the ground below will be:

**(a)** twice as long for the lighter ball as for the heavier one.
**(b)** longer for the lighter ball, but not twice as long.
**(c)** twice as long for the heavier ball as for the lighter one.
**(d)** longer for the heavier ball, but not twice as long.
**(e)** nearly the same for both balls.

CONTENTS

2–1 Reference Frames and Displacement
2–2 Average Velocity
2–3 Instantaneous Velocity
2–4 Acceleration
2–5 Motion at Constant Acceleration
2–6 Solving Problems
2–7 Freely Falling Objects
2–8 Graphical Analysis of Linear Motion

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This Chapter and the next deal with kinematics.

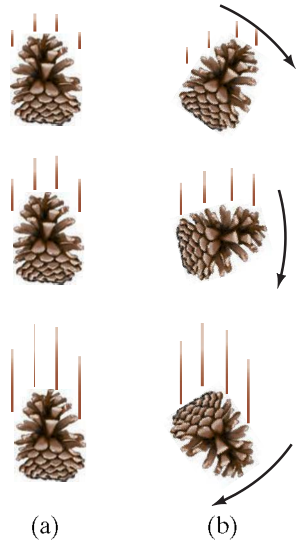



**FIGURE 2–1** A falling pinecone undergoes (a) pure translation; (b) it is rotating as well as translating.

For now we only discuss objects that move without rotating (Fig. 2–1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion. In Chapter 3 we will describe translational motion in two (or three) dimensions along paths that are not straight. (Rotation, shown in Fig. 2–1b, is discussed in Chapter 8.)

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical **point** with no spatial extent (no size). A point particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

## 2–1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\text{km/h} + 5\,\text{km/h} = 85\,\text{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

**FIGURE 2–2** A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

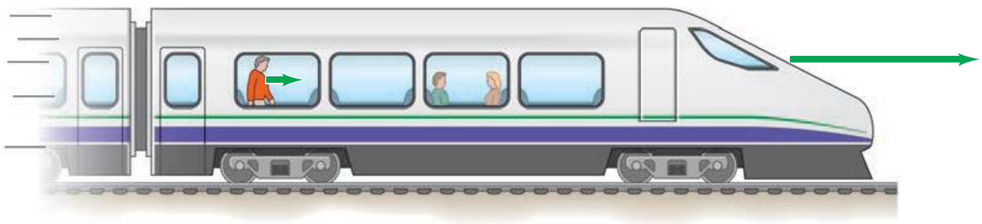
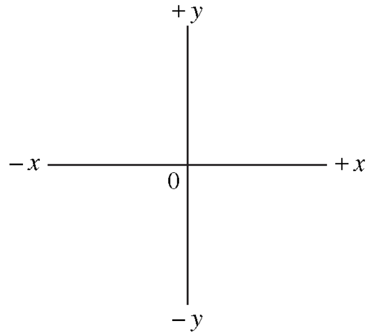


**FIGURE 2–3** Standard set of $xy$ coordinate axes, sometimes called "rectangular coordinates."




When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the $x$ and $y$ axes, as we like for convenience. The $x$ and $y$ axes are always perpendicular to each other. The **origin** is where $x = 0$, $y = 0$. Objects positioned to the right of the origin of coordinates (0) on the $x$ axis have an $x$ coordinate which we almost always choose to be positive; then points to the left of 0 have a negative $x$ coordinate. The position along the $y$ axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its $x$ and $y$ coordinates. In three dimensions, a $z$ axis perpendicular to the $x$ and $y$ axes is added.

For one-dimensional motion, we often choose the $x$ axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its $x$ coordinate. If the motion is vertical, as for a dropped object, we usually use the $y$ axis.

We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point*. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

**CAUTION**

*The displacement may not equal the total distance traveled*

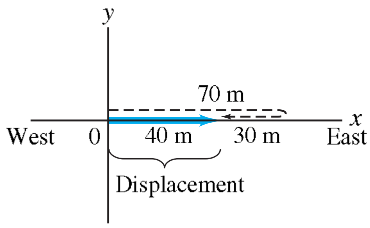


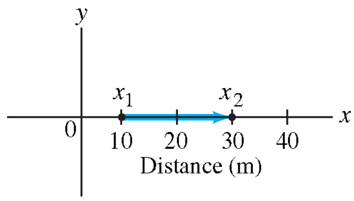

**FIGURE 2–4** A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will be positive (typically to the right along the $x$ axis). Vectors that point in the opposite direction will have a negative sign in front of their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it $t_1$, the object is on the $x$ axis at the position $x_1$ in the coordinate system shown in Fig. 2–5. At some later time, $t_2$, suppose the object has moved to position $x_2$. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

**FIGURE 2–5** The arrow represents the displacement $x_2 - x_1$. Distances are in meters.




$$\Delta x = x_2 - x_1,$$

where the symbol $\Delta$ (Greek letter delta) means "change in." Then $\Delta x$ means "the change in $x$," or "change in position," which is the displacement. The **change in** any quantity means *the final value of that quantity, minus the initial value*. Suppose $x_1 = 10.0\,\text{m}$ and $x_2 = 30.0\,\text{m}$, as in Fig. 2–5. Then

$$\Delta x = x_2 - x_1 = 30.0\,\text{m} - 10.0\,\text{m} = 20.0\,\text{m},$$

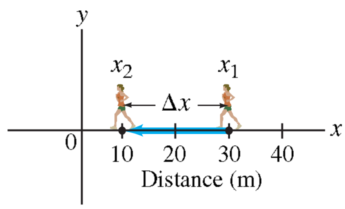
so the displacement is 20.0 m in the positive direction, Fig. 2–5.

Now consider an object moving to the left as shown in Fig. 2–6. Here the object, a person, starts at $x_1 = 30.0\,\text{m}$ and walks to the left to the point $x_2 = 10.0\,\text{m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\,\text{m} - 30.0\,\text{m} = -20.0\,\text{m},$$

and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the $x$ axis, a vector pointing to the right is positive, whereas a vector pointing to the left has a negative sign.

**FIGURE 2–6** For the displacement $\Delta x = x_2 - x_1 = 10.0\,\text{m} - 30.0\,\text{m}$, the displacement vector points left.




**EXERCISE A** An ant starts at $x = 20\,\text{cm}$ on a piece of graph paper and walks along the $x$ axis to $x = -20\,\text{cm}$. It then turns around and walks back to $x = -10\,\text{cm}$. Determine (a) the ant's displacement and (b) the total distance traveled.

# 2–2 Average Velocity

An important aspect of the motion of a moving object is how *fast* it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(2–1)}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. Velocity is therefore a *vector*.

There is a second difference between speed and velocity: namely, the *average velocity* is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

**CAUTION**
*Average speed is not necessarily equal to the magnitude of the average velocity*

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 2–4, where a person walked 70 m east and then 30 m west. The total distance traveled was $70\,\text{m} + 30\,\text{m} = 100\,\text{m}$, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\,\text{m}}{70\,\text{s}} = 1.4\,\text{m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\,\text{m}}{70\,\text{s}} = 0.57\,\text{m/s}.$$

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it $t_1$, the object is on the $x$ axis at position $x_1$ in a coordinate system, and at some later time, $t_2$, suppose it is at position $x_2$. The **elapsed time** (= change in time) is $\Delta t = t_2 - t_1$; during this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the **average velocity**, defined as *the displacement divided by the elapsed time*, can be written

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \text{[average velocity]} \quad \textbf{(2–2)}$$

where $v$ stands for velocity and the bar ($^{-}$) over the $v$ is a standard symbol meaning "average."

**PROBLEM SOLVING**
*+ or − sign can signify the direction for linear motion*

For one-dimensional motion in the usual case of the $+x$ axis to the right, note that if $x_2$ is less than $x_1$, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the $x$ axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

**CAUTION**
*Time interval = elapsed time*

It is always important to choose (and state) the *elapsed time*, or **time interval**, $t_2 - t_1$, the time that passes during our chosen period of observation.

**EXAMPLE 2–1** **Runner's average velocity.** The position of a runner is plotted as moving along the $x$ axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0\,\text{m}$ to $x_2 = 30.5\,\text{m}$, as shown in Fig. 2–7. What is the runner's average velocity?

**APPROACH** We want to find the average velocity, which is the displacement divided by the elapsed time.

**SOLUTION** The displacement is

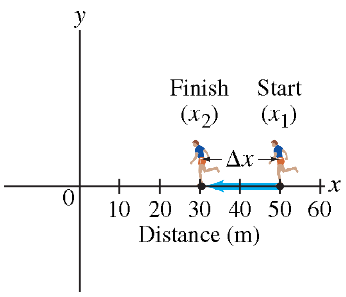
$$\begin{aligned}\Delta x &= x_2 - x_1 \\ &= 30.5\,\text{m} - 50.0\,\text{m} = -19.5\,\text{m}.\end{aligned}$$

The elapsed time, or time interval, is given as $\Delta t = 3.00\,\text{s}$. The average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\,\text{m}}{3.00\,\text{s}} = -6.50\,\text{m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the $x$ axis, as indicated by the arrow in Fig. 2–7. The runner's average velocity is 6.50 m/s to the left.

**FIGURE 2–7** Example 2–1. A person runs from $x_1 = 50.0\,\text{m}$ to $x_2 = 30.5\,\text{m}$. The displacement is $-19.5\,\text{m}$.



**EXAMPLE 2–2** **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

**APPROACH** We want to find the distance traveled, so we solve Eq. 2–2 for $\Delta x$.

**SOLUTION** In Eq. 2–2, $\bar{v} = \Delta x/\Delta t$, we multiply both sides by $\Delta t$ and obtain

$$\Delta x = \bar{v}\,\Delta t = (18\,\text{km/h})(2.5\,\text{h}) = 45\,\text{km}.$$

**EXAMPLE 2–3** **Car changes speed.** A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200-km trip?

**APPROACH** At 50 km/h, the car takes 2.0 h to travel 100 km. At 100 km/h it takes only 1.0 h to travel 100 km. We use the defintion of average velocity, Eq. 2–2.

**SOLUTION** Average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100\,\text{km} + 100\,\text{km}}{2.0\,\text{h} + 1.0\,\text{h}} = 67\,\text{km/h}.$$

**NOTE** Averaging the two speeds, $(50\,\text{km/h} + 100\,\text{km/h})/2 = 75\,\text{km/h}$, gives a wrong answer. Can you see why? You must use the definition of $\bar{v}$, Eq. 2–2.

# 2–3 Instantaneous Velocity

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer, Fig. 2–8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval*. That is, Eq. 2–2 is to be evaluated in the limit of $\Delta t$ becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, $v$, for one-dimensional motion as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad \text{[instantaneous velocity]} \quad \textbf{(2–3)}$$

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of $\Delta t$ approaching zero.[†]

For instantaneous velocity we use the symbol $v$, whereas for average velocity we use $\bar{v}$, with a bar above. In the rest of this book, when we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous speed* always equals the magnitude of the instantaneous velocity. Why? Because distance traveled and the magnitude of the displacement become the same when they become infinitesimally small.

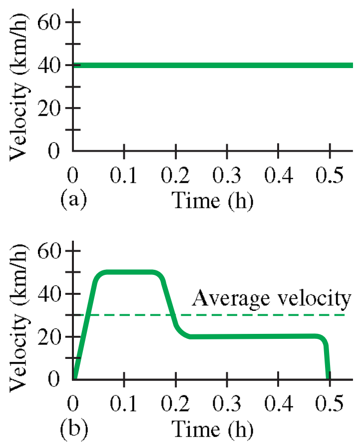
If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 2–9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 2–9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\,\text{km}/0.50\,\text{h} = 30\,\text{km/h}$.

Graphs are often useful for analysis of motion; we discuss additional insights graphs can provide as we go along, especially in Section 2–8.

**EXERCISE B** What is your instantaneous speed at the instant you turn around to move in the opposite direction? (*a*) Depends on how quickly you turn around; (*b*) always zero; (*c*) always negative; (*d*) none of the above.


**FIGURE 2–8** Car speedometer showing mi/h in white, and km/h in orange.

**FIGURE 2–9** Velocity of a car as a function of time: (a) at constant velocity; (b) with velocity varying in time.




[†]We do not simply set $\Delta t = 0$ in this definition, for then $\Delta x$ would also be zero, and we would have an undetermined number. Rather, we consider the *ratio* $\Delta x/\Delta t$, as a whole. As we let $\Delta t$ approach zero, $\Delta x$ approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.

# 2–4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how *rapidly* the velocity of an object is changing.

*Average acceleration* is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the **average acceleration**, $\bar{a}$, over a time interval $\Delta t = t_2 - t_1$, during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \qquad \text{[average acceleration]} \quad \textbf{(2–4)}$$

We saw that velocity is a vector (it has magnitude and direction), so acceleration is a vector too. But for one dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis. (Usually, right is +, left is −.)

The **instantaneous acceleration**, $a$, can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \qquad \text{[instantaneous acceleration]} \quad \textbf{(2–5)}$$

Here $\Delta v$ is the very small change in velocity during the very short time interval $\Delta t$.

**EXAMPLE 2–4** **Average acceleration.** A car accelerates on a straight road from rest to 75 km/h in 5.0 s, Fig. 2–10. What is the magnitude of its average acceleration?

**APPROACH** Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 75\,\text{km/h}$.

**SOLUTION** From Eq. 2–4, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{75\,\text{km/h} - 0\,\text{km/h}}{5.0\,\text{s}} = 15\,\frac{\text{km/h}}{\text{s}}.$$

This is read as "fifteen kilometers per hour per second" and means that, on average, the velocity changed by 15 km/h during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 15 km/h. During the next second its velocity increased by another 15 km/h, reaching a velocity of 30 km/h at $t = 2.0\,\text{s}$, and so on. See Fig. 2–10.

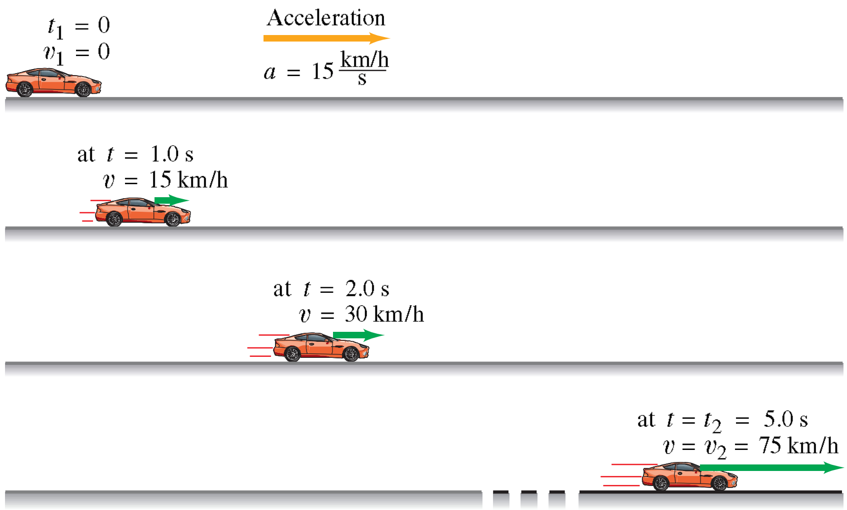



**FIGURE 2–10** Example 2–4. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0\,\text{s}$, $t = 2.0\,\text{s}$, and at the end of our time interval, $t_2 = 5.0\,\text{s}$. The green arrows represent the velocity vectors, whose length represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow, whose magnitude is constant and equals 15 km/h/s or 4.2 m/s$^2$ (see top of next page). Distances are not to scale.

Our result in Example 2–4 contains two different time units: hours and seconds. We usually prefer to use only seconds. To do so we can change km/h to m/s (see Section 1–6, and Example 1–5):

$$75\,\text{km/h} = \left(75\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 21\,\text{m/s}.$$

Then

$$\bar{a} = \frac{21\,\text{m/s} - 0.0\,\text{m/s}}{5.0\,\text{s}} = 4.2\,\frac{\text{m/s}}{\text{s}} = 4.2\,\frac{\text{m}}{\text{s}^2}.$$

We almost always write the units for acceleration as $\text{m/s}^2$ (meters per second squared) instead of m/s/s. This is possible because:

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}\cdot\text{s}} = \frac{\text{m}}{\text{s}^2}.$$

Note that *acceleration tells us how quickly the velocity changes*, whereas *velocity tells us how quickly the position changes.*

**CAUTION**
*Distinguish velocity from acceleration*

**CONCEPTUAL EXAMPLE 2–5** **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

**CAUTION**
*If v or a is zero, is the other zero too?*

**RESPONSE** A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0,\ v \neq 0$.

**EXAMPLE 2–6** **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive $x$ axis (Fig. 2–11). Then the driver steps on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0\,\text{m/s}$, and it takes 5.0 s to slow down to $v_2 = 5.0\,\text{m/s}$, what was the car's average acceleration?

**APPROACH** We put the given initial and final velocities, and the elapsed time, into Eq. 2–4 for $\bar{a}$.

**SOLUTION** In Eq. 2–4, we call the initial time $t_1 = 0$, and set $t_2 = 5.0\,\text{s}$:

$$\bar{a} = \frac{5.0\,\text{m/s} - 15.0\,\text{m/s}}{5.0\,\text{s}} = -2.0\,\text{m/s}^2.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative $x$ direction)—even though the velocity is always pointing to the right. We say that the acceleration is $2.0\,\text{m/s}^2$ to the left, and it is shown in Fig. 2–11 as an orange arrow.

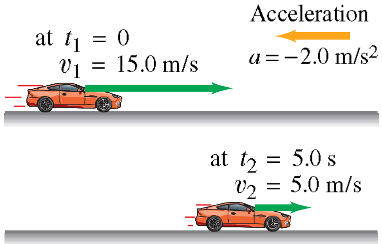


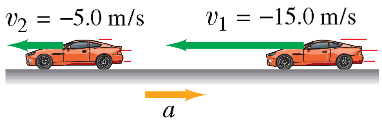

**FIGURE 2–11** Example 2–6, showing the position of the car at times $t_1$ and $t_2$, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left because the car slows down as it moves to the right.

## Deceleration

When an object is slowing down, we can say it is **decelerating**. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive $x$ axis is positive; if the object is slowing down (as in Fig. 2–11), the acceleration *is* negative. But the same car moving to the left (decreasing $x$), and slowing down, has positive acceleration that points to the right, as shown in Fig. 2–12. We have a deceleration whenever the magnitude of the velocity is decreasing; thus the *velocity and acceleration point in opposite directions* when there is deceleration.

**FIGURE 2–12** The car of Example 2–6, now moving to the *left* and decelerating. The acceleration is $a = (v_2 - v_1)/\Delta t$, or

$$a = \frac{(-5.0\,\text{m/s}) - (-15.0\,\text{m/s})}{5.0\,\text{s}}$$
$$= \frac{-5.0\,\text{m/s} + 15.0\,\text{m/s}}{5.0\,\text{s}} = +2.0\,\text{m/s}^2.$$




**EXERCISE C** A car moves along the $x$ axis. What is the sign of the car's acceleration if it is moving in the positive $x$ direction with (*a*) increasing speed or (*b*) decreasing speed? What is the sign of the acceleration if the car moves in the negative $x$ direction with (*c*) increasing speed or (*d*) decreasing speed?

## 2–5 Motion at Constant Acceleration

We now examine motion in a straight line when the magnitude of the acceleration is constant. In this case, the instantaneous and average accelerations are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate $x$, $v$, $a$, and $t$ when $a$ is constant, allowing us to determine any one of these variables if we know the others. We can then solve many interesting Problems.

Notation in physics varies from book to book; and different instructors use different notation. We are now going to change our notation, to simplify it a bit for our discussion here of motion at **constant acceleration**. First we choose the initial time in any discussion to be zero, and we call it $t_0$. That is, $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at $t_0$.) We can then let $t_2 = t$ be the elapsed time. The initial position $(x_1)$ and the initial velocity $(v_1)$ of an object will now be represented by $x_0$ and $v_0$, since they represent $x$ and $v$ at $t = 0$. At time $t$ the position and velocity will be called $x$ and $v$ (rather than $x_2$ and $v_2$). The average velocity during the time interval $t - t_0$ will be (Eq. 2–2)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is $a = \Delta v/\Delta t$ (Eq. 2–4), so

$$a = \frac{v - v_0}{t}.$$

**FIGURE 2–13** An accelerating motorcycle.

A common problem is to determine the velocity of an object after any elapsed time $t$, when we are given the object's constant acceleration. We can solve such problems† by solving for $v$ in the last equation: first we multiply both sides by $t$,

$$at = v - v_0 \quad \text{or} \quad v - v_0 = at.$$

Then, adding $v_0$ to both sides, we obtain

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \textbf{(2–6)}$$

If an object, such as a motorcycle (Fig. 2–13), starts from rest $(v_0 = 0)$ and accelerates at $4.0\,\text{m/s}^2$, after an elapsed time $t = 6.0\,\text{s}$ its velocity will be $v = 0 + at = (4.0\,\text{m/s}^2)(6.0\,\text{s}) = 24\,\text{m/s}$.

Next, let us see how to calculate the position $x$ of an object after a time $t$ when it undergoes constant acceleration. The definition of average velocity (Eq. 2–2) is $\bar{v} = (x - x_0)/t$, which we can rewrite by multiplying both sides by $t$:

$$x = x_0 + \bar{v}t. \qquad \textbf{(2–7)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

**CAUTION**
*Average velocity, but only if $a$ = constant*

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \textbf{(2–8)}$$

(Careful: Equation 2–8 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 2–6 and find, starting with Eq. 2–7,

$$\begin{aligned} x &= x_0 + \bar{v}t \\ &= x_0 + \left(\frac{v_0 + v}{2}\right)t \\ &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \end{aligned}$$

or

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \textbf{(2–9)}$$

Equations 2–6, 2–8, and 2–9 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful

†Appendix A–4 summarizes simple algebraic manipulations.

in situations where the time $t$ is not known. We substitute Eq. 2–8 into Eq. 2–7:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$

Next we solve Eq. 2–6 for $t$, obtaining (see Appendix A–4 for a quick review)

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for $v^2$ and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(2–10)}$$

which is the other useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration $a$ is constant. We collect these *kinematic equations for constant acceleration* here in one place for future reference (the tan background screen emphasizes their usefulness):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(2–11a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(2–11b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(2–11c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(2–11d)}$$

*Kinematic equations for constant acceleration (we'll use them a lot)*

These useful equations are not valid unless $a$ is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that $x$ represents position (not distance), also that $x - x_0$ is the displacement, and that $t$ is the elapsed time. Equations 2–11 are useful also when $a$ is approximately constant to obtain reasonable estimates.

**EXAMPLE 2–7** **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\,\text{m/s}^2$. (*a*) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (*b*) If not, what minimum length must the runway have?

**PHYSICS APPLIED**
*Airport design*

**APPROACH** Assuming the plane's acceleration is constant, we use the kinematic equations for constant acceleration. In (*a*), we want to find $v$, and what we are given is shown in the Table in the margin.

| Known | Wanted |
|---|---|
| $x_0 = 0$ | $v$ |
| $v_0 = 0$ | |
| $x = 150\,\text{m}$ | |
| $a = 2.00\,\text{m/s}^2$ | |

**SOLUTION** (*a*) Of the above four equations, Eq. 2–11c will give us $v$ when we know $v_0$, $a$, $x$, and $x_0$:

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 0 + 2(2.00\,\text{m/s}^2)(150\,\text{m}) = 600\,\text{m}^2/\text{s}^2 \\ v &= \sqrt{600\,\text{m}^2/\text{s}^2} = 24.5\,\text{m/s}. \end{aligned}$$

**PROBLEM SOLVING**
*Equations 2–11 are valid only when the acceleration is constant, which we assume in this Example*

This runway length is *not* sufficient, because the minimum speed is not reached.

