Πίνακες

Monday, October 12, 2020 11:44 PM

$$\begin{pmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} & \cancel{a_{14}} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$

3 γραμμές
4 στήλες

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix} \quad A \in R^{m\times n}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad A \in R^{3\times 3}$$

a_{11}, a_{22}, a_{33} διαγώνια στοιχεία

a_{ii} $\quad i=j$

$A = B$, $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$

$a_{ij} = b_{ij} \quad \forall\ i \neq j$

$a_{11} = b_{11}$

$a_{12} = b_{12}$

$a_{21} = b_{11}$

$\vdots$

$A \in^{2\times 3} = B \in^{2\times 3}$

$ax = b$

1×4

Πίνακας γραμμή $\quad A \in R^{1\times 4}$

$$A = (1\ -2\ 0)$$

Πίνακας στήλη $\quad A \in R^{m\times 1}$

$$A = \begin{pmatrix} -9 \\ 2 \\ 10 \end{pmatrix}$$

Ανάστροφος $\quad A^T$

$$A = \begin{pmatrix} 1 & -9 \\ 2 & 3 \\ 10 & -6 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 2 & 10 \\ -9 & 3 & -6 \end{pmatrix}$$

$(A^T)^T = A \qquad A \in R^{3\times 2} \qquad A^T \in R^{2\times 3}$

$A_{4\times 2}$ ← στήλες

↑ γραμμές

a_{ij}

↑ γραμμές ↑ στήλες

Τετραγωνικός πίνακας

ίσες σε πλήθος γραμμές με στήλες

Συμμετρικός πίνακας

Συμμετρικοί πίνακες

$$\begin{pmatrix} a_{11} & a_{12} & \\ a_{21} & a_{22} & \\ & & a_{33} \end{pmatrix}$$

$a_{ij} = a_{ji}$ $\quad i \neq j$

$a_{35} = a_{53}$

2

Monday, October 19, 2020
11:36 AM

$$\begin{pmatrix} 5 & 2_{12} & 3_{13} & 3 \\ 2_{21} & 7 & 5 & 3 \\ 3_{31} & 5 & 9 & 6 \\ 3 & 3 & 6 & 8 \end{pmatrix}$$

$a_{ij} = a_{ji}$

$a_{12} = a_{21}$

$A = A^T$

$$\begin{pmatrix} \frac{5}{a_{11}} & \frac{2}{a_{12}} & \overline{a_{13}} & \overline{a_{14}} \\ \frac{2}{a_{21}} & \frac{7}{a_{22}} & \overline{a_{23}} & \overline{a_{24}} \\ \overline{a_{31}} & \overline{a_{32}} & \overline{a_{33}} & \overline{a_{34}} \\ \overline{a_{41}} & \overline{a_{42}} & \overline{a_{43}} & \overline{a_{44}} \end{pmatrix}$$

$a_{ij} = a_{ji}$

$a_{21} = a_{12}$

$a_{75} = a_{57}$

$a_{ij} = -a_{ji}$ αντισυμμετρικοί

$$A = \begin{pmatrix} 1 & 9 & -4 \\ -9 & 3 & -6 \\ 4 & 6 & 5 \end{pmatrix}$$

Διαγώνιον

$a_{ij} = 0 \quad i \neq j$

$a_{ij} = a \in \mathbb{R} \quad i = j$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Άνω Τριγωνικόν

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$$

$a_{ij} = 0,$

$i > j$

$$\begin{pmatrix} \ddots & 0 & 0 & a \\ 0 & \ddots & & \\ \vdots & & \ddots & \\ 0 & \cdots & & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \cdots & 0 & \cdots & a \\ 0 & \cdots & a & \cdots & 0 \\ 0 & \cdots & & 0 & 0 \end{pmatrix}$$

Κάτω Τριγωνικόν

$$\begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \end{pmatrix}$$

$a_{ij} = 0 \qquad i < j$

$$\begin{pmatrix} & & 0 \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Μοναδιαίοι πίνακες

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3 \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Πρόσθεση Πινάκων