(*b*) Now we want to find the minimum runway length, $x - x_0$, for a plane to reach $v = 27.8\,\text{m/s}$, given $a = 2.00\,\text{m/s}^2$. We again use Eq. 2–11c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\,\text{m/s})^2 - 0}{2(2.00\,\text{m/s}^2)} = 193\,\text{m}.$$

A 200-m runway is more appropriate for this plane.

**NOTE** We did this Example as if the plane were a particle, so we round off our answer to 200 m.

**EXERCISE D** A car starts from rest and accelerates at a constant $10\,\mathrm{m/s^2}$ during a $\frac{1}{4}$-mile (402 m) race. How fast is the car going at the finish line? (*a*) 8040 m/s; (*b*) 90 m/s; (*c*) 81 m/s; (*d*) 804 m/s.

## 2–6 Solving Problems

Before doing more worked-out Examples, let us look at how to approach problem solving. First, it is important to note that physics is *not* a collection of equations to be memorized. Simply searching for an equation that might work can lead you to a wrong result and will not help you understand physics (Fig. 2–14). A better approach is to use the following (rough) procedure, which we present as a special "Problem Solving Strategy." (Other such Problem Solving Strategies will be found throughout the book.)

**FIGURE 2–14** Read the book, study carefully, and work the Problems using your reasoning abilities.

**PROBLEM SOLVING**

1. Read and **reread** the whole problem carefully before trying to solve it.
2. Decide what **object** (or objects) you are going to study, and for what **time interval**. You can often choose the initial time to be $t = 0$.
3. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the *x* axis to the right as positive.]
4. Write down what quantities are "**known**" or "given," and then what you *want* to know. Consider quantities both at the beginning and at the end of the chosen time interval. You may need to "translate" language into physical terms, such as "starts from rest" means $v_0 = 0$.
5. Think about which **principles of physics** apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which **equations** (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 2–11 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. Sometimes several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
7. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures (Section 1–4).
8. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of 10, as discussed in Section 1–7. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
9. A very important aspect of doing problems is keeping track of **units**. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

**EXAMPLE 2–8** **Acceleration of a car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\,\mathrm{m/s^2}$?

**APPROACH** We follow the Problem Solving Strategy on the previous page, step by step.

**SOLUTION**

1. **Reread** the problem. Be sure you understand what it asks for (here, a time interval: "how long does it take").
2. The **object** under study is the car. We need to choose the **time interval** during which we look at the car's motion: we choose $t = 0$, the initial time, to be the moment the car starts to accelerate from rest $(v_0 = 0)$; the time $t$ is the instant the car has traveled the full 30.0-m width of the intersection.
3. **Draw** a **diagram**: the situation is shown in Fig. 2–15, where the car is shown moving along the positive $x$ axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.
4. The "**knowns**" and the "wanted" information are shown in the Table in the margin. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$. The wanted time $t$ is how long it takes the car to travel 30.0 m.
5. The **physics**: the car, starting from rest $(\text{at } t_0 = 0)$, increases in speed as it covers more distance. The acceleration is constant, so we can use the kinematic equations, Eqs. 2–11.
6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 2–11b is perfect since the only unknown quantity is $t$. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 2–11b $\left(x = x_0 + v_0 t + \tfrac{1}{2}at^2\right)$, we have
$$x = \tfrac{1}{2}at^2.$$
We solve for $t$ by multiplying both sides by $\dfrac{2}{a}$:
$$\frac{2x}{a} = t^2.$$
Taking the square root, we get $t$:
$$t = \sqrt{\frac{2x}{a}}.$$
7. The **calculation**:
$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\mathrm{m})}{2.00\,\mathrm{m/s^2}}} = 5.48\,\mathrm{s}.$$
This is our answer. Note that the units come out correctly.
8. We can check the **reasonableness** of the answer by doing an alternate calculation: we first find the final velocity
$$v = at = (2.00\,\mathrm{m/s^2})(5.48\,\mathrm{s}) = 10.96\,\mathrm{m/s},$$
and then find the distance traveled
$$x = x_0 + \bar{v}t = 0 + \tfrac{1}{2}(10.96\,\mathrm{m/s} + 0)(5.48\,\mathrm{s}) = 30.0\,\mathrm{m},$$
which checks with our given distance.
9. We checked the **units** in step 7, and they came out correctly (seconds).

**NOTE** In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm\,5.48\,\mathrm{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\,\mathrm{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

We explicitly followed the steps of the Problem Solving Strategy in Example 2–8. In upcoming Examples, we will use our usual "Approach" and "Solution" to avoid being wordy.

**PROBLEM SOLVING**
*"Starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$]*

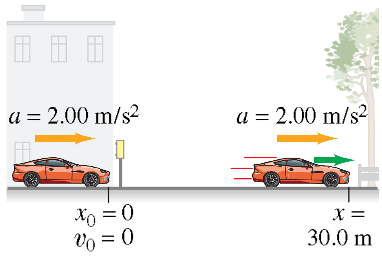


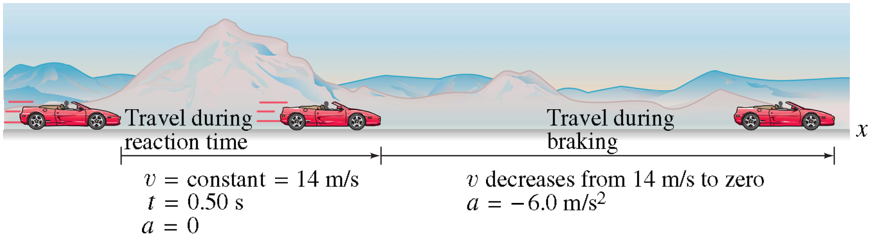

**FIGURE 2–15** Example 2–8.

| Known | Wanted |
|---|---|
| $x_0 = 0$ | $t$ |
| $x = 30.0\,\mathrm{m}$ | |
| $a = 2.00\,\mathrm{m/s^2}$ | |
| $v_0 = 0$ | |

**PROBLEM SOLVING**
*Check your answer*

**PROBLEM SOLVING**
*"Unphysical" solutions*

**FIGURE 2–16** Example 2–9: stopping distance for a braking car.




**PHYSICS APPLIED**

*Car stopping distances*

**EXAMPLE 2–9 ESTIMATE Braking distances.** Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$. (2) The second time interval is the actual braking period when the vehicle slows down $(a \neq 0)$ and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the deceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\,\mathrm{m/s^2}$ to $8\,\mathrm{m/s^2}$. Calculate the total stopping distance for an initial velocity of 50 km/h $(= 14\,\mathrm{m/s} \approx 31\,\mathrm{mi/h})$ and assume the acceleration of the car is $-6.0\,\mathrm{m/s^2}$ (the minus sign appears because the velocity is taken to be in the positive $x$ direction and its magnitude is decreasing). Reaction time for normal drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

**APPROACH** During the "reaction time," part (1), the car moves at constant speed of 14 m/s, so $a = 0$. Once the brakes are applied, part (2), the acceleration is $a = -6.0\,\mathrm{m/s^2}$ and is constant over this time interval. For both parts $a$ is constant, so we can use Eqs. 2–11.

*Part 1: Reaction time*

| Known | Wanted |
|---|---|
| $t = 0.50\,\mathrm{s}$ | $x$ |
| $v_0 = 14\,\mathrm{m/s}$ | |
| $v = 14\,\mathrm{m/s}$ | |
| $a = 0$ | |
| $x_0 = 0$ | |

**SOLUTION** Part (1). We take $x_0 = 0$ for the first time interval, when the driver is reacting (0.50 s): the car travels at a constant speed of 14 m/s so $a = 0$. See Fig. 2–16 and the Table in the margin. To find $x$, the position of the car at $t = 0.50\,\mathrm{s}$ (when the brakes are applied), we cannot use Eq. 2–11c because $x$ is multiplied by $a$, which is zero. But Eq. 2–11b works:

$$x = v_0 t + 0 = (14\,\mathrm{m/s})(0.50\,\mathrm{s}) = 7.0\,\mathrm{m}.$$

Thus the car travels 7.0 m during the driver's reaction time, until the instant the brakes are applied. We will use this result as input to part (2).

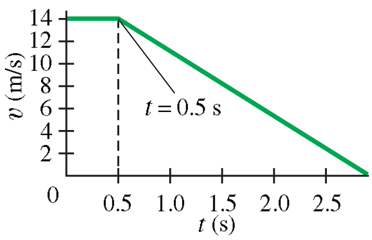
*Part 2: Braking*

| Known | Wanted |
|---|---|
| $x_0 = 7.0\,\mathrm{m}$ | $x$ |
| $v_0 = 14\,\mathrm{m/s}$ | |
| $v = 0$ | |
| $a = -6.0\,\mathrm{m/s^2}$ | |

Part (2). During the second time interval, the brakes are applied and the car is brought to rest. The initial position is $x_0 = 7.0\,\mathrm{m}$ (result of part (1)), and other variables are shown in the second Table in the margin. Equation 2–11a doesn't contain $x$; Eq. 2–11b contains $x$ but also the unknown $t$. Equation 2–11c, $v^2 - v_0^2 = 2a(x - x_0)$, is what we want; after setting $x_0 = 7.0\,\mathrm{m}$, we solve for $x$, the final position of the car (when it stops):

$$\begin{aligned} x &= x_0 + \frac{v^2 - v_0^2}{2a} \\ &= 7.0\,\mathrm{m} + \frac{0 - (14\,\mathrm{m/s})^2}{2(-6.0\,\mathrm{m/s^2})} = 7.0\,\mathrm{m} + \frac{-196\,\mathrm{m^2/s^2}}{-12\,\mathrm{m/s^2}} \\ &= 7.0\,\mathrm{m} + 16\,\mathrm{m} = 23\,\mathrm{m}. \end{aligned}$$

**FIGURE 2–17** Example 2–9. Graph of $v$ vs. $t$.




The car traveled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop, for a total distance traveled of 23 m. Figure 2–17 shows a graph of $v$ vs. $t$: $v$ is constant from $t = 0$ until $t = 0.50\,\mathrm{s}$, and after $t = 0.50\,\mathrm{s}$ it decreases linearly to zero.

**NOTE** From the equation above for $x$, we see that the stopping distance after the driver hit the brakes $(= x - x_0)$ increases with the *square* of the initial speed, not just linearly with speed. If you are traveling twice as fast, it takes four times the distance to stop.

**FIGURE 2–18** Painting of Galileo demonstrating to the Grand Duke of Tuscany his argument for the action of gravity being uniform acceleration. He used an inclined plane to slow down the action. A ball rolling down the plane still accelerates. Tiny bells placed at equal distances along the inclined plane would ring at shorter time intervals as the ball "fell," indicating that the speed was increasing.

**FIGURE 2–19** Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

# 2–7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 2–18), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is. *The speed of a falling object is* not *proportional to its mass.*

Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that *all objects would fall with the same constant acceleration in the absence of air or other resistance*. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–19); that is, $d \propto t^2$. We can see this from Eq. 2–11b for constant acceleration; but Galileo was the first to derive this mathematical relation.

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper flat and horizontal in one hand, and a heavier object like a baseball in the other, and release them at the same time as in Fig. 2–20a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad, you will find (see Fig. 2–20b) that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–21). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the *content* of his science (astronomical discoveries, inertia, free fall) but also for his new methods of *doing* science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

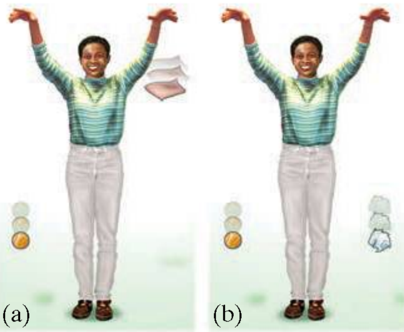
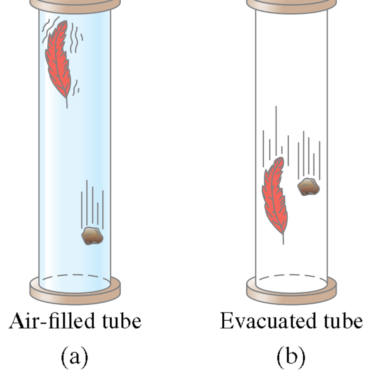



**FIGURE 2–20** (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

**FIGURE 2–21** A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.



Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

> **at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.**

We call this acceleration the **acceleration due to gravity** at the surface of the Earth, and we give it the symbol $g$. Its magnitude is approximately

$$g = 9.80\,\mathrm{m/s^2}. \qquad \left[\begin{array}{l}\text{acceleration due to gravity}\\ \text{at surface of Earth}\end{array}\right]$$

In British units $g$ is about $32\,\mathrm{ft/s^2}$. Actually, $g$ varies slightly according to latitude and elevation on the Earth's surface, but these variations are so small that we will ignore them for most purposes. (Acceleration of gravity in space beyond the Earth's surface is treated in Chapter 5.) The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector, as is any acceleration, and its direction is downward toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–11, where for $a$ we use the value of $g$ given above. Also, since the motion is vertical we will substitute $y$ in place of $x$, and $y_0$ in place of $x_0$. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

**PROBLEM SOLVING**
*You can choose y to be positive either up or down*

**EXERCISE E** Return to the Chapter-Opening Question, page 21, and answer it again now, assuming minimal air resistance. Try to explain why you may have answered differently the first time.

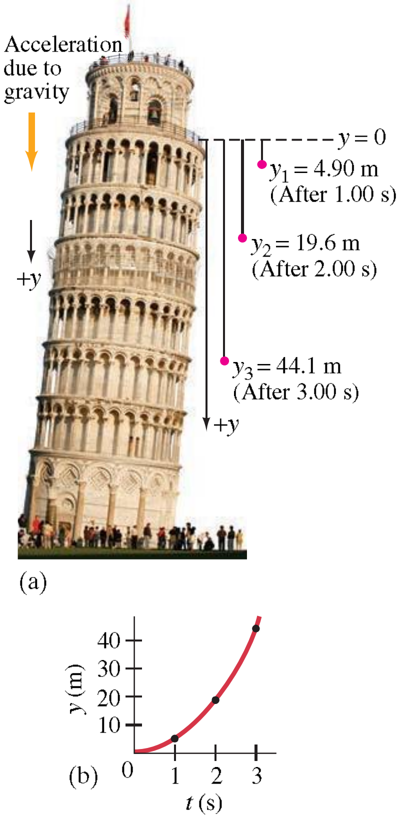



**FIGURE 2–22** Example 2–10. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–19.) (b) Graph of $y$ vs. $t$.

**EXAMPLE 2–10** **Falling from a tower.** Suppose that a ball is dropped ($v_0 = 0$) from a tower. How far will it have fallen after a time $t_1 = 1.00\,\mathrm{s}$, $t_2 = 2.00\,\mathrm{s}$, and $t_3 = 3.00\,\mathrm{s}$? Ignore air resistance.

**APPROACH** Let us take $y$ as positive downward, so the acceleration is $a = g = +9.80\,\mathrm{m/s^2}$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position $y$ of the ball after three different time intervals. Equation 2–11b, with $x$ replaced by $y$, relates the given quantities ($t$, $a$, and $v_0$) to the unknown $y$.

**SOLUTION** We set $t = t_1 = 1.00\,\mathrm{s}$ in Eq. 2–11b:

$$\begin{aligned} y_1 &= v_0 t_1 + \tfrac{1}{2}a t_1^2 \\ &= 0 + \tfrac{1}{2}a t_1^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s})^2 = 4.90\,\mathrm{m}. \end{aligned}$$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\,\mathrm{s}$. Similarly, after 2.00 s $(= t_2)$, the ball's position is

$$y_2 = \tfrac{1}{2}a t_2^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s})^2 = 19.6\,\mathrm{m}.$$

Finally, after 3.00 s $(= t_3)$, the ball's position is (see Fig. 2–22)

$$y_3 = \tfrac{1}{2}a t_3^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(3.00\,\mathrm{s})^2 = 44.1\,\mathrm{m}.$$

**NOTE** Whenever we say "dropped," it means $v_0 = 0$. Note also the graph of $y$ vs. $t$ (Fig. 2–22b): the curve is not straight but bends upward because $y$ is proportional to $t^2$.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

**EXAMPLE 2–11** **Thrown down from a tower.** Suppose the ball in Example 2–10 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

**APPROACH** Again we use Eq. 2–11b, but now $v_0$ is not zero, it is $v_0 = 3.00\,\mathrm{m/s}$.

**SOLUTION** (*a*) At $t_1 = 1.00\,\mathrm{s}$, the position of the ball as given by Eq. 2–11b is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(1.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s})^2 = 7.90\,\mathrm{m}.$$

At $t_2 = 2.00\,\mathrm{s}$ (time interval $t = 0$ to $t = 2.00\,\mathrm{s}$), the position is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\mathrm{m/s})(2.00\,\mathrm{s}) + \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s})^2 = 25.6\,\mathrm{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

(*b*) The velocity is obtained from Eq. 2–11a:

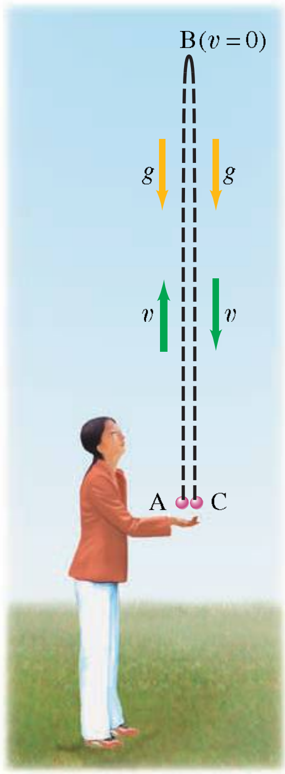
$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 12.8\,\mathrm{m/s} \quad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= 3.00\,\mathrm{m/s} + (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 22.6\,\mathrm{m/s}. \quad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

In Example 2–10, when the ball was dropped $(v_0 = 0)$, the first term $(v_0)$ in these equations was zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s}) = 9.80\,\mathrm{m/s} \quad [\text{at } t_1 = 1.00\,\mathrm{s}] \\ &= (9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s}) = 19.6\,\mathrm{m/s}. \quad [\text{at } t_2 = 2.00\,\mathrm{s}] \end{aligned}$$

**NOTE** For both Examples 2–10 and 2–11, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any instant is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

**FIGURE 2–23** An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.




**EXAMPLE 2–12** **Ball thrown upward.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate how high it goes. Ignore air resistance.

**APPROACH** We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–23) and until it comes back to the hand again. Let us choose $y$ to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 2–10 and 2–11, and so illustrates our options.) The acceleration due to gravity is downward and so will have a negative sign, $a = -g = -9.80\,\mathrm{m/s^2}$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–23), where its speed is zero for an instant; then it descends, with increasing speed.

**SOLUTION** We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–23) we have $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. At time $t$ (maximum height), $v = 0$, $a = -9.80\,\mathrm{m/s^2}$, and we wish to find $y$. We use Eq. 2–11c, replacing $x$ with $y$: $v^2 = v_0^2 + 2ay$. We solve this equation for $y$:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 11.5\,\mathrm{m}.$$

The ball reaches a height of 11.5 m above the hand.

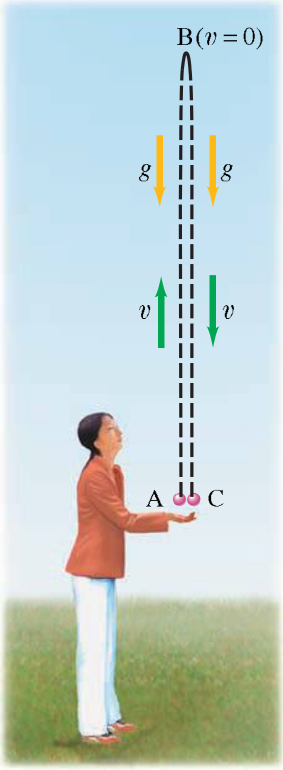



**FIGURE 2–23** (Repeated.) An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.

**EXAMPLE 2–13** **Ball thrown upward, II.** In Fig. 2–23, Example 2–12, how long is the ball in the air before it comes back to the hand?

**APPROACH** We need to choose a time interval to calculate how long the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2–23) in one step and use Eq. 2–11b. We can do this because $y$ is position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$.

**SOLUTION** We use Eq. 2–11b with $a = -9.80\,\mathrm{m/s^2}$ and find

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$0 = 0 + (15.0\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

This equation can be factored (we factor out one $t$):

$$(15.0\,\mathrm{m/s} - 4.90\,\mathrm{m/s^2}\,t)t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\,\mathrm{m/s}}{4.90\,\mathrm{m/s^2}} = 3.06\,\mathrm{s}.$$

The first solution $(t = 0)$ corresponds to the initial point (A) in Fig. 2–23, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\,\mathrm{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

**NOTE** We have ignored air resistance in these last two Examples, which could be significant, so our result is only an approximation to a real, practical situation.

We did not consider the throwing action in these Examples. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* $g$. We consider only the time when the ball is in the air and the acceleration is equal to $g$.

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 2–8, in which case we ignore the "unphysical" solution. But in Example 2–13, both solutions to our equation in $t^2$ are physically meaningful: $t = 0$ and $t = 3.06\,\mathrm{s}$.

**CAUTION**
*Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both*

**CONCEPTUAL EXAMPLE 2–14** **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–23).

**RESPONSE** Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Fig. 2–23 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–23), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero for an instant (zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\mathrm{m/s^2}$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. Remember: the acceleration of gravity always points down toward the Earth, even when the object is moving up.

**CAUTION**
*(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down*
*(2) $a \neq 0$ even at the highest point of a trajectory*

**EXAMPLE 2–15** **Ball thrown upward, III.** Let us consider again the ball thrown upward of Examples 2–12 and 2–13, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–23), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

**APPROACH** Again we assume the acceleration is constant, so we can use Eqs. 2–11. We have the maximum height of 11.5 m and initial speed of 15.0 m/s from Example 2–12. Again we take $y$ as positive upward.

**SOLUTION** (*a*) We consider the time interval between the throw $(t = 0, v_0 = 15.0\,\mathrm{m/s})$ and the top of the path $(y = +11.5\,\mathrm{m},\ v = 0)$, and we want to find $t$. The acceleration is constant at $a = -g = -9.80\,\mathrm{m/s^2}$. Both Eqs. 2–11a and 2–11b contain the time $t$ with other quantities known. Let us use Eq. 2–11a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ gives $0 = v_0 + at$, which we rearrange to solve for $t$: $at = -v_0$ or

$$\begin{aligned} t &= -\frac{v_0}{a} \\ &= -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}. \end{aligned}$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in Example 2–13]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(*b*) Now we consider the time interval from the throw $(t = 0,\ v_0 = 15.0\,\mathrm{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\mathrm{s}$ (as calculated in Example 2–13), and we want to find $v$ when $t = 3.06\,\mathrm{s}$:

$$\begin{aligned} v &= v_0 + at \\ &= 15.0\,\mathrm{m/s} - (9.80\,\mathrm{m/s^2})(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}. \end{aligned}$$

**NOTE** The ball has the same speed (magnitude of velocity) when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). And, as we saw in part (*a*), the time is the same up as down. Thus the motion is *symmetrical* about the maximum height.

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive (see Fig. 2–24) and undergoing $3.00\,g$'s would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

**PROBLEM SOLVING**
*Acceleration in g's*

**FIGURE 2–24** Several planes, in formation, are just coming out of a downward dive.

**EXERCISE F** Two balls are thrown from a cliff. One is thrown directly up, the other directly down. Both balls have the same initial speed, and both hit the ground below the cliff but at different times. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance.

## Additional Example—Using the Quadratic Formula

**EXAMPLE 2–16 Ball thrown upward at edge of cliff.** Suppose that the person of Examples 2–12, 2–13, and 2–15 throws the ball upward at 15.0 m/s while standing on the edge of a cliff, so that the ball can fall to the base of the cliff 50.0 m below, as shown in Fig. 2–25a. (*a*) How long does it take the ball to reach the base of the cliff? (*b*) What is the total distance traveled by the ball? Ignore air resistance (likely to be significant, so our result is an approximation).

**APPROACH** We again use Eq. 2–11b, with $y$ as + upward, but this time we set $y = -50.0\,\mathrm{m}$, the bottom of the cliff, which is 50.0 m below the initial position $(y_0 = 0)$; hence the minus sign.

**SOLUTION** (*a*) We use Eq. 2–11b with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, $y_0 = 0$, and $y = -50.0\,\mathrm{m}$:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$-50.0\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})t - \tfrac{1}{2}(9.80\,\mathrm{m/s^2})t^2.$$

To solve any quadratic equation of the form

$$at^2 + bt + c = 0,$$

where $a$, $b$, and $c$ are constants ($a$ is *not* acceleration here), we use the **quadratic formula** (see Appendix A–4):

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our $y$ equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})t - (50.0\,\mathrm{m}) = 0.$$

Using the quadratic formula, we find as solutions

$$t = 5.07\,\mathrm{s}$$

and

$$t = -2.01\,\mathrm{s}.$$

The first solution, $t = 5.07\,\mathrm{s}$, is the answer we are seeking: the time it takes the ball to rise to its highest point and then fall to the base of the cliff. To rise and fall back to the top of the cliff took 3.06 s (Example 2–13); so it took an additional 2.01 s to fall to the base. But what is the meaning of the other solution, $t = -2.01\,\mathrm{s}$? This is a time before the throw, when our calculation begins, so it isn't relevant here. It is outside our chosen time interval, and so is an *unphysical* solution (also in Example 2–8).

(*b*) From Example 2–12, the ball moves up 11.5 m, falls 11.5 m back down to the top of the cliff, and then down another 50.0 m to the base of the cliff, for a total distance traveled of 73.0 m. [Note that the *displacement*, however, was −50.0 m.] Figure 2–25b shows the $y$ vs. $t$ graph for this situation.

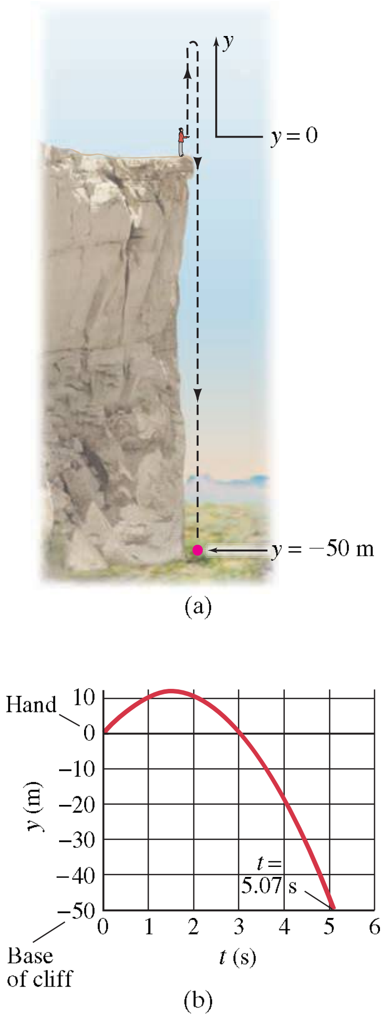



**FIGURE 2–25** Example 2–16. (a) A person stands on the edge of a cliff. A ball is thrown upward, then falls back down past the thrower to the base of the cliff, 50.0 m below. (b) The $y$ vs. $t$ graph.

**CAUTION**

*Sometimes a solution to a quadratic equation does not apply to the actual physical conditions of the Problem*

## 2–8 Graphical Analysis of Linear Motion

### Velocity as Slope

Analysis of motion using graphs can give us additional insight into kinematics. Let us draw a graph of $x$ vs. $t$, making the choice that at $t = 0$, the position of an object is $x = 0$, and the object is moving at a constant velocity, $v = \bar{v} = 11\,\mathrm{m/s}$ $(40\,\mathrm{km/h})$. Our graph starts at $x = 0$, $t = 0$ (the origin). The graph of the position increases linearly in time because, by Eq. 2–2, $\Delta x = \bar{v}\,\Delta t$ and $\bar{v}$ is a constant. So the graph of $x$ vs. $t$ is a straight line, as shown in Fig. 2–26. The small (shaded) triangle on the graph indicates the **slope** of the straight line:

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

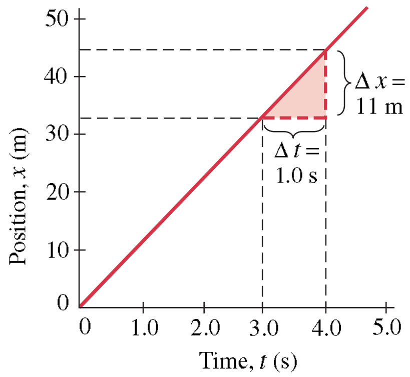
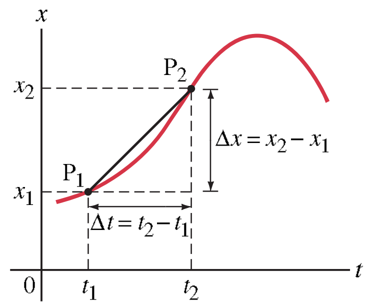



**FIGURE 2–26** Graph of position vs. time for an object moving at a constant velocity of $11\,\mathrm{m/s}$.

We see, using the definition of average velocity (Eq. 2–2), that the *slope of the x vs. t graph is equal to the velocity*. And, as can be seen from the small triangle on the graph, $\Delta x/\Delta t = (11\,\mathrm{m})/(1.0\,\mathrm{s}) = 11\,\mathrm{m/s}$, which is the given velocity.

If the object's velocity changes in time, we might have an $x$ vs. $t$ graph like that shown in Fig. 2–27. (Note that this graph is different from showing the "path" of an object on an $x$ vs. $y$ plot.) Suppose the object is at position $x_1$ at time $t_1$, and at position $x_2$ at time $t_2$. $P_1$ and $P_2$ represent these two points on the graph. A straight line drawn from point $P_1(x_1, t_1)$ to point $P_2(x_2, t_2)$ forms the hypotenuse of a right triangle whose sides are $\Delta x$ and $\Delta t$. The ratio $\Delta x/\Delta t$ is the **slope** of the straight line $P_1P_2$. But $\Delta x/\Delta t$ is also the average velocity of the object during the time interval $\Delta t = t_2 - t_1$. Therefore, we conclude that the *average velocity of an object during any time interval* $\Delta t = t_2 - t_1$ *is equal to the slope of the straight line* (or **chord**) connecting the two points $(x_1, t_1)$ and $(x_2, t_2)$ on an $x$ vs. $t$ graph.

**FIGURE 2–27** Graph of an object's position $x$ vs. time $t$. The slope of the straight line $P_1P_2$ represents the average velocity of the object during the time interval $\Delta t = t_2 - t_1$.




Consider now a time intermediate between $t_1$ and $t_2$, call it $t_3$, at which moment the object is at $x_3$ (Fig. 2–28). The slope of the straight line $P_1P_3$ is less than the slope of $P_1P_2$. Thus the average velocity during the time interval $t_3 - t_1$ is less than during the time interval $t_2 - t_1$.

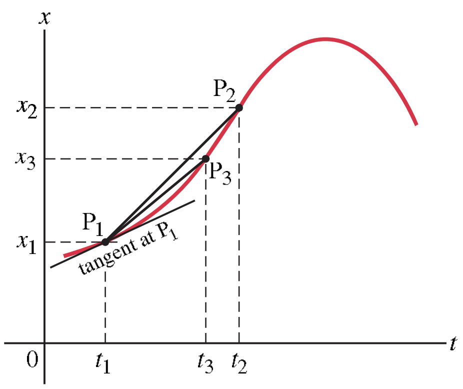



**FIGURE 2–28** Same position vs. time curve as in Fig. 2–27. Note that the average velocity over the time interval $t_3 - t_1$ (which is the slope of $P_1P_3$) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the line tangent to the curve at point $P_1$ equals the *instantaneous* velocity at time $t_1$.

Next let us take point $P_3$ in Fig. 2–28 to be closer and closer to point $P_1$. That is, we let the interval $t_3 - t_1$, which we now call $\Delta t$, to become smaller and smaller. The slope of the line connecting the two points becomes closer and closer to the slope of a line **tangent**† to the curve at point $P_1$. The average velocity (equal to the slope of the chord) thus approaches the slope of the tangent at point $P_1$. The definition of the instantaneous velocity (Eq. 2–3) is the limiting value of the average velocity as $\Delta t$ approaches zero. Thus the *instantaneous velocity equals the slope of the tangent to the curve of x vs. t* at any chosen point (which we can simply call "the slope of the curve" at that point).

**PROBLEM SOLVING**
*Velocity equals slope of x vs. t graph at any instant*

†The tangent is a straight line that touches the curve only at the one chosen point, without passing across or through the curve at that point.

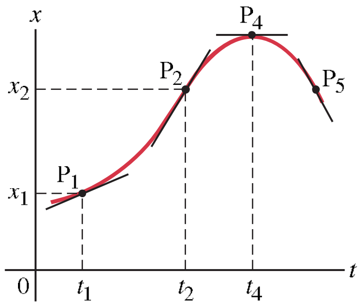
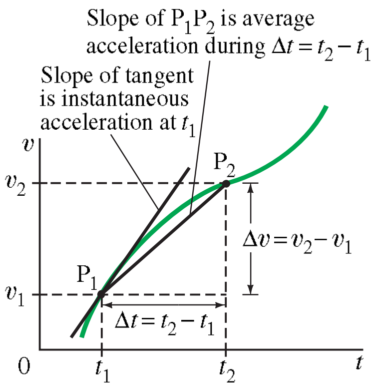



**FIGURE 2–29** Same $x$ vs. $t$ curve as in Figs. 2–27 and 2–28, but here showing the slope at four different points: At $P_4$, the slope is zero, so $v = 0$. At $P_5$ the slope is negative, so $v < 0$.

We can obtain the velocity of an object at any instant from its graph of $x$ vs. $t$. For example, in Fig. 2–29 (which shows the same graph as in Figs. 2–27 and 2–28), as our object moves from $x_1$ to $x_2$, the slope continually increases, so the velocity is increasing. For times after $t_2$, the slope begins to decrease and reaches zero $(v = 0)$ where $x$ has its maximum value, at point $P_4$ in Fig. 2–29. Beyond point $P_4$, the slope is negative, as for point $P_5$. The velocity is therefore negative, which makes sense since $x$ is now decreasing—the particle is moving toward decreasing values of $x$, to the left on a standard $xy$ plot.

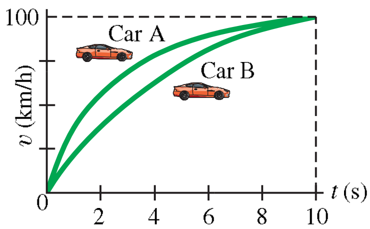
**FIGURE 2–30** A graph of velocity $v$ vs. time $t$. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line $P_1P_2$: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time $t_1$ is the slope of the $v$ vs. $t$ curve at that instant.




## Slope and Acceleration

We can also draw a graph of the *velocity*, $v$, vs. time, $t$, as shown in Fig. 2–30. Then the average acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points $P_1$ and $P_2$ as shown. [Compare this to the position vs. time graph of Fig. 2–27 for which the slope of the straight line represents the average velocity.] The instantaneous acceleration at any time, say $t_1$, is the slope of the tangent to the $v$ vs. $t$ curve at that time, which is also shown in Fig. 2–30. Using this fact for the situation graphed in Fig. 2–30, as we go from time $t_1$ to time $t_2$ the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

**FIGURE 2–31** (below) Example 2–17.




**CONCEPTUAL EXAMPLE 2–17** **Analyzing with graphs.** Figure 2–31 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (*a*) the average acceleration; (*b*) the instantaneous acceleration; and (*c*) the total distance traveled for the two cars.

**RESPONSE** (*a*) Average acceleration is $\Delta v/\Delta t$. Both cars have the same $\Delta v$ (100 km/h) over the same time interval $\Delta t = 10.0$ s, so the average acceleration is the same for both cars. (*b*) Instantaneous acceleration is the slope of the tangent to the $v$ vs. $t$ curve. For the first 4 s or so, the top curve (car A) is steeper than the bottom curve, so car A has a greater acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration for this period. (*c*) Except at $t = 0$ and $t = 10.0$ s, car A is always going faster than car B. Since it is going faster, it will go farther in the same time.

# Summary

[*The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary* cannot *serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.*]

**Kinematics** deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

**Average speed** is the distance traveled divided by the **elapsed time** or **time interval**, $\Delta t$ (the time period over which we choose to make our observations). An object's **average velocity** over a particular time interval is

$$\bar{v} = \frac{\Delta x}{\Delta t}, \qquad \text{(2–2)}$$

where $\Delta x$ is the displacement during the time interval $\Delta t$.

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval $\Delta t$ is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \quad \textbf{(2–4)}$$

where $\Delta v$ is the change of velocity during the time interval $\Delta t$. **Instantaneous acceleration** is the average acceleration taken over an infinitesimally short time interval.

If an object has position $x_0$ and velocity $v_0$ at time $t = 0$ and moves in a straight line with **constant acceleration**, the velocity $v$ and position $x$ at a later time $t$ are related to the acceleration $a$, the initial position $x_0$, and the initial velocity $v_0$ by Eqs. 2–11:

$$\begin{aligned} v &= v_0 + at, \\ x &= x_0 + v_0 t + \tfrac{1}{2}at^2, \\ v^2 &= v_0^2 + 2a(x - x_0), \\ \bar{v} &= \frac{v + v_0}{2}. \end{aligned} \quad \textbf{(2–11)}$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\,\mathrm{m/s^2}$ if air resistance can be ignored. We can apply Eqs. 2–11 for constant acceleration to objects that move up or down freely near the Earth's surface.

The slope of a curve at any point on a graph is the slope of the tangent to the curve at that point. On a graph of position vs. time, the **slope** is equal to the instantaneous velocity. On a graph of velocity vs. time, the slope is the acceleration.

# Questions

1. Does a car speedometer measure speed, velocity, or both? Explain.
2. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant? Explain.
3. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.
4. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.
5. Can an object have a northward velocity and a southward acceleration? Explain.
6. Can the velocity of an object be negative when its acceleration is positive? What about vice versa? If yes, give examples in each case.
7. Give an example where both the velocity and acceleration are negative.
8. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.
9. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.
10. A baseball player hits a ball straight up into the air. It leaves the bat with a speed of 120 km/h. In the absence of air resistance, how fast would the ball be traveling when it is caught at the same height above the ground as it left the bat? Explain.
11. As a freely falling object speeds up, what is happening to its acceleration—does it increase, decrease, or stay the same? (*a*) Ignore air resistance. (*b*) Consider air resistance.
12. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed for the entire trip from A to C equal to 80 km/h? Explain why or why not.
13. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.
14. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.
15. Which of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table? Explain your answers.
16. Describe in words the motion plotted in Fig. 2–32 in terms of velocity, acceleration, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

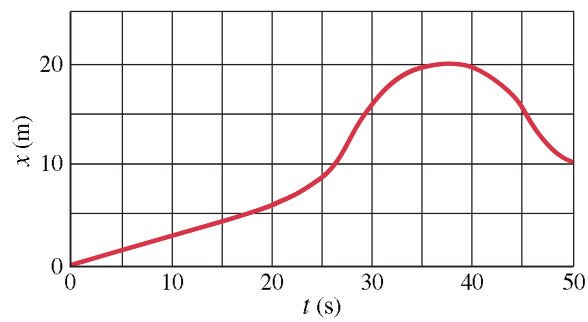



**FIGURE 2–32** Question 16.

17. Describe in words the motion of the object graphed in Fig. 2–33.

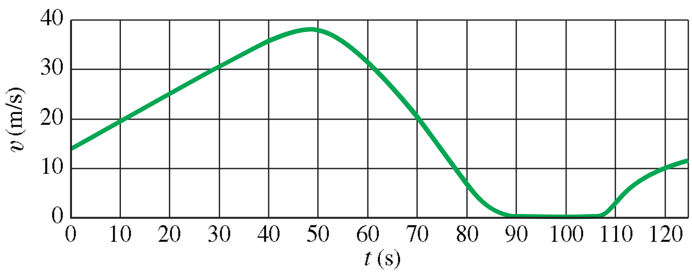



**FIGURE 2–33** Question 17.

# MisConceptual Questions

[List all answers that are valid.]

**1.** Which of the following should be part of solving any problem in physics? Select all that apply:
(*a*) Read the problem carefully.
(*b*) Draw a picture of the situation.
(*c*) Write down the variables that are given.
(*d*) Think about which physics principles to apply.
(*e*) Determine which equations can be used to apply the correct physics principles.
(*f*) Check the units when you have completed your calculation.
(*g*) Consider whether your answer is reasonable.

**2.** In which of the following cases does a car have a negative velocity and a positive acceleration? A car that is traveling in the
(*a*) $-x$ direction at a constant 20 m/s.
(*b*) $-x$ direction increasing in speed.
(*c*) $+x$ direction increasing in speed.
(*d*) $-x$ direction decreasing in speed.
(*e*) $+x$ direction decreasing in speed.

**3.** At time $t = 0$ an object is traveling to the right along the $+x$ axis at a speed of 10.0 m/s with acceleration $-2.0\ \mathrm{m/s^2}$. Which statement is true?
(*a*) The object will slow down, eventually coming to a complete stop.
(*b*) The object cannot have a negative acceleration and be moving to the right.
(*c*) The object will continue to move to the right, slowing down but never coming to a complete stop.
(*d*) The object will slow down, momentarily stopping, then pick up speed moving to the left.

**4.** A ball is thrown straight up. What are the velocity and acceleration of the ball at the highest point in its path?
(*a*) $v = 0,\ a = 0$.
(*b*) $v = 0,\ a = 9.8\ \mathrm{m/s^2}$ up.
(*c*) $v = 0,\ a = 9.8\ \mathrm{m/s^2}$ down.
(*d*) $v = 9.8$ m/s up, $a = 0$.
(*e*) $v = 9.8$ m/s down, $a = 0$.

**5.** You drop a rock off a bridge. When the rock has fallen 4 m, you drop a second rock. As the two rocks continue to fall, what happens to their velocities?
(*a*) Both increase at the same rate.
(*b*) The velocity of the first rock increases faster than the velocity of the second.
(*c*) The velocity of the second rock increases faster than the velocity of the first.
(*d*) Both velocities stay constant.

**6.** You drive 4 km at 30 km/h and then another 4 km at 50 km/h. What is your average speed for the whole 8-km trip?
(*a*) More than 40 km/h.
(*b*) Equal to 40 km/h.
(*c*) Less than 40 km/h.
(*d*) Not enough information.

**7.** A ball is dropped from the top of a tall building. At the same instant, a second ball is thrown upward from ground level. When the two balls pass one another, one on the way up, the other on the way down, compare the magnitudes of their acceleration:
(*a*) The acceleration of the dropped ball is greater.
(*b*) The acceleration of the ball thrown upward is greater.
(*c*) The acceleration of both balls is the same.
(*d*) The acceleration changes during the motion, so you cannot predict the exact value when the two balls pass each other.
(*e*) The accelerations are in opposite directions.

**8.** A ball is thrown downward at a speed of 20 m/s. Choosing the $+y$ axis pointing up and neglecting air resistance, which equation(s) could be used to solve for other variables? The acceleration due to gravity is $g = 9.8\ \mathrm{m/s^2}$ downward.
(*a*) $v = (20\ \mathrm{m/s}) - gt$.
(*b*) $y = y_0 + (-20\ \mathrm{m/s})t - (1/2)gt^2$.
(*c*) $v^2 = (20\ \mathrm{m/s})^2 - 2g(y - y_0)$.
(*d*) $(20\ \mathrm{m/s}) = (v + v_0)/2$.
(*e*) All of the above.

**9.** A car travels along the $x$ axis with increasing speed. We don't know if to the left or the right. Which of the graphs in Fig. 2–34 most closely represents the motion of the car?

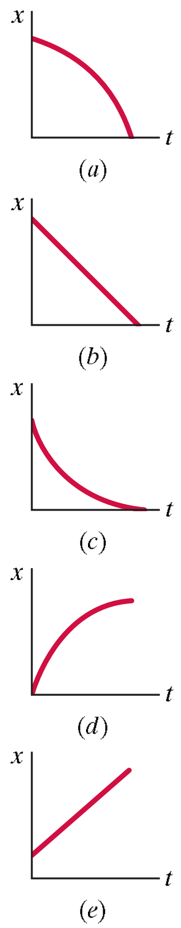



**FIGURE 2–34** MisConceptual Question 9.

For assigned homework and other learning materials, go to the MasteringPhysics website.

# Problems

*[The Problems at the end of each Chapter are ranked* I, II, *or* III *according to estimated difficulty, with level* I *Problems being easiest. Level* III *are meant as challenges for the best students. The Problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—Problems often depend on earlier material. Next is a set of "General Problems" not arranged by Section and not ranked. Finally, there are "Search and Learn" Problems that require rereading parts of the Chapter and sometimes earlier Chapters.]*

(Note: *In Problems, assume a number like 6.4 is accurate to* ± 0.1; *and 950 is* ± 10 *unless 950 is said to be "precisely" or "very nearly" 950, in which case assume 950* ± 1. *See Section 1–4.)*

## 2–1 to 2–3 Speed and Velocity

**1.** (I) If you are driving 95 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

**2.** (I) What must your car's average speed be in order to travel 235 km in 2.75 h?

**3.** (I) A particle at $t_1 = -2.0\,\text{s}$ is at $x_1 = 4.8\,\text{cm}$ and at $t_2 = 4.5\,\text{s}$ is at $x_2 = 8.5\,\text{cm}$. What is its average velocity over this time interval? Can you calculate its average speed from these data? Why or why not?

**4.** (I) A rolling ball moves from $x_1 = 8.4\,\text{cm}$ to $x_2 = -4.2\,\text{cm}$ during the time from $t_1 = 3.0\,\text{s}$ to $t_2 = 6.1\,\text{s}$. What is its average velocity over this time interval?

**5.** (I) A bird can fly 25 km/h. How long does it take to fly 3.5 km?

**6.** (II) According to a rule-of-thumb, each five seconds between a lightning flash and the following thunder gives the distance to the flash in miles. (*a*) Assuming that the flash of light arrives in essentially no time at all, estimate the speed of sound in m/s from this rule. (*b*) What would be the rule for kilometers?

**7.** (II) You are driving home from school steadily at 95 km/h for 180 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 4.5 h. (*a*) How far is your hometown from school? (*b*) What was your average speed?

**8.** (II) A horse trots away from its trainer in a straight line, moving 38 m away in 9.0 s. It then turns abruptly and gallops halfway back in 1.8 s. Calculate (*a*) its average speed and (*b*) its average velocity for the entire trip, using "away from the trainer" as the positive direction.

**9.** (II) A person jogs eight complete laps around a 400-m track in a total time of 14.5 min. Calculate (*a*) the average speed and (*b*) the average velocity, in m/s.

**10.** (II) Every year the Earth travels about $10^9$ km as it orbits the Sun. What is Earth's average speed in km/h?

**11.** (II) A car traveling 95 km/h is 210 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

**12.** (II) Calculate the average speed and average velocity of a complete round trip in which the outgoing 250 km is covered at 95 km/h, followed by a 1.0-h lunch break, and the return 250 km is covered at 55 km/h.

**13.** (II) Two locomotives approach each other on parallel tracks. Each has a speed of 155 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Fig. 2–35.)