$A, B \in \mathbb{R}^{m \times n}$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & & \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}$$

$C = A + B \in \mathbb{R}^{m \times n}$

$$C = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}$$

$$C = \lambda \cdot A \qquad \lambda \in \mathbb{R} \qquad c_{ij} = \lambda a_{ij}$$

$$C = \lambda \cdot A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \dots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \dots & \lambda a_{2n} \\ \vdots & & & \\ \lambda a_{m1} & \lambda a_{m2} & \dots & \lambda a_{mn} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 5 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1+5 & 0+1 & -1+0 \\ 2+(-2) & 1+0 & 0+1 \\ -1+1 & 0+2 & 1+1 \end{pmatrix} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 12 & 1 & 5 \end{pmatrix} + \begin{pmatrix} 9 & 1 & -3 \\ -2 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 1 & -4 \\ 10 & -3 & 6 \end{pmatrix}$$

$$2 \cdot \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 & 2\cdot 0 & 2\cdot(-1) \\ 2(-1) & 2\cdot 1 & 2\cdot 5 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ -2 & 2 & 10 \end{pmatrix}$$

$$a \cdot \begin{pmatrix} 0 & 1 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 0 & a \\ a & -3a \end{pmatrix}$$

1) $\lambda(A+B) = \lambda A + \lambda B \qquad \lambda \in \mathbb{R}$

2) $(\lambda+\mu)A = \lambda A + \mu A \qquad \lambda, \mu \in \mathbb{R}$

3) $(\lambda\mu)A = \lambda(\mu A)$ $\lambda, \mu \in \mathbb{R}$

4) $1 \cdot A = A$

Πολλαπλασιασμός Πινάκων

$$\underset{\_\times\_}{=} A_{\_\times n} \quad B_{n\times \_}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{pmatrix} =$$

$$= a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + \dots + a_{1n}b_{n1} =$$

$$= \sum_{k=1}^{n} a_{1k} \cdot b_{k1} \quad (=) \quad c_{11}$$ στοιχείο σε έναν πίνακα C

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \dots + x_n$$

$C_{m\times 1}$ $\quad$ $C_{m\times 1}$

$$C = A \cdot B = A_{m\times n} \cdot B_{n\times 1}$$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & & \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + \dots + a_{1n}b_{n1} \\ \dots + a_{22}b_{21} + a_{23}b_{31} + \dots + a_{2n}b_{n1} \end{pmatrix} \quad \begin{pmatrix} \sum_{k=1}^{n} a_{1k}b_{k1} \\ \sum a_{2k}b_{k1} \end{pmatrix}$$

$$\begin{pmatrix} 1 \to a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}+\dots+a_{1n}b_{n1} \\ 1 \to a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31}+\dots+a_{2n}b_{n1} \\ 1 \to a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31}+\dots+a_{3n}b_{n1} \\ \vdots \\ m1 \to a_{m1}b_{11}+a_{m2}b_{21}+a_{m3}b_{31}+\dots+a_{mn}b_{n1} \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{n} a_{1k}b_{k1} \\ \sum_{k=1}^{n} a_{2k}b_{k1} \\ \sum_{k=1}^{n} a_{3k}b_{k1} \\ \vdots \\ \sum_{k=1}^{n} a_{mk}b_{k1} \end{pmatrix}$$

$$m\times r = A_{m\times n}\cdot B_{n\times r}$$

$$C_{ij} = a_{i1}b_{1j}+a_{i2}b_{2j}+a_{i3}b_{3j}+\dots+a_{in}b_{nj}$$

$$i = 1 \to m \qquad j = 1 \to r$$

$$C_{3\times3} = A_{3\times3}\, B_{3\times3}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