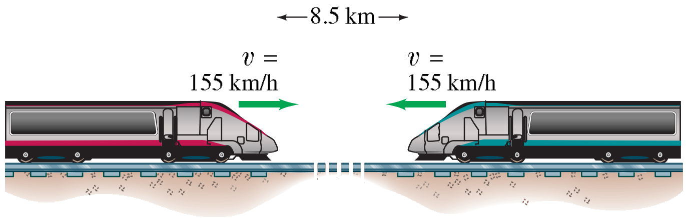



**FIGURE 2–35** Problem 13.

**14.** (II) Digital bits on a 12.0-cm diameter audio CD are encoded along an outward spiraling path that starts at radius $R_1 = 2.5\,\text{cm}$ and finishes at radius $R_2 = 5.8\,\text{cm}$. The distance between the centers of neighboring spiral-windings is $1.6\,\mu\text{m}$ $(= 1.6 \times 10^{-6}\,\text{m})$. (*a*) Determine the total length of the spiraling path. [*Hint*: Imagine "unwinding" the spiral into a straight path of width $1.6\,\mu\text{m}$, and note that the original spiral and the straight path both occupy the same area.] (*b*) To read information, a CD player adjusts the rotation of the CD so that the player's readout laser moves along the spiral path at a constant speed of about 1.2 m/s. Estimate the maximum playing time of such a CD.

**15.** (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.80 s after the ball is released from his hands. What is the speed of the ball, assuming the speed of sound is 340 m/s?

**16.** (III) An automobile traveling 95 km/h overtakes a 1.30-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it, and how far will the car have traveled in this time? See Fig. 2–36. What are the results if the car and train are traveling in opposite directions?

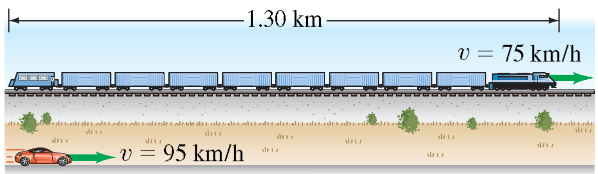



**FIGURE 2–36** Problem 16.

## 2–4 Acceleration

**17.** (I) A sports car accelerates from rest to 95 km/h in 4.3 s. What is its average acceleration in $\text{m/s}^2$?

**18.** (I) A sprinter accelerates from rest to 9.00 m/s in 1.38 s. What is her acceleration in (*a*) $\text{m/s}^2$; (*b*) $\text{km/h}^2$?

**19.** (II) A sports car moving at constant velocity travels 120 m in 5.0 s. If it then brakes and comes to a stop in 4.0 s, what is the magnitude of its acceleration (assumed constant) in $\text{m/s}^2$, and in $g$'s $(g = 9.80\,\text{m/s}^2)$?

**20.** (II) At highway speeds, a particular automobile is capable of an acceleration of about $1.8\,\mathrm{m/s^2}$. At this rate, how long does it take to accelerate from 65 km/h to 120 km/h?

**21.** (II) A car moving in a straight line starts at $x = 0$ at $t = 0$. It passes the point $x = 25.0\,\mathrm{m}$ with a speed of 11.0 m/s at $t = 3.00\,\mathrm{s}$. It passes the point $x = 385\,\mathrm{m}$ with a speed of 45.0 m/s at $t = 20.0\,\mathrm{s}$. Find (*a*) the average velocity, and (*b*) the average acceleration, between $t = 3.00\,\mathrm{s}$ and $t = 20.0\,\mathrm{s}$.

## 2–5 and 2–6 Motion at Constant Acceleration

**22.** (I) A car slows down from 28 m/s to rest in a distance of 88 m. What was its acceleration, assumed constant?

**23.** (I) A car accelerates from 14 m/s to 21 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

**24.** (I) A light plane must reach a speed of 35 m/s for takeoff. How long a runway is needed if the (constant) acceleration is $3.0\,\mathrm{m/s^2}$?

**25.** (II) A baseball pitcher throws a baseball with a speed of 43 m/s. Estimate the average acceleration of the ball during the throwing motion. In throwing the baseball, the pitcher accelerates it through a displacement of about 3.5 m, from behind the body to the point where it is released (Fig. 2–37).

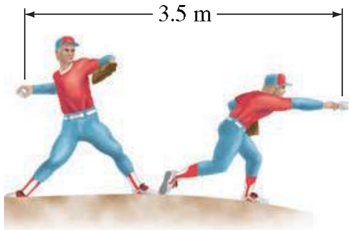


**FIGURE 2–37** Problem 25.

**26.** (II) A world-class sprinter can reach a top speed (of about 11.5 m/s) in the first 18.0 m of a race. What is the average acceleration of this sprinter and how long does it take her to reach that speed?

**27.** (II) A car slows down uniformly from a speed of 28.0 m/s to rest in 8.00 s. How far did it travel in that time?

**28.** (II) In coming to a stop, a car leaves skid marks 65 m long on the highway. Assuming a deceleration of $4.00\,\mathrm{m/s^2}$, estimate the speed of the car just before braking.

**29.** (II) A car traveling 75 km/h slows down at a constant $0.50\,\mathrm{m/s^2}$ just by "letting up on the gas." Calculate (*a*) the distance the car coasts before it stops, (*b*) the time it takes to stop, and (*c*) the distance it travels during the first and fifth seconds.

**30.** (II) Determine the stopping distances for an automobile going a constant initial speed of 95 km/h and human reaction time of 0.40 s: (*a*) for an acceleration $a = -3.0\,\mathrm{m/s^2}$; (*b*) for $a = -6.0\,\mathrm{m/s^2}$.

**31.** (II) A driver is traveling 18.0 m/s when she sees a red light ahead. Her car is capable of decelerating at a rate of $3.65\,\mathrm{m/s^2}$. If it takes her 0.350 s to get the brakes on and she is 20.0 m from the intersection when she sees the light, will she be able to stop in time? How far from the beginning of the intersection will she be, and in what direction?

**32.** (II) A 75-m-long train begins uniform acceleration from rest. The front of the train has a speed of 18 m/s when it passes a railway worker who is standing 180 m from where the front of the train started. What will be the speed of the last car as it passes the worker? (See Fig. 2–38.)

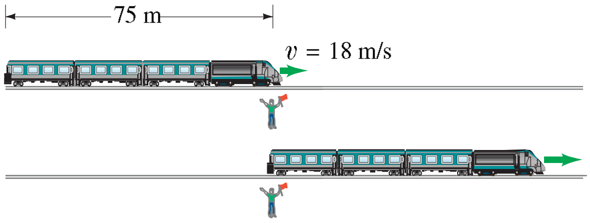


**FIGURE 2–38** Problem 32.

**33.** (II) A space vehicle accelerates uniformly from 85 m/s at $t = 0$ to 162 m/s at $t = 10.0\,\mathrm{s}$. How far did it move between $t = 2.0\,\mathrm{s}$ and $t = 6.0\,\mathrm{s}$?

**34.** (III) A fugitive tries to hop on a freight train traveling at a constant speed of 5.0 m/s. Just as an empty box car passes him, the fugitive starts from rest and accelerates at $a = 1.4\,\mathrm{m/s^2}$ to his maximum speed of 6.0 m/s, which he then maintains. (*a*) How long does it take him to catch up to the empty box car? (*b*) What is the distance traveled to reach the box car?

**35.** (III) Mary and Sally are in a foot race (Fig. 2–39). When Mary is 22 m from the finish line, she has a speed of 4.0 m/s and is 5.0 m behind Sally, who has a speed of 5.0 m/s. Sally thinks she has an easy win and so, during the remaining portion of the race, decelerates at a constant rate of $0.40\,\mathrm{m/s^2}$ to the finish line. What constant acceleration does Mary now need during the remaining portion of the race, if she wishes to cross the finish line side-by-side with Sally?

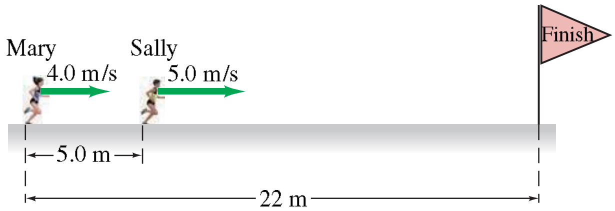


**FIGURE 2–39** Problem 35.

**36.** (III) An unmarked police car traveling a constant 95 km/h is passed by a speeder traveling 135 km/h. Precisely 1.00 s after the speeder passes, the police officer steps on the accelerator; if the police car's acceleration is $2.60\,\mathrm{m/s^2}$, how much time passes before the police car overtakes the speeder (assumed moving at constant speed)?

## 2–7 Freely Falling Objects (neglect air resistance)

**37.** (I) A stone is dropped from the top of a cliff. It is seen to hit the ground below after 3.55 s. How high is the cliff?

**38.** (I) Estimate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing."

**39.** (II) A ball player catches a ball 3.4 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

**40.** (II) A baseball is hit almost straight up into the air with a speed of 25 m/s. Estimate (*a*) how high it goes, (*b*) how long it is in the air. (*c*) What factors make this an estimate?

**41.** (II) The best rebounders in basketball have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

**42.** (II) An object starts from rest and falls under the influence of gravity. Draw graphs of (*a*) its speed and (*b*) the distance it has fallen, as a function of time from $t = 0$ to $t = 5.00\,s$. Ignore air resistance.

**43.** (II) A stone is thrown vertically upward with a speed of 24.0 m/s. (*a*) How fast is it moving when it is at a height of 13.0 m? (*b*) How much time is required to reach this height? (*c*) Why are there two answers to (*b*)?

**44.** (II) For an object falling freely from rest, show that the distance traveled *during* each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). (This was first shown by Galileo.) See Figs. 2–19 and 2–22.

**45.** (II) A rocket rises vertically, from rest, with an acceleration of $3.2\,m/s^2$ until it runs out of fuel at an altitude of 775 m. After this point, its acceleration is that of gravity, downward. (*a*) What is the velocity of the rocket when it runs out of fuel? (*b*) How long does it take to reach this point? (*c*) What maximum altitude does the rocket reach? (*d*) How much time (total) does it take to reach maximum altitude? (*e*) With what velocity does it strike the Earth? (*f*) How long (total) is it in the air?

**46.** (II) A helicopter is ascending vertically with a speed of 5.40 m/s. At a height of 105 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? [*Hint*: What is $v_0$ for the package?]

**47.** (II) Roger sees water balloons fall past his window. He notices that each balloon strikes the sidewalk 0.83 s after passing his window. Roger's room is on the third floor, 15 m above the sidewalk. (*a*) How fast are the balloons traveling when they pass Roger's window? (*b*) Assuming the balloons are being released from rest, from what floor are they being released? Each floor of the dorm is 5.0 m high.

**48.** (II) Suppose you adjust your garden hose nozzle for a fast stream of water. You point the nozzle vertically upward at a height of 1.8 m above the ground (Fig. 2–40). When you quickly turn off the nozzle, you hear the water striking the ground next to you for another 2.5 s. What is the water speed as it leaves the nozzle?


**FIGURE 2–40** Problem 48.

**49.** (III) A falling stone takes 0.31 s to travel past a window 2.2 m tall (Fig. 2–41). From what height above the top of the window did the stone fall?

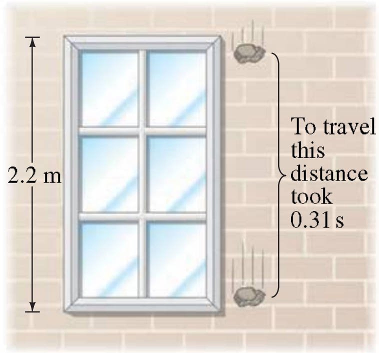



**FIGURE 2–41** Problem 49.

**50.** (III) A rock is dropped from a sea cliff, and the sound of it striking the ocean is heard 3.4 s later. If the speed of sound is 340 m/s, how high is the cliff?

## 2–8 Graphical Analysis

**51.** (II) Figure 2–42 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

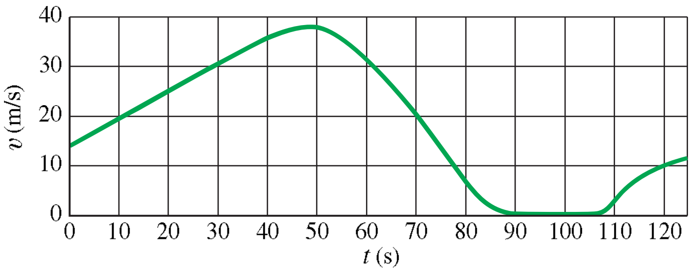



**FIGURE 2–42** Problem 51.

**52.** (II) A sports car accelerates approximately as shown in the velocity–time graph of Fig. 2–43. (The short flat spots in the curve represent manual shifting of the gears.) Estimate the car's average acceleration in (*a*) second gear and (*b*) fourth gear.

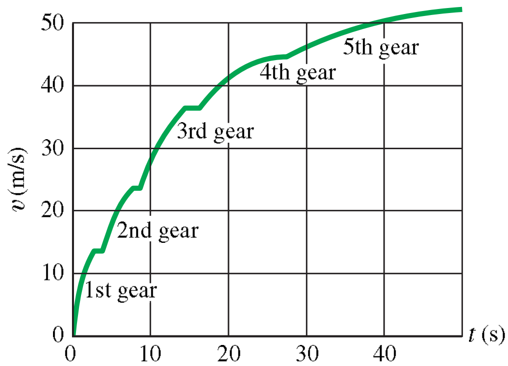



**FIGURE 2–43** Problem 52. The velocity of a car as a function of time, starting from a dead stop. The flat spots in the curve represent gear shifts.

53. (II) The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 2–44. What is its instantaneous velocity (*a*) at $t = 10.0\,\text{s}$ and (*b*) at $t = 30.0\,\text{s}$? What is its average velocity (*c*) between $t = 0$ and $t = 5.0\,\text{s}$, (*d*) between $t = 25.0\,\text{s}$ and $t = 30.0\,\text{s}$, and (*e*) between $t = 40.0\,\text{s}$ and $t = 50.0\,\text{s}$?

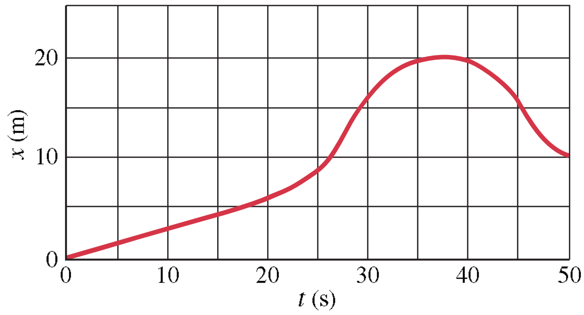



**FIGURE 2–44** Problems 53, 54, and 55.

54. (II) In Fig. 2–44, (*a*) during what time periods, if any, is the velocity constant? (*b*) At what time is the velocity greatest? (*c*) At what time, if any, is the velocity zero? (*d*) Does the object move in one direction or in both directions during the time shown?

55. (III) Sketch the $v$ vs. $t$ graph for the object whose displacement as a function of time is given by Fig. 2–44.

# General Problems

56. The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

57. A person who is properly restrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 "*g*'s" $(1.00\,g = 9.80\,\text{m/s}^2)$. Assuming uniform deceleration at 30 *g*'s, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 95 km/h.

58. A person jumps out a fourth-story window 18.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 2–45. (*a*) What was the average deceleration experienced by the survivor when she was slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, to generate a smaller deceleration): would you stiffen or loosen the net? Explain.

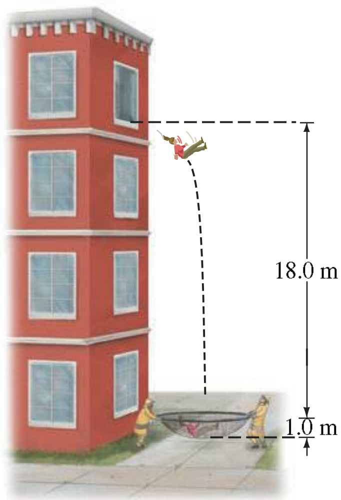



**FIGURE 2–45** Problem 58.

59. A bicyclist in the Tour de France crests a mountain pass as he moves at 15 km/h. At the bottom, 4.0 km farther, his speed is 65 km/h. Estimate his average acceleration (in $\text{m/s}^2$) while riding down the mountain.

60. Consider the street pattern shown in Fig. 2–46. Each intersection has a traffic signal, and the speed limit is 40 km/h. Suppose you are driving from the west at the speed limit. When you are 10.0 m from the first intersection, all the lights turn green. The lights are green for 13.0 s each. (*a*) Calculate the time needed to reach the third stoplight. Can you make it through all three lights without stopping? (*b*) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of $2.00\,\text{m/s}^2$ to the speed limit. Can the second car make it through all three lights without stopping? By how many seconds would it make it, or not make it?

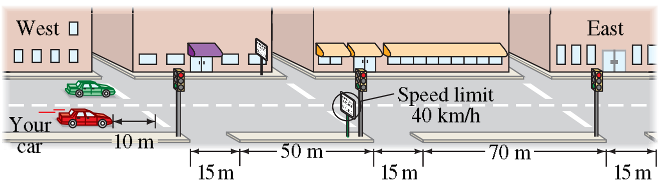



**FIGURE 2–46** Problem 60.

61. An airplane travels 2100 km at a speed of 720 km/h, and then encounters a tailwind that boosts its speed to 990 km/h for the next 2800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Does Eq. 2–11d apply?]

62. A stone is dropped from the roof of a high building. A second stone is dropped 1.30 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

63. A person jumps off a diving board 4.0 m above the water's surface into a deep pool. The person's downward motion stops 2.0 m below the surface of the water. Estimate the average deceleration of the person while under the water.

**64.** In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting the ball downhill, see Fig. 2–47) is more difficult than from a downhill lie. To see why, assume that on a particular green the ball decelerates constantly at $1.8\,\mathrm{m/s^2}$ going downhill, and constantly at $2.6\,\mathrm{m/s^2}$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

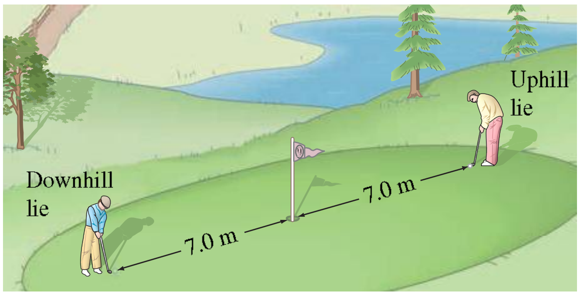



**FIGURE 2–47** Problem 64.

**65.** A stone is thrown vertically upward with a speed of 15.5 m/s from the edge of a cliff 75.0 m high (Fig. 2–48). (*a*) How much later does it reach the bottom of the cliff? (*b*) What is its speed just before hitting? (*c*) What total distance did it travel?

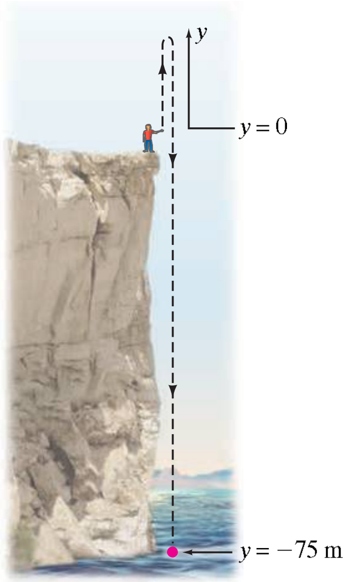



**FIGURE 2–48** Problem 65.

**66.** In the design of a **rapid transit system**, it is necessary to balance the average speed of a train against the distance between station stops. The more stops there are, the slower the train's average speed. To get an idea of this problem, calculate the time it takes a train to make a 15.0-km trip in two situations: (*a*) the stations at which the trains must stop are 3.0 km apart (a total of 6 stations, including those at the ends); and (*b*) the stations are 5.0 km apart (4 stations total). Assume that at each station the train accelerates at a rate of $1.1\,\mathrm{m/s^2}$ until it reaches 95 km/h, then stays at this speed until its brakes are applied for arrival at the next station, at which time it decelerates at $-2.0\,\mathrm{m/s^2}$. Assume it stops at each intermediate station for 22 s.

**67.** A person driving her car at 35 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning to red, and she is 28 m away from the near side of the intersection (Fig. 2–49). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is $-5.8\,\mathrm{m/s^2}$, whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

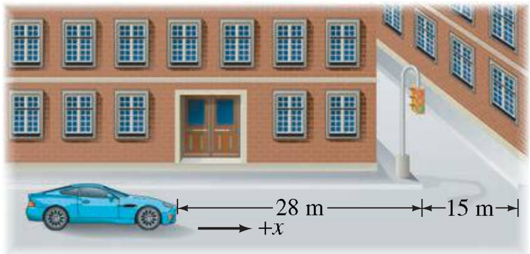



**FIGURE 2–49** Problem 67.

**68.** A car is behind a truck going 18 m/s on the highway. The car's driver looks for an opportunity to pass, guessing that his car can accelerate at $0.60\,\mathrm{m/s^2}$ and that he has to cover the 20-m length of the truck, plus 10-m extra space at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably at the speed limit, 25 m/s (55 mph). He estimates that the car is about 500 m away. Should he attempt the pass? Give details.

**69.** Agent Bond is standing on a bridge, 15 m above the road below, and his pursuers are getting too close for comfort. He spots a flatbed truck approaching at 25 m/s, which he measures by knowing that the telephone poles the truck is passing are 25 m apart in this region. The roof of the truck is 3.5 m above the road, and Bond quickly calculates how many poles away the truck should be when he drops down from the bridge onto the truck, making his getaway. How many poles is it?

**70.** A conveyor belt is used to send burgers through a grilling machine. If the grilling machine is 1.2 m long and the burgers require 2.8 min to cook, how fast must the conveyor belt travel? If the burgers are spaced 25 cm apart, what is the rate of burger production (in burgers/min)?

**71.** Two students are asked to find the height of a particular building using a barometer. Instead of using the barometer as an altitude measuring device, they take it to the roof of the building and drop it off, timing its fall. One student reports a fall time of 2.0 s, and the other, 2.3 s. What % difference does the 0.3 s make for the estimates of the building's height?

**72.** Two children are playing on two trampolines. The first child bounces up one-and-a-half times higher than the second child. The initial speed up of the second child is 4.0 m/s. (*a*) Find the maximum height the second child reaches. (*b*) What is the initial speed of the first child? (*c*) How long was the first child in the air?

**73.** If there were no air resistance, how long would it take a free-falling skydiver to fall from a plane at 3200 m to an altitude of 450 m, where she will open her parachute? What would her speed be at 450 m? (In reality, the air resistance will restrict her speed to perhaps 150 km/h.)

**74.** You stand at the top of a cliff while your friend stands on the ground below you. You drop a ball from rest and see that she catches it 1.4 s later. Your friend then throws the ball up to you, such that it just comes to rest in your hand. What is the speed with which your friend threw the ball?

**75.** On an audio compact disc (CD), digital bits of information are encoded sequentially along a spiral path. Each bit occupies about 0.28 $\mu$m. A CD player's readout laser scans along the spiral's sequence of bits at a constant speed of about 1.2 m/s as the CD spins. (*a*) Determine the number $N$ of digital bits that a CD player reads every second. (*b*) The audio information is sent to each of the two loudspeakers 44,100 times per second. Each of these samplings requires 16 bits, and so you might expect the required bit rate for a CD player to be

$$N_0 = 2\left(44{,}100\,\frac{\text{samplings}}{\text{s}}\right)\left(16\,\frac{\text{bits}}{\text{sampling}}\right) = 1.4\times10^6\,\frac{\text{bits}}{\text{s}},$$

where the 2 is for the 2 loudspeakers (the 2 stereo channels). Note that $N_0$ is less than the number $N$ of bits actually read per second by a CD player. The excess number of bits $(= N - N_0)$ is needed for encoding and error-correction. What percentage of the bits on a CD are dedicated to encoding and error-correction?

## Search and Learn

**1.** Discuss two conditions given in Section 2–7 for being able to use a constant acceleration of magnitude $g = 9.8\ \text{m/s}^2$. Give an example in which one of these conditions would not be met and would not even be a reasonable approximation of motion.

**2.** In a lecture demonstration, a 3.0-m-long vertical string with ten bolts tied to it at equal intervals is dropped from the ceiling of the lecture hall. The string falls on a tin plate, and the class hears the clink of each bolt as it hits the plate. (*a*) The sounds will not occur at equal time intervals. Why? (*b*) Will the time between clinks increase or decrease as the string falls? (*c*) How could the bolts be tied so that the clinks occur at equal intervals? (Assume the string is vertical with the bottom bolt touching the tin plate when the string is released.)

**3.** The position of a ball rolling in a straight line is given by $x = 2.0 - 3.6t + 1.7t^2$, where $x$ is in meters and $t$ in seconds. (*a*) What do the numbers 2.0, 3.6, and 1.7 refer to? (*b*) What are the units of each of these numbers? (*c*) Determine the position of the ball at $t = 1.0$ s, 2.0 s, and 3.0 s. (*d*) What is the average velocity over the interval $t = 1.0$ s to $t = 3.0$ s?

## ANSWERS TO EXERCISES

**A:** (*a*) displacement = −30 cm; (*b*) total distance = 50 cm.
**B:** (*b*).
**C:** (*a*) +; (*b*) −; (*c*) −; (*d*) +.
**D:** (*b*).
**E:** (*e*).
**F:** (*c*).


This snowboarder flying through the air shows an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity, $\vec{\mathbf{g}}$. Galileo analyzed the motion of objects in 2 dimensions under the action of gravity near the Earth's surface (now called "projectile motion") into its horizontal and vertical components.

We will discuss vectors and how to add them. Besides analyzing projectile motion, we will also see how to work with relative velocity.

CHAPTER 3

# Kinematics in Two Dimensions; Vectors

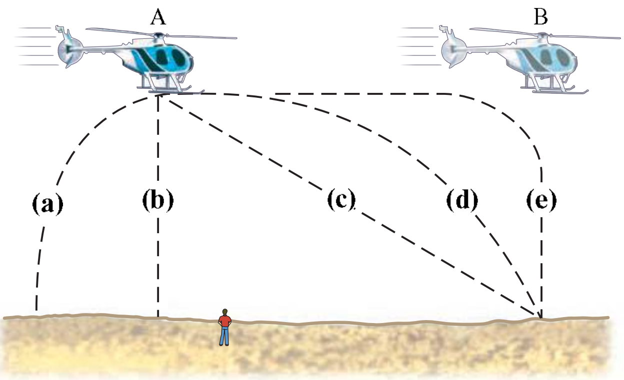
CONTENTS


3–1 Vectors and Scalars
3–2 Addition of Vectors—Graphical Methods
3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar
3–4 Adding Vectors by Components
3–5 Projectile Motion
3–6 Solving Projectile Motion Problems
*3–7 Projectile Motion Is Parabolic
3–8 Relative Velocity


**CHAPTER-OPENING QUESTION—Guess now!**

[*Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.*]

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?




In Chapter 2 we dealt with motion along a straight line. We now consider the motion of objects that move in paths in two (or three) dimensions. In particular, we discuss an important type of motion known as *projectile motion*: objects projected outward near the Earth's surface, such as struck baseballs and golf balls, kicked footballs, and other projectiles. Before beginning our discussion of motion in two dimensions, we will need a new tool, vectors, and how to add them.

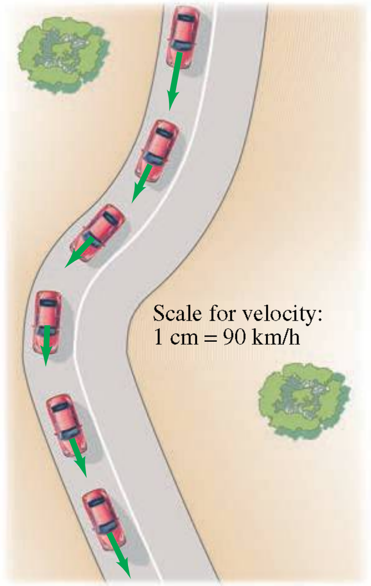



**FIGURE 3–1** Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

# 3–1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

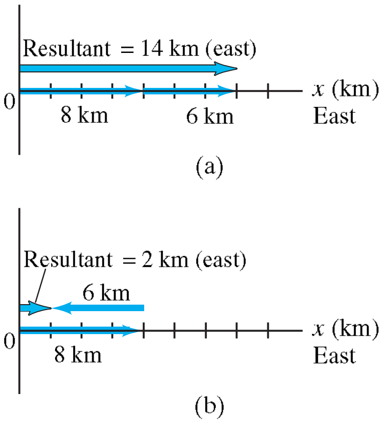
Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The length of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 3–1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 3–1 by measuring the length of the corresponding arrow and using the scale shown $(1\text{ cm} = 90\text{ km/h})$.

When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write $\vec{\mathbf{v}}$. If we are concerned only with the magnitude of the vector, we will write simply $v$, in italics, as we do for other symbols.

# 3–2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol $\vec{\mathbf{D}}$, and velocity vectors, $\vec{\mathbf{v}}$. But the results will apply for other vectors we encounter later.

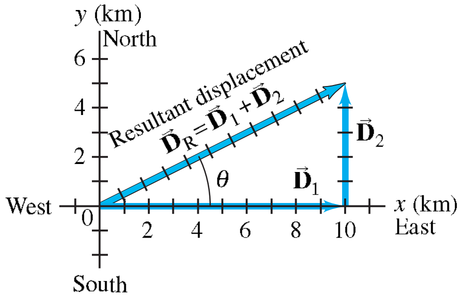
**FIGURE 3–2** Combining vectors in one dimension.




We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be $8\text{ km} + 6\text{ km} = 14\text{ km}$ east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3–2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3–2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: $8\text{ km} - 6\text{ km} = 2\text{ km}$.

But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive $y$ axis points north and the positive $x$ axis points east, Fig. 3–3. On this graph, we draw an arrow, labeled $\vec{\mathbf{D}}_1$, to represent the 10.0-km displacement to the east. Then we draw a second arrow, $\vec{\mathbf{D}}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3–3.

**FIGURE 3–3** A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$, which are shown as arrows. Also shown is the resultant displacement vector, $\vec{\mathbf{D}}_R$, which is the vector sum of $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. Measurement on the graph with ruler and protractor shows that $\vec{\mathbf{D}}_R$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.



After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\vec{\mathbf{D}}_R$ in Fig. 3–3. (The subscript R stands for resultant.) Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle $\theta = 27°$ north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive *x* axis. The magnitude (length) of $\vec{\mathbf{D}}_R$ can also be obtained using the theorem of Pythagoras in this case, because $D_1$, $D_2$, and $D_R$ form a right triangle with $D_R$ as the hypotenuse. Thus

$$D_R = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\text{ km})^2 + (5.0\text{ km})^2}$$
$$= \sqrt{125\text{ km}^2} = 11.2\text{ km}.$$

You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

The resultant displacement vector, $\vec{\mathbf{D}}_R$, is the sum of the vectors $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. That is,

$$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_R \leq (D_1 + D_2),$$

where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3–3), $D_R = 11.2\text{ km}$, whereas $D_1 + D_2$ equals 15 km, which is the total distance traveled. Note also that we cannot set $\vec{\mathbf{D}}_R$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 = (11.2\text{ km}, 27°\text{ N of E})$.

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it $\vec{\mathbf{D}}_1$—to scale.
2. Next draw the second vector, $\vec{\mathbf{D}}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

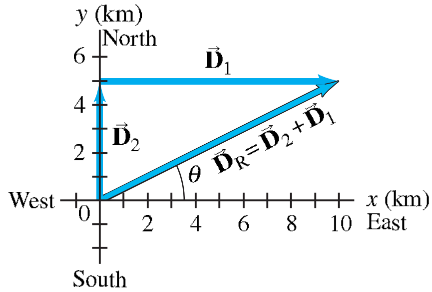
The length of the resultant vector represents its magnitude. Note that vectors can be moved parallel to themselves on paper (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). That is, now using $\vec{\mathbf{V}}$ to represent any type of vector,

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1.$$

[Mathematicians call this equation the *commutative* property of vector addition.]

**FIGURE 3–4** If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3–3.)



The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 3–5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors.

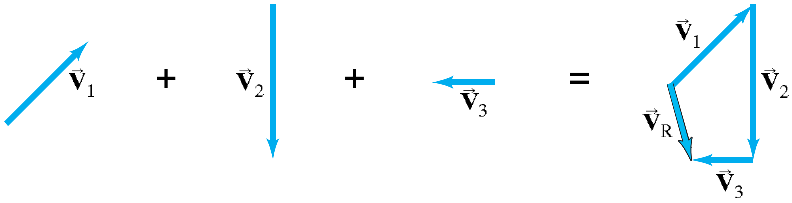



**FIGURE 3–5** The resultant of three vectors: $\vec{V}_R = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$.

A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3–6b. The resultant is the diagonal drawn from the common origin. In Fig. 3–6a, the tail-to-tip method is shown, and we can see that both methods yield the same result.

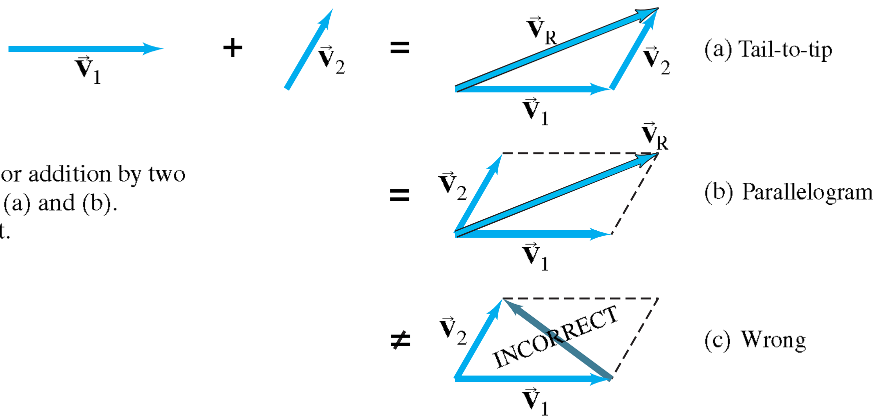



**FIGURE 3–6** Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

**CAUTION**

*Be sure to use the correct diagonal on the parallelogram to get the resultant*

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3–6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\vec{V}_2 - \vec{V}_1$, as we will see in the next Section.)

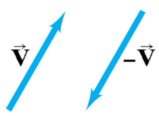
**CONCEPTUAL EXAMPLE 3–1** **Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

**RESPONSE** The sum can take on any value from $6.0\ (= 3.0 + 3.0)$ where the vectors point in the same direction, to $0\ (= 3.0 - 3.0)$ when the vectors are antiparallel. Magnitudes between 0 and 6.0 occur when the two vectors are at an angle other than 0° and 180°.

**EXERCISE A** If the two vectors of Example 3–1 are perpendicular to each other, what is the resultant vector length?

## 3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

**FIGURE 3–7** The negative of a vector is a vector having the same length but opposite direction.




Given a vector $\vec{V}$, we define the *negative* of this vector $(-\vec{V})$ to be a vector with the same magnitude as $\vec{V}$ but opposite in direction, Fig. 3–7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

We can now define the subtraction of one vector from another: the difference between two vectors $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$ is defined as

$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + (-\vec{\mathbf{V}}_1).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

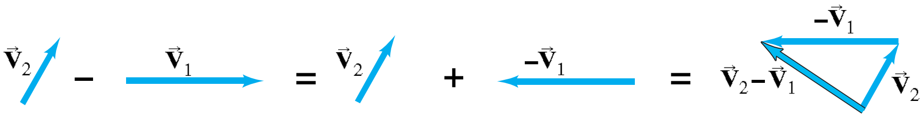
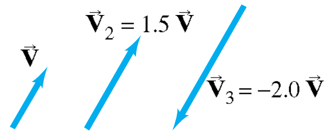



**FIGURE 3–8** Subtracting two vectors: $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$.

A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar $c$. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude $cV$. That is, multiplication of a vector by a positive scalar $c$ changes the magnitude of the vector by a factor $c$ but doesn't alter the direction. If $c$ is a negative scalar (such as $-2.0$), the magnitude of the product $c\vec{\mathbf{V}}$ is changed by the factor $|c|$ (where $|c|$ means the magnitude of $c$), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 3–9.

**FIGURE 3–9** Multiplying a vector $\vec{\mathbf{V}}$ by a scalar $c$ gives a vector whose magnitude is $c$ times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if $c$ is negative).




**EXERCISE B** What does the "incorrect" vector in Fig. 3–6c represent? (*a*) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1$; (*b*) $\vec{\mathbf{V}}_1 - \vec{\mathbf{V}}_2$; (*c*) something else (specify).

# 3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

## Components

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions, such as the $x$ and $y$ axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive $x$ axis to be to the east and the positive $y$ axis north. This vector $\vec{\mathbf{V}}$ is resolved into its $x$ and $y$ components by drawing dashed lines (AB and AC) out from the tip (A) of the vector, making them perpendicular to the $x$ and $y$ axes. Then the lines 0B and 0C represent the $x$ and $y$ components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 3–10b. These *vector components* are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. In this book we usually show vector components as arrows, like vectors, but dashed. The *scalar components*, $V_x$ and $V_y$, are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative $x$ or $y$ axis. As can be seen in Fig. 3–10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$.

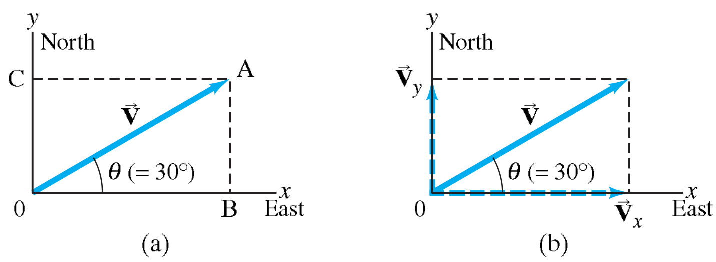



**FIGURE 3–10** Resolving a vector $\vec{\mathbf{V}}$ into its components along a chosen set of $x$ and $y$ axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.

To add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.

Given any angle $\theta$, as in Fig. 3–11a, a right triangle can be constructed by drawing a line perpendicular to one of its sides, as in Fig. 3–11b. The longest side of a right triangle, opposite the right angle, is called the hypotenuse, which we label $h$. The side opposite the angle $\theta$ is labeled $o$, and the side adjacent is labeled $a$. We let $h$, $o$, and $a$ represent the lengths of these sides, respectively.

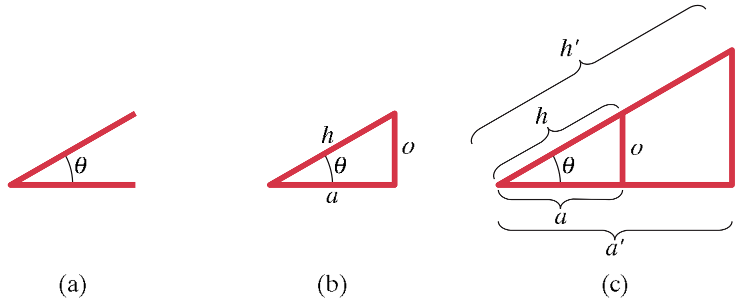



**FIGURE 3–11** Starting with an angle $\theta$ as in (a), we can construct right triangles of different sizes, (b) and (c), but the ratio of the lengths of the sides does not depend on the size of the triangle.

We now define the three trigonometric functions, sine, cosine, and tangent (abbreviated sin, cos, tan), in terms of the right triangle, as follows:

$$\begin{aligned} \sin\theta &= \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h} \\ \cos\theta &= \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h} \\ \tan\theta &= \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}. \end{aligned} \qquad \textbf{(3–1)}$$

If we make the triangle bigger, but keep the same angles, then the ratio of the length of one side to the other, or of one side to the hypotenuse, remains the same. That is, in Fig. 3–11c we have: $a/h = a'/h'$; $o/h = o'/h'$; and $o/a = o'/a'$. Thus the values of sine, cosine, and tangent do not depend on how big the triangle is. They depend only on the size of the angle. The values of sine, cosine, and tangent for different angles can be found using a scientific calculator, or from the Table in Appendix A.

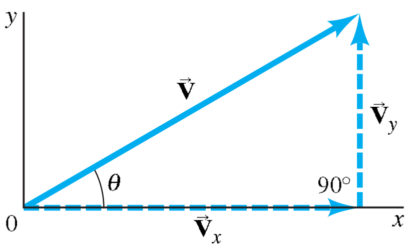
A useful trigonometric identity is

$$\sin^2\theta + \cos^2\theta = 1 \qquad \textbf{(3–2)}$$

which follows from the Pythagorean theorem ($o^2 + a^2 = h^2$ in Fig. 3–11). That is:

$$\sin^2\theta + \cos^2\theta = \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2 + a^2}{h^2} = \frac{h^2}{h^2} = 1.$$

(See Appendix A and inside the rear cover for other details on trigonometric functions and identities.)




$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

**FIGURE 3–12** Finding the components of a vector using trigonometric functions. The equations are valid only if $\theta$ is the angle $\vec{\mathbf{V}}$ makes with the positive $x$ axis.

The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–12, where a vector and its two components are thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in Fig. 3–12, where $\theta$ is the angle $\vec{\mathbf{V}}$ makes with the $+x$ axis. If we multiply the definition of $\sin\theta = V_y/V$ by $V$ on both sides, we get

$$V_y = V\sin\theta. \qquad \textbf{(3–3a)}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \qquad \textbf{(3–3b)}$$

Note that if $\theta$ is not the angle the vector makes with the positive $x$ axis, Eqs. 3–3 are not valid.

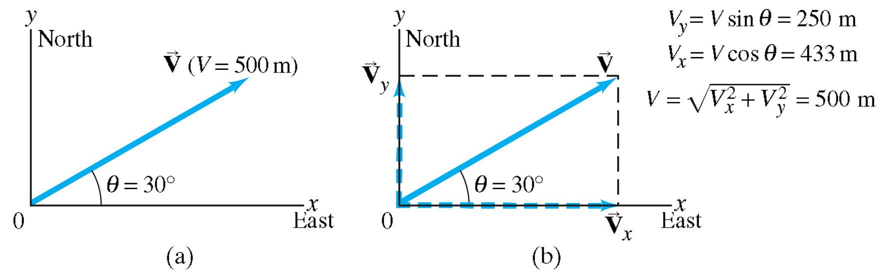



**FIGURE 3–13** (a) Vector $\vec{\mathbf{V}}$ represents a displacement of 500 m at a 30° angle north of east. (b) The components of $\vec{\mathbf{V}}$ are $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$, whose magnitudes are given on the right in the diagram.

Using Eqs. 3–3, we can calculate $V_x$ and $V_y$ for any vector, such as that illustrated in Fig. 3–10 or Fig. 3–12. Suppose $\vec{\mathbf{V}}$ represents a displacement of 500 m in a direction 30° north of east, as shown in Fig. 3–13. Then $V = 500\,\mathrm{m}$. From a calculator or Tables, $\sin 30° = 0.500$ and $\cos 30° = 0.866$. Then

$$\begin{aligned} V_x &= V\cos\theta = (500\,\mathrm{m})(0.866) = 433\,\mathrm{m}\ (\text{east}), \\ V_y &= V\sin\theta = (500\,\mathrm{m})(0.500) = 250\,\mathrm{m}\ (\text{north}). \end{aligned}$$

There are two ways to specify a vector in a given coordinate system:

1. We can give its components, $V_x$ and $V_y$.
2. We can give its magnitude $V$ and the angle $\theta$ it makes with the positive $x$ axis.

We can shift from one description to the other using Eqs. 3–3, and, for the reverse, by using the theorem of Pythagoras† and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \quad \textbf{(3–4a)}$$

$$\tan\theta = \frac{V_y}{V_x} \quad \textbf{(3–4b)}$$

as can be seen in Fig. 3–12.

## Adding Vectors

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 3–14, that the addition of any two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ to give a resultant, $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, implies that

$$\begin{aligned} V_{Rx} &= V_{1x} + V_{2x} \\ V_{Ry} &= V_{1y} + V_{2y}. \end{aligned} \quad \textbf{(3–5)}$$

That is, the sum of the $x$ components equals the $x$ component of the resultant vector, and the sum of the $y$ components equals the $y$ component of the resultant, as can be verified by a careful examination of Fig. 3–14. Note that we do *not* add $x$ components to $y$ components.

If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3–4.

†In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where $V_z$ is the component along the third, or $z$, axis.

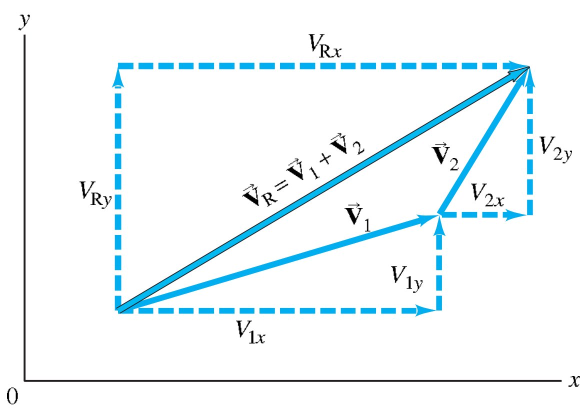



**FIGURE 3–14** The components of $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ are $V_{Rx} = V_{1x} + V_{2x}$ and $V_{Ry} = V_{1y} + V_{2y}$.

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

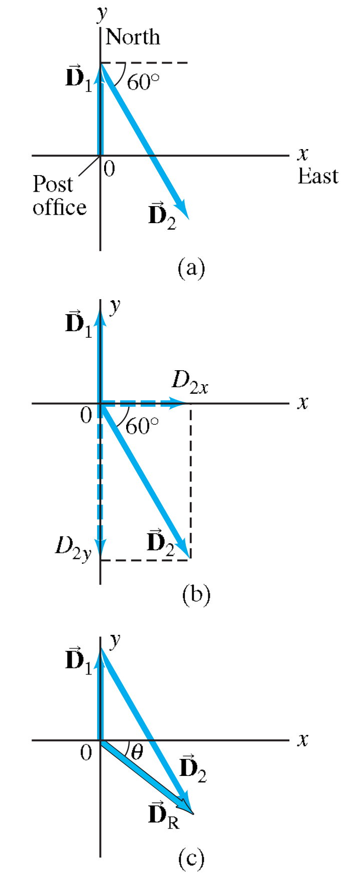



**FIGURE 3–15** Example 3–2. (a) The two displacement vectors, $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$. (b) $\vec{\mathbf{D}}_2$ is resolved into its components. (c) $\vec{\mathbf{D}}_1$ and $\vec{\mathbf{D}}_2$ are added to obtain the resultant $\vec{\mathbf{D}}_R$. The component method of adding the vectors is explained in the Example.

**EXAMPLE 3–2** **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 3–15a). What is her displacement from the post office?

**APPROACH** We choose the positive $x$ axis to be east and the positive $y$ axis to be north, since those are the compass directions used on most maps. The origin of the $xy$ coordinate system is at the post office. We resolve each vector into its $x$ and $y$ components. We add the $x$ components together, and then the $y$ components together, giving us the $x$ and $y$ components of the resultant.