$c_{11} = a_{11}\cdot b_{11}+a_{12}b_{21}+a_{13}b_{31}$

$12 = a_{11}\cdot b_{12}+a_{12}\cdot b_{22}+a_{13}b_{32}$

$13 = a_{11}\cdot b_{13}+a_{12}\cdot b_{23}+a_{13}b_{33}$

$21 = a_{21}\cdot b_{11}+a_{22}b_{21}+a_{23}b_{31}$

$22 = a_{21}\cdot b_{12}+a_{22}b_{22}+a_{23}b_{32}$

$23 = a_{21}\cdot b_{13}+a_{22}b_{23}+a_{23}\cdot b_{33}$

$c_{31} = a_{31}\cdot b_{11}+a_{32}\cdot b_{21}+a_{33}b_{31}$

$c_{32} = a_{31}\cdot b_{12}+a_{32}\cdot b_{22}+a_{33}\cdot b_{32}$

$c_{33} = a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33}$

$\cancel{B\cdot A}$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ -2 & -1 & 0 \end{pmatrix} \qquad B_{3\times2} = \begin{pmatrix} 5 & 1 \\ -2 & 4 \\ 1 & 2 \end{pmatrix}$$

$C = A\cdot B$

3×2

$$\underset{3\times 3}{A} = \begin{pmatrix} \cdot & \cdot & \cdot \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \underset{3\times 2}{B} = \begin{pmatrix} \cdot & \cdot \\ -2 & 0 \\ 1 & 2 \end{pmatrix} \qquad \underset{3\times 2}{C} = A \cdot B$$

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 8 & 2 \\ -4 & 1 \end{pmatrix}$$

$c_{11} = a_{11} \cdot b_{11} + a_{12} b_{21} + a_{13} b_{31} =$
$= 1 \cdot 5 + 0 \cdot (-2) + (-1) \cdot 1 = 4$

$c_{12} = 1 \cdot 1 + 0 \cdot 0 + (-1) \cdot 2 = -1$

$c_{21} = 2 \cdot 5 + 1 \cdot (-2) + 0 \cdot 1 = 8$

$c_{22} = 2 \cdot 1 + 1 \cdot 0 + 0 \cdot 2$
$= 2$

$c_{31} = (-1) \cdot 5 + 0 \cdot (-2) + 1 \cdot 1$
$= -4$

$c_{32} = (-1) \cdot 1 + 0 \cdot 0 + 1 \cdot 2$
$= 1$

$$\overset{A}{\begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}} \overset{B}{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}} = \overset{C}{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}}$$

$A_{3\times 3} \quad B_{3\times 3} \longrightarrow C_{3\times 3}$

$c_{11} = a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} =$
$1 \cdot 1 + (-1) \cdot 2 + 1 \cdot 1 = 1 - 2 + 1 = 0$

$c_{12} = a_{11} \cdot b_{12} + a_{12} \cdot b_{22} + a_{13} \cdot b_{32} =$
$= 1 \cdot 2 + (-1) \cdot 4 + 1 \cdot 2 = 2 - 4 + 2 = 0$

$c_{13} = 1 \cdot 3 + (-1) \cdot 6 + 1 \cdot 3 = 3 - 6 + 3 = 0$

$c_{21} = -3 \cdot 1 + 2 \cdot 2 + (-1) \cdot 1 = -3 + 4 - 1 = 0$

$c_{22} = -3 \cdot 2 + 2 \cdot 4 + (-1) \cdot 2 = -6 + 8 - 2 = 0$

$c_{23} = -3 \cdot 3 + 2 \cdot 6 + (-1) \cdot 3 = -9 + 12 - 3 = 0$

$c_{31} = -2 \cdot 1 + 1 \cdot 2 + 0 \cdot 1 = 0$

$c_{32} = 0 \qquad c_{33} = 0$

$$A \cdot B = C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$B \cdot A =$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

$$A_{m\times n} \, B_{k\times r} = C_{m\times r}$$

$$n = k$$

$$A_{2\times 3} \, B_{3\times 1} = C_{2\times 1}$$

$11 = 1 \cdot 1 + 2 \cdot (-3) + 3(-2) = 1 - 6 - 6 = -11$

$12 = 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 = -1 + 4 + 3 = 6$

$13 = -1$

$21 = -22$

$22 = 12 = 2 \cdot (-1) + 4 \cdot 2 + 6 \cdot 1 = -2 + 8 + 6 = 12$

$23 = -2$

$31 = -11$

$32 = 6$

$c_{33} = -1$

$$A \cdot B \neq BA$$

$A \cdot I = A$; $IA = A$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A \cdot I = \underset{2\times 3}{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}} \underset{3\times 3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} = \underset{2\times 3}{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix}}$$