**SOLUTION** Resolve each displacement vector into its components, as shown in Fig. 3–15b. Since $\vec{\mathbf{D}}_1$ has magnitude 22.0 km and points north, it has only a $y$ component:

$$D_{1x} = 0, \qquad D_{1y} = 22.0\,\text{km}.$$

$\vec{\mathbf{D}}_2$ has both $x$ and $y$ components:

$$D_{2x} = +(47.0\,\text{km})(\cos 60°) = +(47.0\,\text{km})(0.500) = +23.5\,\text{km}$$
$$D_{2y} = -(47.0\,\text{km})(\sin 60°) = -(47.0\,\text{km})(0.866) = -40.7\,\text{km}.$$

Notice that $D_{2y}$ is negative because this vector component points along the negative $y$ axis. The resultant vector, $\vec{\mathbf{D}}_R$, has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0\,\text{km} + 23.5\,\text{km} = +23.5\,\text{km}$$
$$D_{Ry} = D_{1y} + D_{2y} = 22.0\,\text{km} + (-40.7\,\text{km}) = -18.7\,\text{km}.$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5\,\text{km}, \qquad D_{Ry} = -18.7\,\text{km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3–4:

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5\,\text{km})^2 + (-18.7\,\text{km})^2} = 30.0\,\text{km}$$

$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7\,\text{km}}{23.5\,\text{km}} = -0.796.$$

A calculator with a key labeled INV TAN, or ARC TAN, or TAN$^{-1}$ gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the $x$ axis, Fig. 3–15c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.

**NOTE** Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.

**PROBLEM SOLVING**
*Identify the correct quadrant by drawing a careful diagram*

As we saw in Example 3–2, any component that points along the negative $x$ or $y$ axis gets a minus sign. The signs of trigonometric functions depend on which "quadrant" the angle falls in: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants; see Appendix A, Fig. A–7. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram, like Fig. 3–15. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.

## PROBLEM SOLVING

### Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

1. **Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
2. **Choose** $x$ and $y$ **axes**. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors, which then will have only one component.)
3. **Resolve** each vector into its $x$ and $y$ **components**, showing each component along its appropriate ($x$ or $y$) axis as a (dashed) arrow.
4. **Calculate each component** (when not given) using sines and cosines. If $\theta_1$ is the angle that vector $\vec{\mathbf{V}}_1$ makes with the positive $x$ axis, then:
$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$
Pay careful attention to **signs**: any component that points along the negative $x$ or $y$ axis gets a minus sign.
5. **Add** the $x$ **components** together to get the $x$ component of the resultant. Similarly for $y$:
$$V_{Rx} = V_{1x} + V_{2x} + \text{any others}$$
$$V_{Ry} = V_{1y} + V_{2y} + \text{any others}.$$
This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).
6. If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3–4:
$$V_R = \sqrt{V_{Rx}^2 + V_{Ry}^2}, \qquad \tan\theta = \frac{V_{Ry}}{V_{Rx}}.$$
The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle $\theta$.

**EXAMPLE 3–3** **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–16a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

**APPROACH** We follow the steps in the Problem Solving Strategy above.

**SOLUTION**

1. **Draw a diagram** such as Fig. 3–16a, where $\vec{\mathbf{D}}_1$, $\vec{\mathbf{D}}_2$, and $\vec{\mathbf{D}}_3$ represent the three legs of the trip, and $\vec{\mathbf{D}}_R$ is the plane's total displacement.
2. **Choose axes**: Axes are also shown in Fig. 3–16a: $x$ is east, $y$ north.
3. **Resolve components**: It is imperative to draw a good diagram. The components are drawn in Fig. 3–16b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–15b, here we draw them "tail-to-tip" style, which is just as valid and may make it easier to see.
4. **Calculate the components**:
$$\begin{aligned}
\vec{\mathbf{D}}_1: D_{1x} &= +D_1\cos 0^\circ = D_1 = 620\text{ km}\\
D_{1y} &= +D_1 \sin 0^\circ = 0\text{ km}\\
\vec{\mathbf{D}}_2: D_{2x} &= +D_2\cos 45^\circ = +(440\text{ km})(0.707) = +311\text{ km}\\
D_{2y} &= -D_2\sin 45^\circ = -(440\text{ km})(0.707) = -311\text{ km}\\
\vec{\mathbf{D}}_3: D_{3x} &= -D_3\cos 53^\circ = -(550\text{ km})(0.602) = -331\text{ km}\\
D_{3y} &= -D_3\sin 53^\circ = -(550\text{ km})(0.799) = -439\text{ km}.
\end{aligned}$$
We have given a minus sign to each component that in Fig. 3–16b points in the $-x$ or $-y$ direction. The components are shown in the Table in the margin.
5. **Add the components**: We add the $x$ components together, and we add the $y$ components together to obtain the $x$ and $y$ components of the resultant:
$$D_{Rx} = D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}$$
$$D_{Ry} = D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km}.$$
The $x$ and $y$ components of the resultant are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
6. **Magnitude and direction**: We can also give the answer as
$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51^\circ.$$
Thus, the total displacement has magnitude 960 km and points 51° below the $x$ axis (south of east), as was shown in our original sketch, Fig. 3–16a.

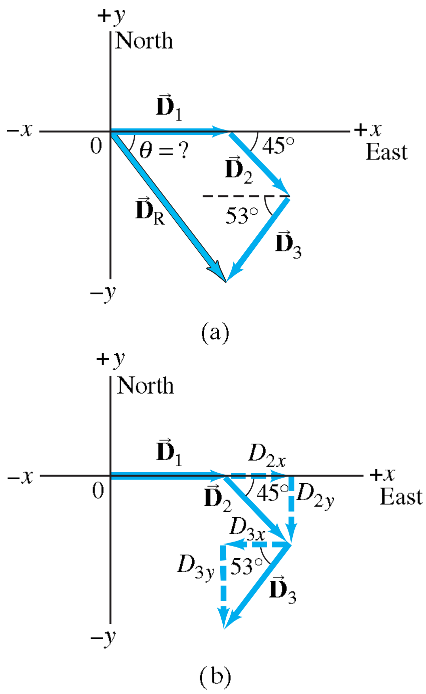
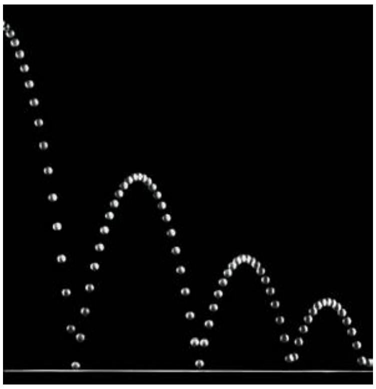



**FIGURE 3–16** Example 3–3.

| Vector | Components $x$ (km) | $y$ (km) |
|---|---|---|
| $\vec{\mathbf{D}}_1$ | 620 | 0 |
| $\vec{\mathbf{D}}_2$ | 311 | −311 |
| $\vec{\mathbf{D}}_3$ | −331 | −439 |
| $\vec{\mathbf{D}}_R$ | 600 | −750 |



(a)

(b)

**FIGURE 3–17** Photographs of (a) a bouncing ball and (b) a thrown basketball, each showing the characteristic "parabolic" path of projectile motion.

## 3–5 Projectile Motion

In Chapter 2, we studied the one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–17), which we can describe as taking place in two dimensions if there is no wind.

Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\,\mathrm{m/s^2}$, and we assume it is constant.†

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an $xy$ coordinate system (so $x_0 = y_0 = 0$).

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal ($x$) direction, $\vec{\mathbf{v}}_{x0}$. See Fig. 3–18, where an object falling vertically is also shown for comparison. The velocity vector $\vec{\mathbf{v}}$ at each instant points in the direction of the ball's motion at that instant and is thus always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of velocity and acceleration separately, and we can apply the kinematic equations (Eqs. 2–11a through 2–11c) to the $x$ and $y$ components of the motion.

First we examine the vertical ($y$) component of the motion. At the instant the ball leaves the table's top $(t = 0)$, it has only an $x$ component of velocity. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration $g$, the acceleration due to gravity. Thus $v_y$ is initially zero $(v_{y0} = 0)$ but increases continually in the downward direction (until the ball hits the ground). Let us take $y$ to be positive upward. Then the acceleration due to gravity is in the $-y$ direction, so $a_y = -g$. From Eq. 2–11a (using $y$ in place of $x$) we can write $v_y = v_{y0} + a_y t = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by Eq. 2–11b written in terms of $y$: $y = y_0 + v_{y0} + \frac{1}{2}a_y t^2$. Given $y_0 = 0$, $v_{y0} = 0$, and $a_y = -g$, then $y = -\frac{1}{2}gt^2$.

†This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

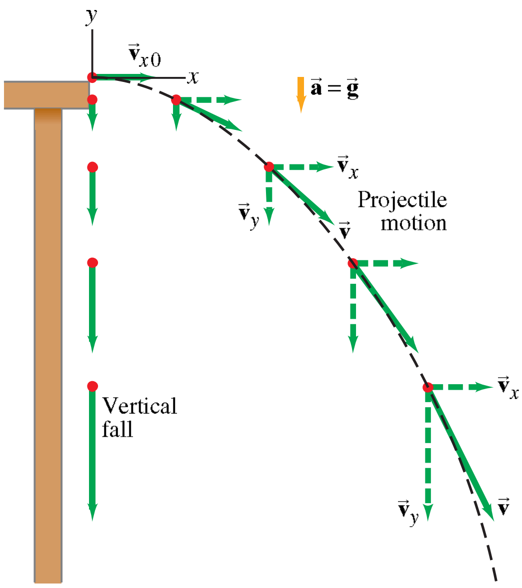



**FIGURE 3–18** Projectile motion of a small ball projected horizontally with initial velocity $\vec{\mathbf{v}} = \vec{\mathbf{v}}_{x0}$. The dashed black line represents the path of the object. The velocity vector $\vec{\mathbf{v}}$ is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison; $v_y$ is the same at each instant for the falling object and the projectile.)

In the horizontal direction, on the other hand, there is no acceleration (we are ignoring air resistance). With $a_x = 0$, the horizontal component of velocity, $v_x$, remains constant, equal to its initial value, $v_{x0}$, and thus has the same magnitude at each point on the path. The horizontal displacement (with $a_x = 0$) is given by $x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$. The two vector components, $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, can be added vectorially at any instant to obtain the velocity $\vec{\mathbf{v}}$ at that time (that is, for each point on the path), as shown in Fig. 3–18.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 3–18. Figure 3–19 is a multiple-exposure photograph of an experiment that confirms this.

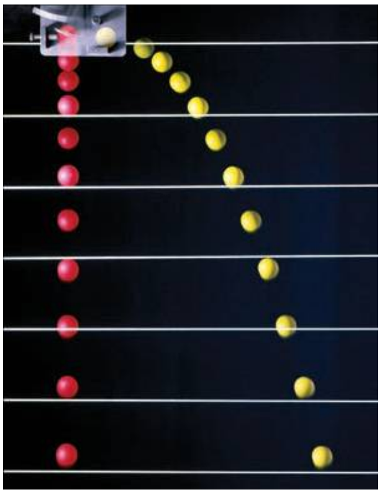


**FIGURE 3–19** Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

**EXERCISE C** Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster ball or the slower one?

If an object is projected at an upward angle, as in Fig. 3–20, the analysis is similar, except that now there is an initial vertical component of velocity, $v_{y0}$. Because of the downward acceleration of gravity, the upward component of velocity $v_y$ gradually decreases with time until the object reaches the highest point on its path, at which point $v_y = 0$. Subsequently the object moves downward (Fig. 3–20) and $v_y$ increases in the downward direction, as shown (that is, becoming more negative). As before, $v_x$ remains constant.

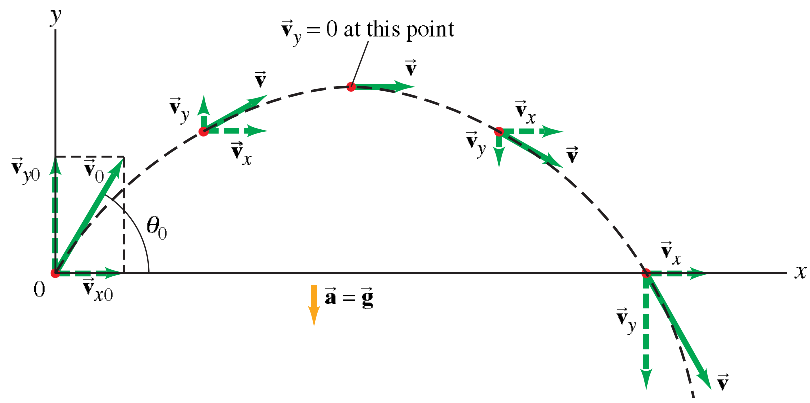


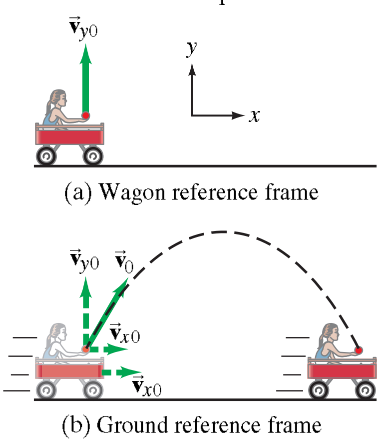

**FIGURE 3–20** Path of a projectile launched with initial velocity $\vec{\mathbf{v}}_0$ at angle $\theta_0$ to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The figure does not show where the projectile hits the ground (at that point, projectile motion ceases).

**EXERCISE D** Where in Fig. 3–20 is (i) $\vec{\mathbf{v}} = 0$, (ii) $v_y = 0$, and (iii) $v_x = 0$?

**CONCEPTUAL EXAMPLE 3–4** **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 3–21. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 3–21a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

**RESPONSE** The child throws the apple straight up from her own reference frame with initial velocity $\vec{\mathbf{v}}_{y0}$ (Fig. 3–21a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon, $\vec{\mathbf{v}}_{x0}$. Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 3–21b. The apple experiences no horizontal acceleration, so $\vec{\mathbf{v}}_{x0}$ will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (*b*).

**FIGURE 3–21** Example 3–4.




**EXERCISE E** Return to the Chapter-Opening Question, page 49, and answer it again now. Try to explain why you may have answered differently the first time. Describe the role of the helicopter in this example of projectile motion.

# 3–6 Solving Projectile Motion Problems

We now work through several Examples of projectile motion quantitatively. We use the kinematic equations (2–11a through 2–11c) separately for the vertical and horizontal components of the motion. These equations are shown separately for the $x$ and $y$ components of the motion in Table 3–1, for the general case of two-dimensional motion at constant acceleration. Note that $x$ and $y$ are the respective displacements, that $v_x$ and $v_y$ are the components of the velocity, and that $a_x$ and $a_y$ are the components of the acceleration, each of which is constant. The subscript 0 means "at $t = 0$."

**TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions**

| *x* component (horizontal) | | *y* component (vertical) |
|---|---|---|
| $v_x = v_{x0} + a_x t$ | (Eq. 2–11a) | $v_y = v_{y0} + a_y t$ |
| $x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$ | (Eq. 2–11b) | $y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$ |
| $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$ | (Eq. 2–11c) | $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ |

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes $y$ is positive upward, so $a_y = -g = -9.80\,\mathrm{m/s^2}$.

**TABLE 3–2 Kinematic Equations for Projectile Motion**
**($y$ positive upward; $a_x = 0$, $a_y = -g = -9.80\,\mathrm{m/s^2}$)**

| Horizontal Motion ($a_x = 0$, $v_x$ = constant) | | Vertical Motion† ($a_y = -g$ = constant) |
|---|---|---|
| $v_x = v_{x0}$ | (Eq. 2–11a) | $v_y = v_{y0} - gt$ |
| $x = x_0 + v_{x0} t$ | (Eq. 2–11b) | $y = y_0 + v_{y0} t - \frac{1}{2} g t^2$ |
| | (Eq. 2–11c) | $v_y^2 = v_{y0}^2 - 2g(y - y_0)$ |

† If $y$ is taken positive downward, the minus ($-$) signs in front of $g$ become $+$ signs.

If the projection angle $\theta_0$ is chosen relative to the $+x$ axis (Fig. 3–20), then

$$v_{x0} = v_0 \cos\theta_0, \qquad \text{and} \qquad v_{y0} = v_0 \sin\theta_0 .$$

**PROBLEM SOLVING**
*Choice of time interval*

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set $\vec{\mathbf{a}} = \vec{\mathbf{g}}$.

PROBLEM SOLVING

## Projectile Motion

Our approach to solving Problems in Section 2–6 also applies here. Solving Problems involving projectile motion can require creativity, and cannot be done just by following some rules. Certainly you must avoid just plugging numbers into equations that seem to "work."

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening to the object.
3. **Choose** an origin and an $xy$ **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the $x$ and $y$ analyses. The $x$ and $y$ motions are connected by the common time, $t$.
5. **Examine** the horizontal ($x$) and vertical ($y$) **motions** separately. If you are given the initial velocity, you may want to resolve it into its $x$ and $y$ components.
6. List the **known** and **unknown** quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\,\mathrm{m/s^2}$, and using the $-$ or $+$ sign, depending on whether you choose $y$ positive up or down. Remember that $v_x$ never changes throughout the trajectory, and that $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. A little planning goes a long way. **Apply** the **relevant equations** (Table 3–2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–4).

**EXAMPLE 3–5** **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

**APPROACH** We explicitly follow the steps of the Problem Solving Strategy on the previous page.

**SOLUTION**

**1.** and **2. Read, choose the object, and draw a diagram.** Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 3–22.

**3. Choose a coordinate system.** We choose the $y$ direction to be positive upward, with the top of the cliff as $y_0 = 0$. The $x$ direction is horizontal with $x_0 = 0$ at the point where the motorcycle leaves the cliff.

**4. Choose a time interval.** We choose our time interval to begin $(t = 0)$ just as the motorcycle leaves the cliff top at position $x_0 = 0,\ y_0 = 0$. Our time interval ends just before the motorcycle touches the ground below.

**5. Examine *x* and *y* motions.** In the horizontal ($x$) direction, the acceleration $a_x = 0$, so the velocity is constant. The value of $x$ when the motorcycle reaches the ground is $x = +90.0\,\text{m}$. In the vertical direction, the acceleration is the acceleration due to gravity, $a_y = -g = -9.80\,\text{m/s}^2$. The value of $y$ when the motorcycle reaches the ground is $y = -50.0\,\text{m}$. The initial velocity is horizontal and is our unknown, $v_{x0}$; the initial vertical velocity is zero, $v_{y0} = 0$.

**6. List knowns and unknowns.** See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity $v_{x0}$ (which stays constant until landing), we also do not know the time $t$ when the motorcycle reaches the ground.

**7. Apply relevant equations.** The motorcycle maintains constant $v_x$ as long as it is in the air. The time it stays in the air is determined by the $y$ motion—when it reaches the ground. So we first find the time using the $y$ motion, and then use this time value in the $x$ equations. To find out how long it takes the motorcycle to reach the ground below, we use Eq. 2–11b (Tables 3–1 and 3–2) for the vertical ($y$) direction with $y_0 = 0$ and $v_{y0} = 0$:

$$\begin{aligned} y &= y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \\ &= 0 + 0 + \tfrac{1}{2}(-g)t^2 \end{aligned}$$

or

$$y = -\tfrac{1}{2}gt^2.$$

We solve for $t$ and set $y = -50.0\,\text{m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\,\text{m})}{-9.80\,\text{m/s}^2}} = 3.19\,\text{s}.$$

To calculate the initial velocity, $v_{x0}$, we again use Eq. 2–11b, but this time for the horizontal ($x$) direction, with $a_x = 0$ and $x_0 = 0$:

$$\begin{aligned} x &= x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \\ &= 0 + v_{x0}t + 0 \end{aligned}$$

or

$$x = v_{x0}t.$$

Then

$$v_{x0} = \frac{x}{t} = \frac{90.0\,\text{m}}{3.19\,\text{s}} = 28.2\,\text{m/s},$$

which is about 100 km/h (roughly 60 mi/h).

**NOTE** In the time interval of the projectile motion, the only acceleration is $g$ in the negative $y$ direction. The acceleration in the $x$ direction is zero.

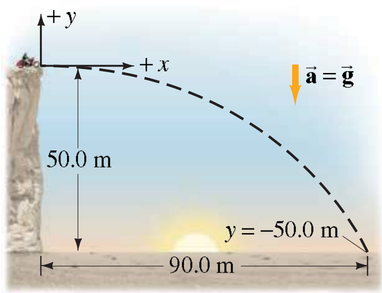



**FIGURE 3–22** Example 3–5.

| Known | Unknown |
|---|---|
| $x_0 = y_0 = 0$ | $v_{x0}$ |
| $x = 90.0\,\text{m}$ | $t$ |
| $y = -50.0\,\text{m}$ | |
| $a_x = 0$ | |
| $a_y = -g = -9.80\,\text{m/s}^2$ | |
| $v_{y0} = 0$ | |

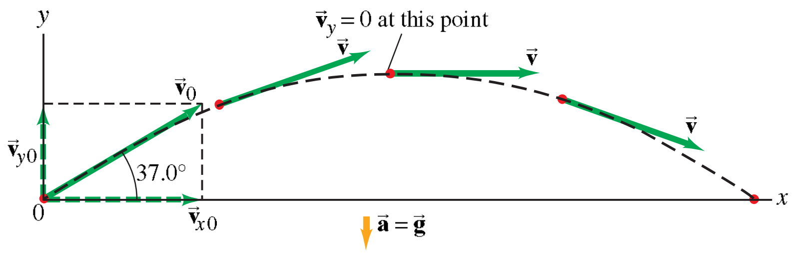



**FIGURE 3–23** Example 3–6.

**PHYSICS APPLIED**
*Sports*

**EXAMPLE 3–6** **A kicked football.** A kicked football leaves the ground at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 3–23. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, and (*c*) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

**APPROACH** This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the $y$ direction as positive upward, and treat the $x$ and $y$ motions separately. The total time in the air is again determined by the $y$ motion. The $x$ motion occurs at constant velocity. The $y$ component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

**SOLUTION** We resolve the initial velocity into its components (Fig. 3–23):

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\text{m/s})(0.799) = 16.0\,\text{m/s}$$
$$v_{y0} = v_0 \sin 37.0° = (20.0\,\text{m/s})(0.602) = 12.0\,\text{m/s}.$$

(*a*) To find the maximum height, we consider a time interval that begins just after the football loses contact with the foot until the ball reaches its maximum height. During this time interval, the acceleration is $g$ downward. At the maximum height, the velocity is horizontal (Fig. 3–23), so $v_y = 0$. This occurs at a time given by $v_y = v_{y0} - gt$ with $v_y = 0$ (see Eq. 2–11a in Table 3–2), so $v_{y0} = gt$ and

$$t = \frac{v_{y0}}{g} = \frac{(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 1.224\,\text{s} \approx 1.22\,\text{s}.$$

From Eq. 2–11b, with $y_0 = 0$, we can solve for $y$ at this time $(t = v_{y0}/g)$:

$$y = v_{y0}t - \tfrac{1}{2}gt^2 = \frac{v_{y0}^2}{g} - \frac{1}{2}\frac{v_{y0}^2}{g} = \frac{v_{y0}^2}{2g} = \frac{(12.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = 7.35\,\text{m}.$$

The maximum height is 7.35 m. [Solving Eq. 2–11c for $y$ gives the same result.]

(*b*) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot $(t = 0,\ y_0 = 0)$ and ending just before the ball touches the ground $(y = 0$ again). We can use Eq. 2–11b with $y_0 = 0$ and also set $y = 0$ (ground level):

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$
$$0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2.$$

This equation can be factored:

$$t\left(\tfrac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, $y_0$), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0\,\text{m/s})}{(9.80\,\text{m/s}^2)} = 2.45\,\text{s},$$

which is the total travel time of the football.

(*c*) The total distance traveled in the $x$ direction is found by applying Eq. 2–11b with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0$ m/s, and $t = 2.45$ s:

$$x = v_{x0}t = (16.0\,\text{m/s})(2.45\,\text{s}) = 39.2\,\text{m}.$$

**PROBLEM SOLVING**
*Symmetry*

**NOTE** In (*b*), the time needed for the whole trip, $t = 2v_{y0}/g = 2.45$ s, is double the time to reach the highest point, calculated in (*a*). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).

**EXERCISE F** In Example 3–6, what is (*a*) the velocity vector at the maximum height, and (*b*) the acceleration vector at maximum height?

In Example 3–6, we treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates (mainly overestimates).

**CONCEPTUAL EXAMPLE 3–7** **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance $d$ away, Fig. 3–24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

**RESPONSE** Both the water balloon and the boy in the tree start falling at the same instant, and in a time $t$ they each fall the same vertical distance $y = \frac{1}{2}gt^2$, much like Fig. 3–19. In the time it takes the water balloon to travel the horizontal distance $d$, the balloon will have the same $y$ position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

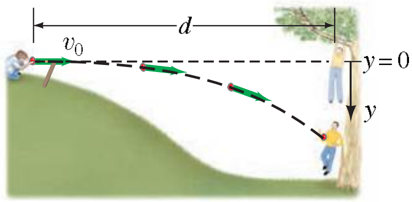



**FIGURE 3–24** Example 3–7.

## Level Horizontal Range

The total distance the football traveled in Example 3–6 is called the horizontal **range** $R$. We now derive a formula for the range, which applies to a projectile that lands at the same level it started ($= y_0$): that is, $y\,(\text{final}) = y_0$ (see Fig. 3–25a). Looking back at Example 3–6 part ($c$), we see that $x = R = v_{x0}t$ where (from part $b$) $t = 2v_{y0}/g$. Thus

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \qquad [y = y_0]$$

where $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$. This can be rewritten, using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ (Appendix A or inside the rear cover):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [\text{only if } y\,(\text{final}) = y_0]$$

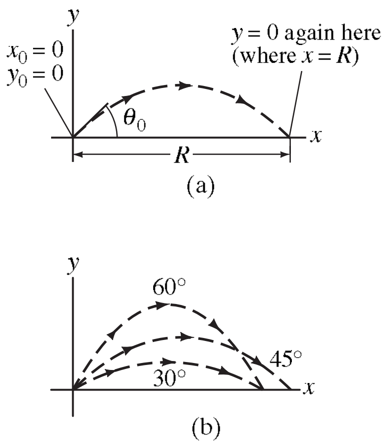
Note that the *maximum* range, for a given initial velocity $v_0$, is obtained when $\sin 2\theta$ takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{max} = v_0^2/g.$$

The maximum range increases by the square of $v_0$, so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

When air resistance is important, the range is less for a given $v_0$, and the maximum range is obtained at an angle smaller than 45°.

**FIGURE 3–25** (a) The range $R$ of a projectile. (b) There are generally two angles $\theta_0$ that will give the same range. If one angle is $\theta_{01}$, the other is $\theta_{02} = 90° - \theta_{01}$. Example 3–8.




**EXAMPLE 3–8** **Range of a cannon ball.** Suppose one of Napoleon's cannons had a muzzle speed, $v_0$, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

**APPROACH** We use the equation just derived for the range, $R = v_0^2 \sin 2\theta_0/g$, with $R = 320$ m.

**SOLUTION** We solve for $\sin 2\theta_0$ in the range formula:

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\,\text{m})(9.80\,\text{m/s}^2)}{(60.0\,\text{m/s})^2} = 0.871.$$

We want to solve for an angle $\theta_0$ that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Thus, $2\theta_0 = 60.6°$ is a solution, so $\theta_0 = 30.3°$. But $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–7), so $\theta_0$ can also be $\theta_0 = 59.7°$. In general we have two solutions (see Fig. 3–25b), which in the present case are given by

$$\theta_0 = 30.3° \quad \text{or} \quad 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

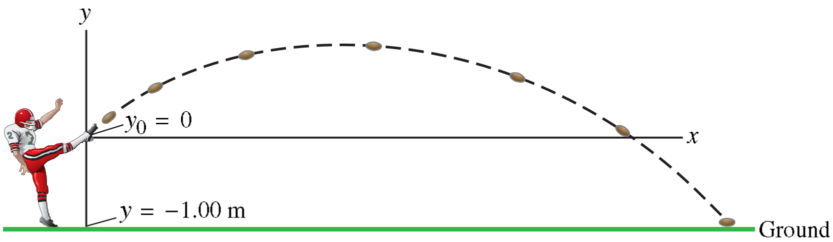



**FIGURE 3–26** Example 3–9: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00\text{ m}$.

**PHYSICS APPLIED**
*Sports*

**PROBLEM SOLVING**
*Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because $y \neq y_0$*

**EXAMPLE 3–9** **A punt.** Suppose the football in Example 3–6 was punted, and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0,\ y_0 = 0$.

**APPROACH** The only difference here from Example 3–6 is that the football hits the ground *below* its starting point of $y_0 = 0$. That is, the ball hits the ground at $y = -1.00\text{ m}$. See Fig. 3–26. Thus we cannot use the range formula which is valid only if $y\,(\text{final}) = y_0$. As in Example 3–6, $v_0 = 20.0\text{ m/s}$, $\theta_0 = 37.0°$.

**SOLUTION** With $y = -1.00\text{ m}$ and $v_{y0} = 12.0\text{ m/s}$ (see Example 3–6), we use the $y$ version of Eq. 2–11b with $a_y = -g$,

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\text{ m} = 0 + (12.0\text{ m/s})t - \left(4.90\text{ m/s}^2\right)t^2.$$

We rearrange this equation into standard form $\left(ax^2 + bx + c = 0\right)$ so we can use the quadratic formula:

$$\left(4.90\text{ m/s}^2\right)t^2 - (12.0\text{ m/s})t - (1.00\text{ m}) = 0.$$

The quadratic formula (Appendix A–4) gives

$$t = \frac{12.0\text{ m/s} \pm \sqrt{(-12.0\text{ m/s})^2 - 4\left(4.90\text{ m/s}^2\right)(-1.00\text{ m})}}{2\left(4.90\text{ m/s}^2\right)}$$
$$= 2.53\text{ s} \quad \text{or} \quad -0.081\text{ s}.$$

The second solution would correspond to a time prior to the kick, so it doesn't apply. With $t = 2.53\text{ s}$ for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using $v_{x0} = 16.0\text{ m/s}$ from Example 3–6):

$$x = v_{x0}t = (16.0\text{ m/s})(2.53\text{ s}) = 40.5\text{ m}.$$

Our assumption in Example 3–6 that the ball leaves the foot at ground level would result in an underestimate of about 1.3 m in the distance our punt traveled.

# *3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that $\vec{\mathbf{g}}$ is constant. To do so, we need to find $y$ as a function of $x$ by eliminating $t$ between the two equations for horizontal and vertical motion (Eq. 2–11b in Table 3–2), and for simplicity we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \qquad \textbf{(3–6)}$$

We see that $y$ as a function of $x$ has the form

$$y = Ax - Bx^2,$$

where $A$ and $B$ are constants for any specific projectile motion. This is the standard equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

**FIGURE 3–27** Examples of projectile motion: a boy jumping, and glowing lava from the volcano Stromboli.

*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.

# 3–8 Relative Velocity

We now consider how observations made in different frames of reference are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the train tracks will measure 80 km/h for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 km/h for the train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of 90 km/h − 75 km/h = 15 km/h.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the **relative velocity**. But if they are not along the same line, we must make use of vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat heads directly across a river, as shown in Fig. 3–28. We let $\vec{\mathbf{v}}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\vec{\mathbf{v}}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\vec{\mathbf{v}}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\vec{\mathbf{v}}_{BW}$ is what the boat's motor produces (against the water), whereas $\vec{\mathbf{v}}_{BS}$ is equal to $\vec{\mathbf{v}}_{BW}$ plus the effect of the current, $\vec{\mathbf{v}}_{WS}$. Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 3–28)

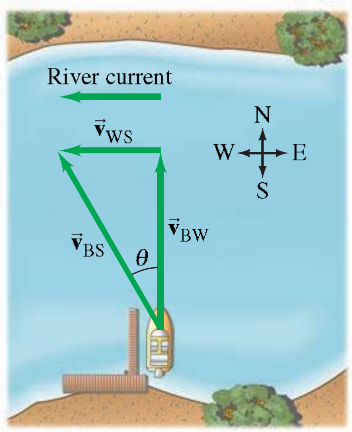



**FIGURE 3–28** A boat heads north directly across a river which flows west. Velocity vectors are shown as green arrows:
$\vec{\mathbf{v}}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\vec{\mathbf{v}}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\vec{\mathbf{v}}_{WS}$ = velocity of **W**ater with respect to the **S**hore (river current).
As it crosses the river, the boat is dragged downstream by the current.

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}. \tag{3–7}$$

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–7 are the same; also, the outer subscripts on the right of Eq. 3–7 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\vec{\mathbf{v}}_{BS}$. By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.†

Equation 3–7 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity $\vec{\mathbf{v}}_{FB}$ relative to the boat, his velocity relative to the shore is $\vec{\mathbf{v}}_{FS} = \vec{\mathbf{v}}_{FB} + \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{\mathbf{v}}_{BA} = -\vec{\mathbf{v}}_{AB}. \tag{3–8}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

†We thus can see, for example, that the equation $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} + \vec{\mathbf{v}}_{WS}$ is wrong: the inner subscripts are not the same, and the outer ones on the right do not correspond to the subscripts on the left.

**EXAMPLE 3–10** **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\,\text{m/s}$. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20\,\text{m/s}$, at what upstream angle must the boat head? (See Fig. 3–29.)

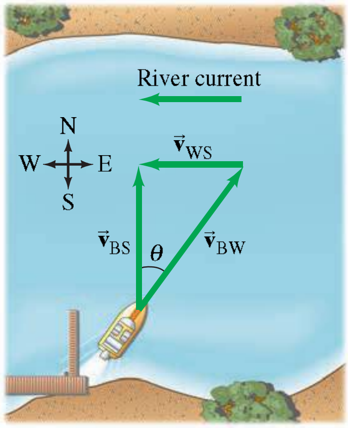



**FIGURE 3–29** Example 3–10.

**APPROACH** If the boat heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's current, the boat must have an upstream (eastward) component of velocity as well as a cross-stream (northward) component. Figure 3–29 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is where the boat is supposed to go. (Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.)

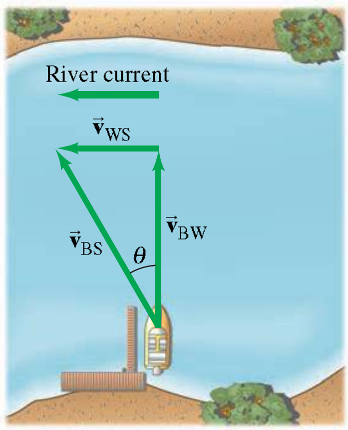
**SOLUTION** Vector $\vec{\mathbf{v}}_{BW}$ points upstream at angle $\theta$ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\,\text{m/s}}{1.85\,\text{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

**EXAMPLE 3–11** **Heading across the river.** The same boat $(v_{BW} = 1.85\,\text{m/s})$ now heads directly across the river whose current is still 1.20 m/s. (*a*) What is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

**FIGURE 3–30** Example 3–11. A boat heading directly across a river whose current moves at 1.20 m/s.




**APPROACH** The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 3–30. The boat's velocity with respect to the shore, $\vec{\mathbf{v}}_{BS}$, is the sum of its velocity with respect to the water, $\vec{\mathbf{v}}_{BW}$, plus the velocity of the water with respect to the shore, $\vec{\mathbf{v}}_{WS}$: just as before,

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

**SOLUTION** (*a*) Since $\vec{\mathbf{v}}_{BW}$ is perpendicular to $\vec{\mathbf{v}}_{WS}$, we can get $v_{BS}$ using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\,\text{m/s})^2 + (1.20\,\text{m/s})^2} = 2.21\,\text{m/s}.$$

We can obtain the angle (note how $\theta$ is defined in Fig. 3–30) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\,\text{m/s})/(1.85\,\text{m/s}) = 0.6486.$$

A calculator with a key INV TAN or ARC TAN or TAN$^{-1}$ gives $\theta = \tan^{-1}(0.6486) = 33.0°$. Note that this angle is not equal to the angle calculated in Example 3–10.

(*b*) The travel time for the boat is determined by the time it takes to cross the river. Given the river's width $D = 110\,\text{m}$, we can use the velocity component in the direction of $D$, $v_{BW} = D/t$. Solving for $t$, we get $t = 110\,\text{m}/1.85\,\text{m/s} = 59.5\,\text{s}$. The boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\,\text{m/s})(59.5\,\text{s}) = 71.4\,\text{m} \approx 71\,\text{m}.$$

**NOTE** There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

# Summary

A quantity such as velocity, that has both a magnitude and a direction, is called a **vector**. A quantity such as mass, that has only a magnitude, is called a **scalar**. On diagrams, vectors are represented by arrows.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first vector to the tip of the last vector. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude $V$ making an angle $\theta$ with the $+x$ axis has components

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta. \qquad (3\text{–}3)$$

Given the components, we can find a vector's magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \qquad (3\text{–}4)$$

**Projectile motion** is the motion of an object in the air near the Earth's surface under the effect of gravity alone. It can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $\vec{\mathbf{g}}$, just as for an object falling vertically under the action of gravity.

The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.

# Questions

**1.** One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are their velocities equal? Explain.

**2.** Can you conclude that a car is not accelerating if its speedometer indicates a steady 60 km/h? Explain.

**3.** Give several examples of an object's motion in which a great distance is traveled but the displacement is zero.

**4.** Can the displacement vector for a particle moving in two dimensions be longer than the length of path traveled by the particle over the same time interval? Can it be less? Discuss.

**5.** During baseball practice, a player hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball? Explain.

**6.** If $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, is $V$ necessarily greater than $V_1$ and/or $V_2$? Discuss.

**7.** Two vectors have length $V_1 = 3.5\text{ km}$ and $V_2 = 4.0\text{ km}$. What are the maximum and minimum magnitudes of their vector sum?

**8.** Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?

**9.** Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?

**10.** Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?

**11.** How could you determine the speed a slingshot imparts to a rock, using only a meter stick, a rock, and the slingshot?

**12.** In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?

**13.** It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bullet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.

**14.** You are on the street trying to hit a friend in his dorm window with a water balloon. He has a similar idea and is aiming at you with *his* water balloon. You aim straight at each other and throw at the same instant. Do the water balloons hit each other? Explain why or why not.

**15.** A projectile is launched at an upward angle of 30° to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch, ignoring air resistance? Explain.

**16.** A projectile has the least speed at what point in its path?

**17.** Two cannonballs, A and B, are fired from the ground with identical initial speeds, but with $\theta_A$ larger than $\theta_B$. (*a*) Which cannonball reaches a higher elevation? (*b*) Which stays longer in the air? (*c*) Which travels farther? Explain.

**18.** A person sitting in an enclosed train car, moving at constant velocity, throws a ball straight up into the air in her reference frame. (*a*) Where does the ball land? What is your answer if the car (*b*) accelerates, (*c*) decelerates, (*d*) rounds a curve, (*e*) moves with constant velocity but is open to the air?

**19.** If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

**20.** Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first? Explain.

**21.** If you stand motionless under an umbrella in a rainstorm where the drops fall vertically, you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

# MisConceptual Questions

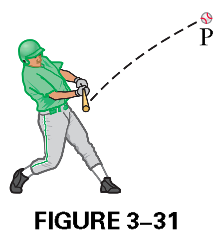
**1.** You are adding vectors of length 20 and 40 units. Which of the following choices is a possible resultant magnitude?
(*a*) 0.
(*b*) 18.
(*c*) 37.
(*d*) 64.
(*e*) 100.

**2.** The magnitude of a component of a vector must be
(*a*) less than or equal to the magnitude of the vector.
(*b*) equal to the magnitude of the vector.
(*c*) greater than or equal to the magnitude of the vector.
(*d*) less than, equal to, or greater than the magnitude of the vector.

**3.** You are in the middle of a large field. You walk in a straight line for 100 m, then turn left and walk 100 m more in a straight line before stopping. When you stop, you are 100 m from your starting point. By how many degrees did you turn?
(*a*) 90°.
(*b*) 120°.
(*c*) 30°.
(*d*) 180°.
(*e*) This is impossible. You cannot walk 200 m and be only 100 m away from where you started.

**4.** A bullet fired from a rifle begins to fall
(*a*) as soon as it leaves the barrel.
(*b*) after air friction reduces its speed.
(*c*) not at all if air resistance is ignored.

**5.** A baseball player hits a ball that soars high into the air. After the ball has left the bat, and while it is traveling upward (at point P in Fig. 3–31), what is the direction of acceleration? Ignore air resistance.




(*a*) (*b*) (*c*)

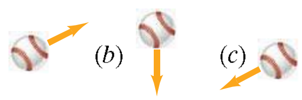


**FIGURE 3–31** MisConceptual Question 5.

**6.** One ball is dropped vertically from a window. At the same instant, a second ball is thrown horizontally from the same window. Which ball has the greater speed at ground level?
(*a*) The dropped ball.
(*b*) The thrown ball.
(*c*) Neither—they both have the same speed on impact.
(*d*) It depends on how hard the ball was thrown.

7. You are riding in an enclosed train car moving at 90 km/h. If you throw a baseball straight up, where will the baseball land?
(*a*) In front of you.
(*b*) Behind you.
(*c*) In your hand.
(*d*) Can't decide from the given information.

8. Which of the three kicks in Fig. 3–32 is in the air for the longest time? They all reach the same maximum height *h*. Ignore air resistance.
(*a*), (*b*), (*c*), or (*d*) all the same time.

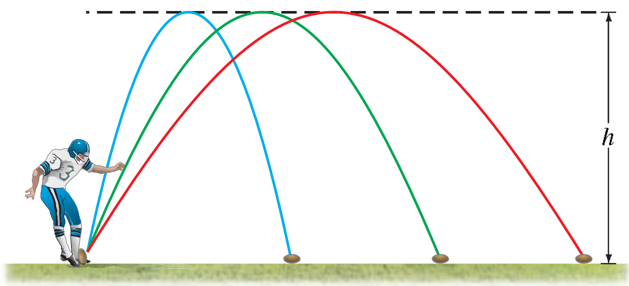



**FIGURE 3–32** MisConceptual Question 8.

9. A baseball is hit high and far. Which of the following statements is true? At the highest point,
(*a*) the magnitude of the acceleration is zero.
(*b*) the magnitude of the velocity is zero.
(*c*) the magnitude of the velocity is the slowest.
(*d*) more than one of the above is true.
(*e*) none of the above are true.

10. A hunter is aiming horizontally at a monkey who is sitting in a tree. The monkey is so terrified when it sees the gun that it falls off the tree. At that very instant, the hunter pulls the trigger. What will happen?
(*a*) The bullet will miss the monkey because the monkey falls down while the bullet speeds straight forward.
(*b*) The bullet will hit the monkey because both the monkey and the bullet are falling downward at the same rate due to gravity.
(*c*) The bullet will miss the monkey because although both the monkey and the bullet are falling downward due to gravity, the monkey is falling faster.
(*d*) It depends on how far the hunter is from the monkey.

11. Which statements are *not* valid for a projectile? Take up as positive.
(*a*) The projectile has the same *x* velocity at any point on its path.
(*b*) The acceleration of the projectile is positive and decreasing when the projectile is moving upwards, zero at the top, and increasingly negative as the projectile descends.
(*c*) The acceleration of the projectile is a constant negative value.
(*d*) The *y* component of the velocity of the projectile is zero at the highest point of the projectile's path.
(*e*) The velocity at the highest point is zero.

12. A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is
(*a*) less than 20 m/s.
(*b*) exactly 20 m/s.
(*c*) more than 20 m/s.

For assigned homework and other learning materials, go to the MasteringPhysics website.

# Problems

## 3–2 to 3–4 Vector Addition

1. (I) A car is driven 225 km west and then 98 km southwest (45°). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.

2. (I) A delivery truck travels 21 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.

3. (I) If $V_x = 9.80$ units and $V_y = -6.40$ units, determine the magnitude and direction of $\vec{\mathbf{V}}$.

4. (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m, 36° north of east; (2) 18 m, 37° east of north; and (3) 26 m, 33° west of south.

5. (II) $\vec{\mathbf{V}}$ is a vector 24.8 units in magnitude and points at an angle of 23.4° above the negative *x* axis. (*a*) Sketch this vector. (*b*) Calculate $V_x$ and $V_y$. (*c*) Use $V_x$ and $V_y$ to obtain (again) the magnitude and direction of $\vec{\mathbf{V}}$. [*Note*: Part (*c*) is a good way to check if you've resolved your vector correctly.]

6. (II) Vector $\vec{\mathbf{V}}_1$ is 6.6 units long and points along the negative *x* axis. Vector $\vec{\mathbf{V}}_2$ is 8.5 units long and points at +55° to the positive *x* axis. (*a*) What are the *x* and *y* components of each vector? (*b*) Determine the sum $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$ (magnitude and angle).

7. (II) Figure 3–33 shows two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, whose magnitudes are $A = 6.8$ units and $B = 5.5$ units. Determine $\vec{\mathbf{C}}$ if (*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, (*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, (c) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}}$. Give the magnitude and direction for each.

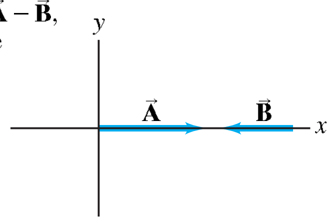



**FIGURE 3–33** Problem 7.

8. (II) An airplane is traveling 835 km/h in a direction 41.5° west of north (Fig. 3–34).
(*a*) Find the components of the velocity vector in the northerly and westerly directions. (*b*) How far north and how far west has the plane traveled after 1.75 h?

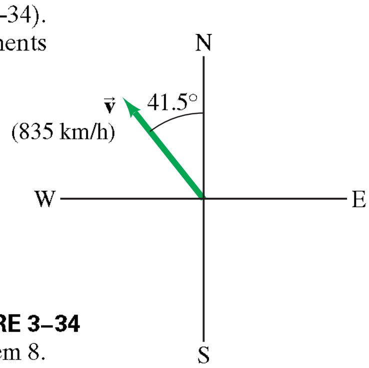



**FIGURE 3–34** Problem 8.

**9.** (II) Three vectors are shown in Fig. 3–35. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with the $+x$ axis.

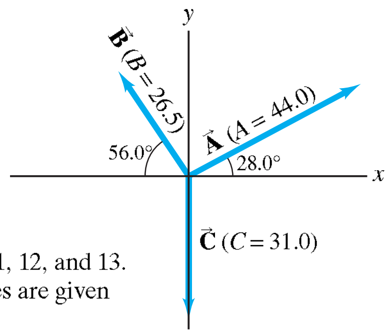



**FIGURE 3–35** Problems 9, 10, 11, 12, and 13. Vector magnitudes are given in arbitrary units.

**10.** (II) (*a*) Given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Fig. 3–35, determine $\vec{\mathbf{B}} - \vec{\mathbf{A}}$. (*b*) Determine $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ without using your answer in (*a*). Then compare your results and see if they are opposite.

**11.** (II) Determine the vector $\vec{\mathbf{A}} - \vec{\mathbf{C}}$, given the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ in Fig. 3–35.

**12.** (II) For the vectors shown in Fig. 3–35, determine (*a*) $\vec{\mathbf{B}} - 3\vec{\mathbf{A}}$, (*b*) $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}$.

**13.** (II) For the vectors given in Fig. 3–35, determine (*a*) $\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}}$, (*b*) $\vec{\mathbf{A}} + \vec{\mathbf{B}} - \vec{\mathbf{C}}$, and (*c*) $\vec{\mathbf{C}} - \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

**14.** (II) Suppose a vector $\vec{\mathbf{V}}$ makes an angle $\phi$ with respect to the $y$ axis. What could be the $x$ and $y$ components of the vector $\vec{\mathbf{V}}$?