$11 = 1 \cdot 1 + 0 \cdot 0 + (-1) \cdot 0 = 1$

$c_{12} = 1 \cdot 0 + 0 \cdot 1 + (-1) \cdot 0 = 0$

$c_{13} = 1 \cdot 0 + 0 \cdot 0 + (-1) \cdot 1 = -1$

$c_{21} = 2 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 = 2$

$c_{22} = 2 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 = 1$

$c_{23} = 2 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$I \cdot \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \underset{2\times 2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \underset{2\times 3}{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}} = \underset{2\times 3}{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix}}$$

$$\begin{pmatrix} \ldots \\ 2 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} \ldots \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \ldots \\ 2 & 1 & 0 \end{pmatrix}$$

$$\underbrace{2\times \underline{2} \qquad \underline{2}\times 3} \qquad\qquad 2\times 3$$

$11 = 1\cdot 1 + 0\cdot 2 = 1$
$12 = 1\cdot 0 + 0\cdot 1 = 0$
$13 = 1\cdot(-1) + 0\cdot 0 = -1$
$21 = 0\cdot 1 + 1\cdot 2 = 2$
$22 = 0\cdot 0 + 1\cdot 1 = 1$
$23 = 0\cdot(-1) + 1\cdot 0 = 0$

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$\underset{2\times 3 \ 3\times 3}{A I_3} = \underset{2\times 2 \ 2\times 3}{I_2 A} = \underset{2\times 3}{A}$$

$A\cdot B = 0 \quad A\cdot B \neq B\cdot A \quad A\cdot B = 0 \quad A\neq 0,\ B\neq 0$

1/ $(A+B)C = AC + BC$

/ $A(B+C) = AB + AC$

/ $A(BC) = (AB)C$

/ $A\cdot I = A$

/ $A\cdot 0 = 0$

/ $(\lambda A)B = \lambda(AB) = A(\lambda B)$

/ $A\cdot B = B\cdot A = I_n \quad B = A^{-1}$ αντίστροφος του A

/ $A^1 = A \quad (A^2 = A\cdot A,\ A^3 = A^2\cdot A,\ \ldots,\ A^n = \underbrace{A\cdot A \cdots A}_{n})$

/ $A^2 = A\cdot A$

/ $A^{n+1} = A^n \cdot A^1$

1/ $A^0 = I$

/ $A^m \cdot A^n = A^{m+n}$

3/ $(A^m)^n = A^{m\cdot n}$

4/ $A^k = A$ ταυτοδύναμος $k = 1, 2, 3\ldots$

/ $A^k = 0$ μηδενοδύναμος

/ $A^k = 0$ μηδενοδύναμος

6/ $A^2 = I$

-/ $(A^{-1})^{-1} = A$

8/ $(A B)^{-1} = B^{-1} A^{-1}$

$$A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} \qquad A^2 = A$$

$$A \cdot A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$A^2 = A$

$A^3 = A^2 \cdot A = A \cdot A = A^2 = A$

<u>Ορίζουσες</u>

$|A|$ $\det(A)$ ορίζουσα του πίνακα A

2×2 $A_{2\times2} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$|A_{2\times2}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{21} \cdot a_{12}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\cdot(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$a_{12}\cdot(-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \ldots$$

$$a_{12}\cdot(-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{22}(-1)^{2+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ a_{32}(-1)^{3+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = - - - -$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11}\cdot(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} +$$

$$a_{12}\cdot(-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{13}(-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix}$$

$$a_{14}\cdot(-1)^{1+4}\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix} = a_{11}\left(a_{22}\cdot(-1)^{1+1}\begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}\right.$$

$$\left. + a_{32}\cdot(-1)^{2+1}\begin{vmatrix} a_{23} & a_{24} \\ a_{43} & a_{44} \end{vmatrix} + a_{42}\cdot(-1)^{3+1}\begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix}\right)$$