**15.** (II) The summit of a mountain, 2450 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction 38.4° west of north. What are the components of the displacement vector from camp to summit? What is its magnitude? Choose the $x$ axis east, $y$ axis north, and $z$ axis up.

**16.** (III) You are given a vector in the $xy$ plane that has a magnitude of 90.0 units and a $y$ component of $-65.0$ units. (*a*) What are the two possibilities for its $x$ component? (*b*) Assuming the $x$ component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the $-x$ direction.

### 3–5 and 3–6 Projectile Motion (neglect air resistance)

**17.** (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?

**18.** (I) A diver running 2.5 m/s dives out horizontally from the edge of a vertical cliff and 3.0 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

**19.** (II) Estimate by what factor a person can jump farther on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.

**20.** (II) A ball is thrown horizontally from the roof of a building 7.5 m tall and lands 9.5 m from the base. What was the ball's initial speed?

**21.** (II) A ball thrown horizontally at 12.2 m/s from the roof of a building lands 21.0 m from the base of the building. How high is the building?

**22.** (II) A football is kicked at ground level with a speed of 18.0 m/s at an angle of 31.0° to the horizontal. How much later does it hit the ground?

**23.** (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.5 m away (Fig. 3–36)? Why are there two different angles? Sketch the two trajectories.

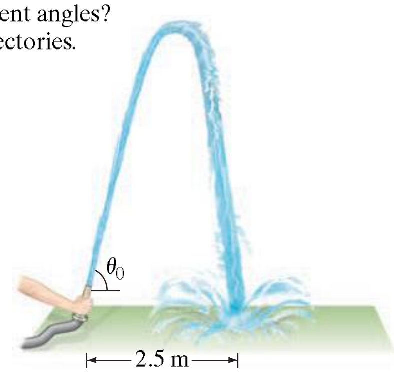



**FIGURE 3–36** Problem 23.

**24.** (II) You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 4.0 s for the dart to land back at the barrel. What is the maximum horizontal range of your gun?

**25.** (II) A grasshopper hops along a level road. On each hop, the grasshopper launches itself at angle $\theta_0 = 45°$ and achieves a range $R = 0.80$ m. What is the average horizontal speed of the grasshopper as it hops along the road? Assume that the time spent on the ground between hops is negligible.

**26.** (II) Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 4.0 m/s and enjoys a free fall until she is 150 m above the valley floor, at which time she opens her parachute (Fig. 3–37). (*a*) How long is the jumper in free fall? Ignore air resistance. (*b*) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?

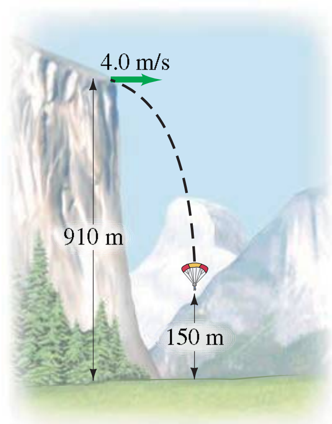



**FIGURE 3–37** Problem 26.

**27.** (II) A projectile is fired with an initial speed of 36.6 m/s at an angle of 42.2° above the horizontal on a long flat firing range. Determine (*a*) the maximum height reached by the projectile, (*b*) the total time in the air, (*c*) the total horizontal distance covered (that is, the range), and (*d*) the speed of the projectile 1.50 s after firing.

**28.** (II) An athlete performing a long jump leaves the ground at a 27.0° angle and lands 7.80 m away. (*a*) What was the takeoff speed? (*b*) If this speed were increased by just 5.0%, how much longer would the jump be?

**29.** (II) A baseball is hit with a speed of 27.0 m/s at an angle of 45.0°. It lands on the flat roof of a 13.0-m-tall nearby building. If the ball was hit when it was 1.0 m above the ground, what horizontal distance does it travel before it lands on the building?

**30.** (II) A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 m below. If the plane is traveling horizontally with a speed of 250 km/h (69.4 m/s), how far in advance of the recipients (horizontal distance) must the goods be dropped (Fig. 3–38)?

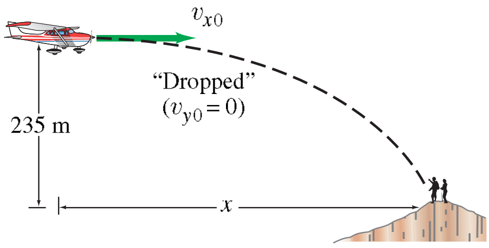



**FIGURE 3–38** Problem 30.

**31.** (III) Suppose the rescue plane of Problem 30 releases the supplies a horizontal distance of 425 m in advance of the mountain climbers. What vertical velocity (up or down) should the supplies be given so that they arrive precisely at the climbers' position (Fig. 3–39)? With what speed do the supplies land?

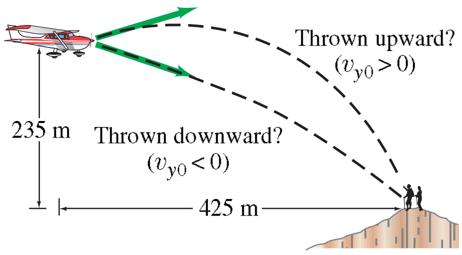



**FIGURE 3–39** Problem 31.

**32.** (III) Show that the time required for a projectile to reach its highest point is equal to the time for it to return to its original height if air resistance is neglible.

**33.** (III) Suppose the kick in Example 3–6 is attempted 36.0 m from the goalposts, whose crossbar is 3.05 m above the ground. If the football is directed perfectly between the goalposts, will it pass over the bar and be a field goal? Show why or why not. If not, from what horizontal distance must this kick be made if it is to score?

**34.** (III) Revisit Example 3–7, and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 3–40) and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

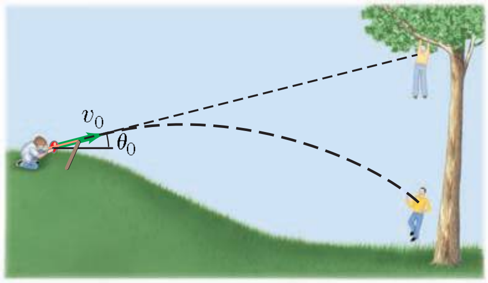



**FIGURE 3–40** Problem 34.

**35.** (III) A stunt driver wants to make his car jump over 8 cars parked side by side below a horizontal ramp (Fig. 3–41). (*a*) With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5 m above the cars and the horizontal distance he must clear is 22 m. (*b*) If the ramp is now tilted upward, so that "takeoff angle" is 7.0° above the horizontal, what is the new minimum speed?

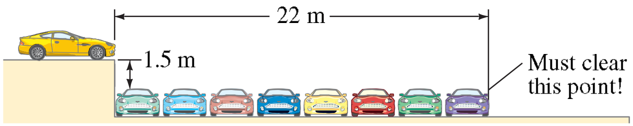



**FIGURE 3–41** Problem 35.

## 3–8 Relative Velocity

**36.** (I) Huck Finn walks at a speed of 0.70 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The heavy raft is traveling down the Mississippi River at a speed of 1.50 m/s relative to the river bank (Fig. 3–42). What is Huck's velocity (speed and direction) relative to the river bank?

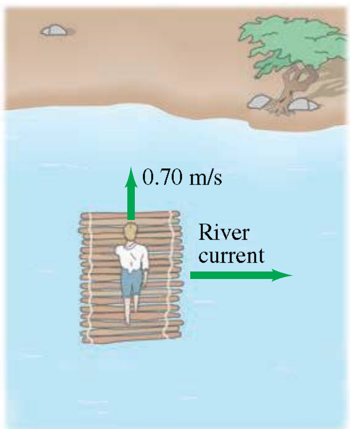



**FIGURE 3–42** Problem 36.

**37.** (II) Two planes approach each other head-on. Each has a speed of 780 km/h, and they spot each other when they are initially 10.0 km apart. How much time do the pilots have to take evasive action?

**38.** (II) A passenger on a boat moving at 1.70 m/s on a still lake walks up a flight of stairs at a speed of 0.60 m/s, Fig. 3–43. The stairs are angled at 45° pointing in the direction of motion as shown. What is the velocity of the passenger relative to the water?

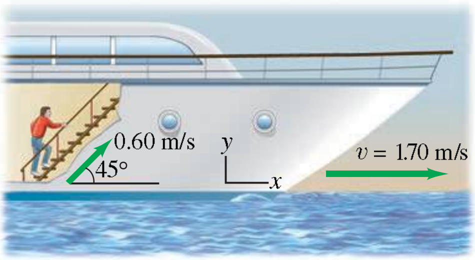



**FIGURE 3–43** Problem 38.

**39.** (II) A person in the passenger basket of a hot-air balloon throws a ball horizontally outward from the basket with speed 10.0 m/s (Fig. 3–44). What initial velocity (magnitude and direction) does the ball have relative to a person standing on the ground (*a*) if the hot-air balloon is rising at 3.0 m/s relative to the ground during this throw, (*b*) if the hot-air balloon is descending at 3.0 m/s relative to the ground?

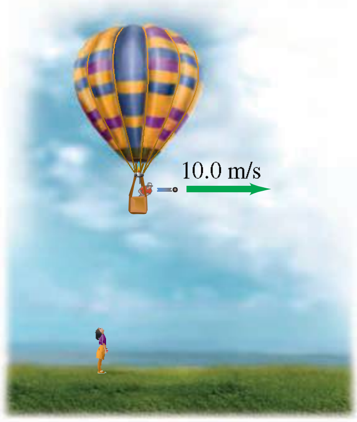



**FIGURE 3–44** Problem 39.

**40.** (II) An airplane is heading due south at a speed of 688 km/h. If a wind begins blowing from the southwest at a speed of 90.0 km/h (average), calculate (*a*) the velocity (magnitude and direction) of the plane, relative to the ground, and (*b*) how far from its intended position it will be after 11.0 min if the pilot takes no corrective action. [*Hint*: First draw a diagram.]

**41.** (II) In what direction should the pilot aim the plane in Problem 40 so that it will fly due south?

**42.** (II) A swimmer is capable of swimming 0.60 m/s in still water. (*a*) If she aims her body directly across a 45-m-wide river whose current is 0.50 m/s, how far downstream (from a point opposite her starting point) will she land? (*b*) How long will it take her to reach the other side?

**43.** (II) A boat, whose speed in still water is 2.50 m/s, must cross a 285-m-wide river and arrive at a point 118 m upstream from where it starts (Fig. 3–45). To do so, the pilot must head the boat at a 45.0° upstream angle. What is the speed of the river's current?

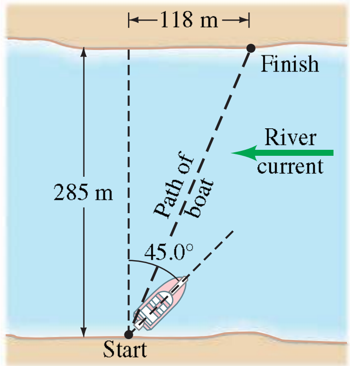



**FIGURE 3–45** Problem 43.

**44.** (II) A child, who is 45 m from the bank of a river, is being carried helplessly downstream by the river's swift current of 1.0 m/s. As the child passes a lifeguard on the river's bank, the lifeguard starts swimming in a straight line (Fig. 3–46) until she reaches the child at a point downstream. If the lifeguard can swim at a speed of 2.0 m/s relative to the water, how long does it take her to reach the child? How far downstream does the lifeguard intercept the child?

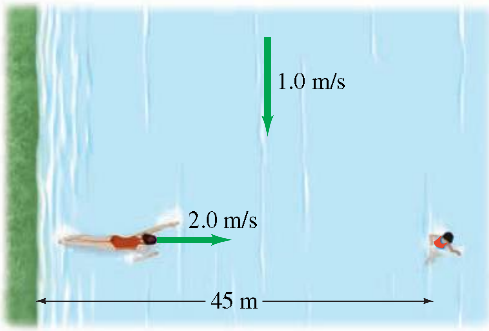



**FIGURE 3–46** Problem 44.

**45.** (III) Two cars approach a street corner at right angles to each other (Fig. 3–47). Car 1 travels at a speed relative to Earth $v_{1E} = 35$ km/h, and car 2 at $v_{2E} = 55$ km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

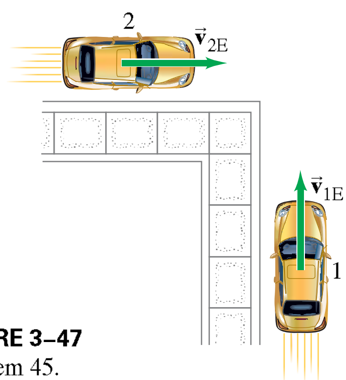



**FIGURE 3–47** Problem 45.

## General Problems

46. Two vectors, $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$, add to a resultant $\vec{\mathbf{V}}_R = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$. Describe $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ if (*a*) $V_R = V_1 + V_2$, (*b*) $V_R^2 = V_1^2 + V_2^2$, (*c*) $V_1 + V_2 = V_1 - V_2$.

47. On mountainous downhill roads, escape routes are sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of 26°, calculate the horizontal and vertical components of the acceleration of a truck that slowed from 110 km/h to rest in 7.0 s. See Fig. 3–48.

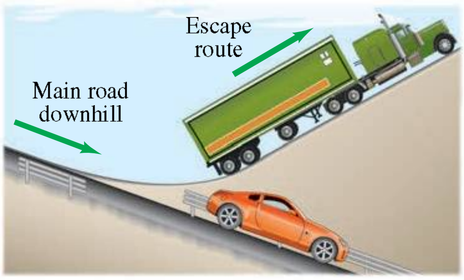



**FIGURE 3–48** Problem 47.

48. A light plane is headed due south with a speed relative to still air of 185 km/h. After 1.00 h, the pilot notices that they have covered only 135 km and their direction is not south but 15.0° east of south. What is the wind velocity?

49. Romeo is throwing pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 8.5 m from the base of the wall (Fig. 3–49). How fast are the pebbles going when they hit her window?

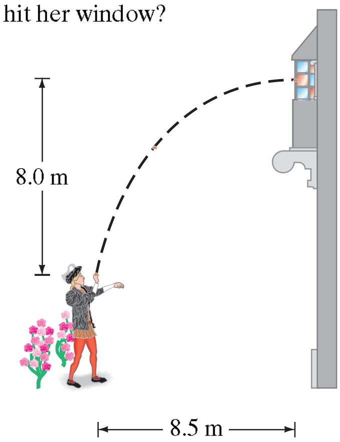



**FIGURE 3–49** Problem 49.

50. *Apollo* astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 32 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)

51. (*a*) A long jumper leaves the ground at 45° above the horizontal and lands 8.0 m away. What is her "takeoff" speed $v_0$? (*b*) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m vertically below. If she long jumps from the edge of the left bank at 45° with the speed calculated in (*a*), how long, or short, of the opposite bank will she land (Fig. 3–50)?

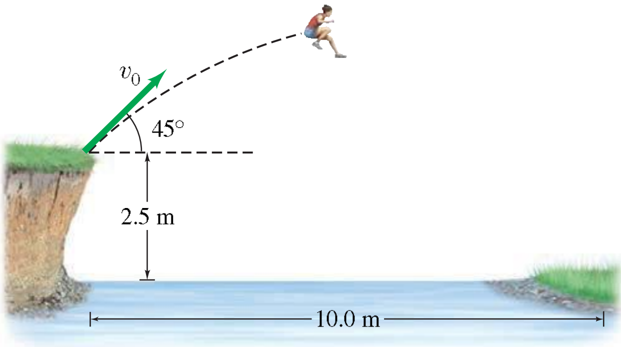



**FIGURE 3–50** Problem 51.

52. A projectile is shot from the edge of a cliff 115 m above ground level with an initial speed of 65.0 m/s at an angle of 35.0° with the horizontal, as shown in Fig. 3–51. (*a*) Determine the time taken by the projectile to hit point P at ground level. (*b*) Determine the distance $X$ of point P from the base of the vertical cliff. At the instant just before the projectile hits point P, find (*c*) the horizontal and the vertical components of its velocity, (*d*) the magnitude of the velocity, and (*e*) the angle made by the velocity vector with the horizontal. (*f*) Find the maximum height above the cliff top reached by the projectile.

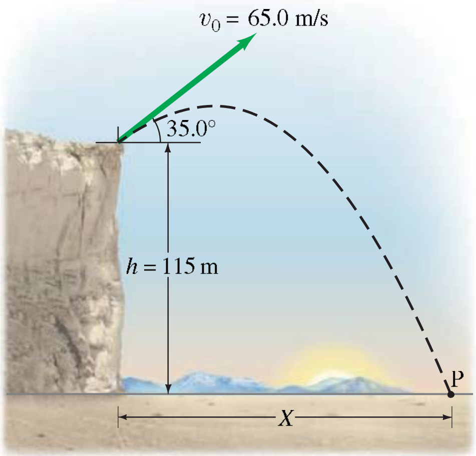



**FIGURE 3–51** Problem 52.

**53.** Raindrops make an angle $\theta$ with the vertical when viewed through a moving train window (Fig. 3–52). If the speed of the train is $v_T$, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?


**FIGURE 3–52** Problem 53.

**54.** A hunter aims directly at a target (on the same level) 38.0 m away. (*a*) If the arrow leaves the bow at a speed of 23.1 m/s, by how much will it miss the target? (*b*) At what angle should the bow be aimed so the target will be hit?

**55.** The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 3–53. What minimum pushoff speed is necessary to clear the rocks? How long are they in the air?

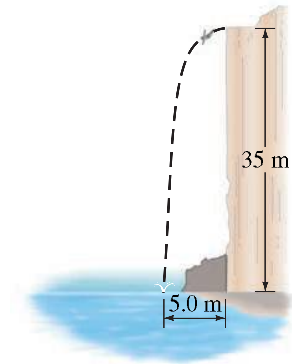



**FIGURE 3–53** Problem 55.

**56.** When Babe Ruth hit a homer over the 8.0-m-high right-field fence 98 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a 36° angle with the ground.

**57.** At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is "launched" from a height of 2.50 m? Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 3–54.

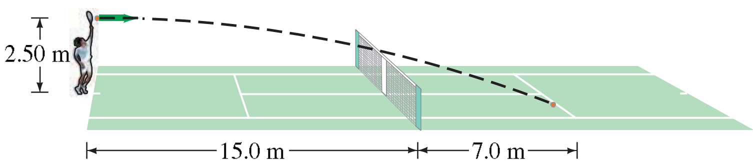



**FIGURE 3–54** Problem 57.

**58.** Spymaster Chris, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact's open car which is traveling 156 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 3–55)?

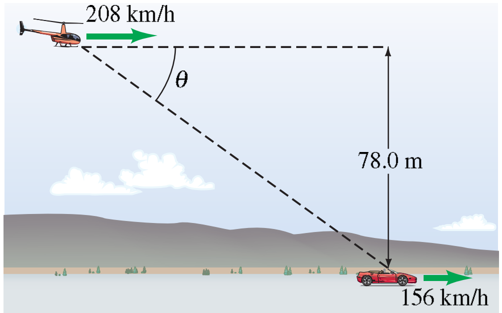



**FIGURE 3–55** Problem 58.

**59.** A boat can travel 2.20 m/s in still water. (*a*) If the boat points directly across a stream whose current is 1.20 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) What will be the position of the boat, relative to its point of origin, after 3.00 s?

**60.** A projectile is launched from ground level to the top of a cliff which is 195 m away and 135 m high (see Fig. 3–56). If the projectile lands on top of the cliff 6.6 s after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

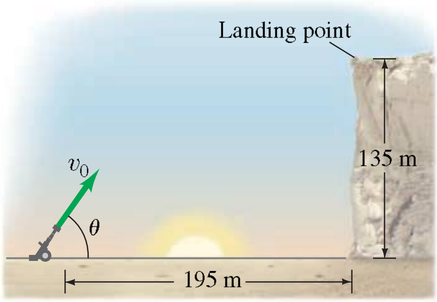



**FIGURE 3–56** Problem 60.

**61.** A basketball is shot from an initial height of 2.40 m (Fig. 3–57) with an initial speed $v_0 = 12\text{ m/s}$ directed at an angle $\theta_0 = 35°$ above the horizontal. (*a*) How far from the basket was the player if he made a basket? (*b*) At what angle to the horizontal did the ball enter the basket?

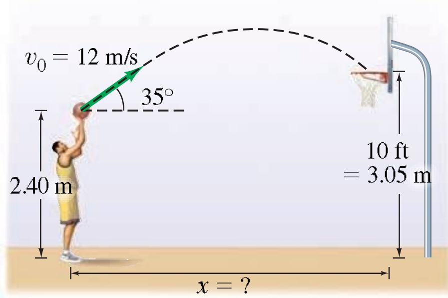



**FIGURE 3–57** Problem 61.

**62.** A rock is kicked horizontally at 15 m/s from a hill with a 45° slope (Fig. 3–58). How long does it take for the rock to hit the ground?

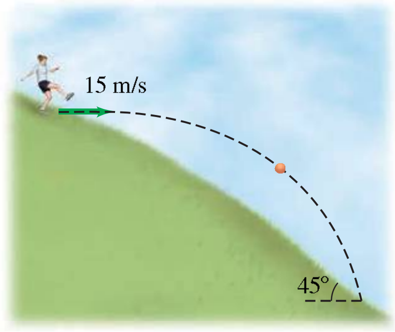



**FIGURE 3–58** Problem 62.

**63.** A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal. (*a*) What are the horizontal and vertical components of the initial velocity? (*b*) If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?

**64** If a baseball pitch leaves the pitcher's hand horizontally at a velocity of 150 km/h, by what % will the pull of gravity change the magnitude of the velocity when the ball reaches the batter, 18 m away? For this estimate, ignore air resistance and spin on the ball.

# Search and Learn

**1.** Here is something to try at a sporting event. Show that the maximum height $h$ attained by an object projected into the air, such as a baseball, football, or soccer ball, is approximately given by

$$h \approx 1.2 t^2 \text{ m},$$

where $t$ is the total time of flight for the object in seconds. Assume that the object returns to the same level as that from which it was launched, as in Fig. 3–59. For example, if you count to find that a baseball was in the air for $t = 5.0$ s, the maximum height attained was $h = 1.2 \times (5.0)^2 = 30$ m. The fun of this relation is that $h$ can be determined without knowledge of the launch speed $v_0$ or launch angle $\theta_0$. Why is that exactly? See Section 3–6.

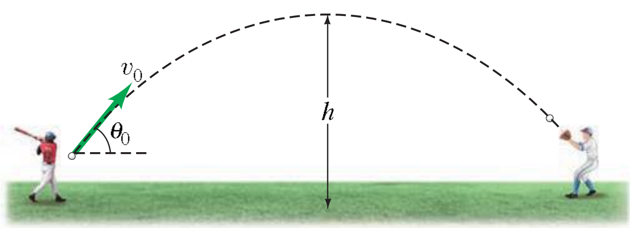



**FIGURE 3–59** Search and Learn 1.

**2.** The initial angle of projectile A is 30°, while that of projectile B is 60°. Both have the same level horizontal range. How do the initial velocities and flight times (elapsed time from launch until landing) compare for A and B?

**3.** You are driving south on a highway at 12 m/s (approximately 25 mi/h) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of 7.0° to the horizontal. Estimate the speed of the vertically falling snowflakes relative to the ground. [*Hint*: Construct a relative velocity diagram similar to Fig. 3–29 or 3–30. Be careful about which angle is the angle given.]

## ANSWERS TO EXERCISES

**A:** $3.0\sqrt{2} \approx 4.2$ units.
**B:** (*a*).
**C:** They hit at the same time.
**D:** (i) Nowhere; (ii) at the highest point; (iii) nowhere.
**E:** (*d*). It provides the initial velocity of the box.
**F:** (*a*) $v = v_{x0} = 16.0$ m/s, horizontal; (*b*) 9.80 m/s$^2$ down.