$$a_{12}\cdot\left(a_{23}\cdot(-1)^{1+2}\begin{vmatrix} a_{31} & a_{34} \\ \cdots & a_{44} \end{vmatrix} + a_{33}\cdot(-1)^{2+2}\begin{vmatrix} a_{21} & a_{24} \\ a_{41} & a_{44} \end{vmatrix} + a_{43}(-1)^{3+2}\begin{vmatrix} a_{21} & a_{24} \\ a_{31} & a_{34} \end{vmatrix}\right)$$

$$a_{12}\cdot\left(a_{23}\cdot(-1)^{1+2}\begin{vmatrix}a_{31}&a_{34}\\a_{41}&a_{44}\end{vmatrix}+a_{33}\cdot(-1)\begin{vmatrix}a_{21}&a_{24}\\a_{41}&a_{44}\end{vmatrix}+a_{43}(-1)\begin{vmatrix}a_{31}&a_{34}\end{vmatrix}\right)$$

$+\ldots$

Η ορίζουσα ενός πίνακα είναι ίση με την ορίζουσα του ανάστροφου A^T

Αν μια στήλη ή γραμμή είναι 0 τότε η ορίζουσα είναι 0.

Αν ο πίνακας έχει 2 γραμμές ή 2 στήλες ίσες ή ανάλογες τότε η ορίζουσα είναι 0

Η ορίζουσα ενός πίνακα αλλάζει πρόσημο (όχι απόλυτη τιμή) αν αντιμεταθέσουμε 2 γραμμές ή στήλες.

Αν σε μια ορίζουσα προσθέσουμε μια γραμμή (στήλη) πολλαπλασιασμένη με έναν αριθμό σε μια άλλη γραμμή (στήλη) η ορίζουσα δεν αλλάζει

Αν πολλαπλασιάσουμε μια γραμμή (στήλη) ενός πίνακα με έναν αριθμό τότε και η ορίζουσα πολλαπλασιάζεται με τον αριθμό αυτό.

$\det(A\cdot B)=\det(A)\cdot\det(B)$

Η ορίζουσα ενός άνω (κάτω) τριγωνικού πίνακα όπως και διαγωνίου είναι ίση με το γινόμενο των διαγωνίων στοιχείων.

$\det(A)=a_{11}\cdot a_{22}\cdot a_{33}\cdots a_{nn}$

Η ορίζουσα του μοναδιαίου πίνακα είναι ίση με 1

$\det(I)=1$

Η ορίζουσα του αντίστροφου ενός πίνακα είναι ίση με το αντίστροφο της ορίζουσας του αρχικού πίνακα A

$\det(A^{-1})=1/\det(A)$.

$$\begin{vmatrix}1&3&4&1\\4&-3&2&0\\3&1&2&0\end{vmatrix}=1\cdot(-1)^{1+4}\begin{vmatrix}4&-3&2\\3&1&2\\2&0&1\end{vmatrix}+\cancelto{0}{0\cdot(-1)^{2+4}\begin{vmatrix}1&3&4\\3&1&2\\2&0&1\end{vmatrix}}$$

$$\begin{vmatrix} 3 & 1 & 2 & 0 \\ 2 & 0 & 1 & 0 \end{vmatrix} \to 0 \qquad \begin{vmatrix} 2 & 0 & 1 \end{vmatrix} \qquad / \; |2 \;\; 0 \;\; 1|$$

$$1 \cdot 0 \cdot (-1)^{3+4} \begin{vmatrix} 1 & 3 & 4 \\ 4 & -3 & 2 \\ 2 & 0 & 1 \end{vmatrix} + 0 \cdot (-1)^{4+4} \begin{vmatrix} 1 & 3 & 4 \\ 4 & -3 & 2 \\ 3 & 1 & 2 \end{vmatrix} =$$

$$\Big(2 \cdot (-1)^{3+1} \begin{vmatrix} -3 & 2 \\ 1 & 2 \end{vmatrix} + 0 \cdot (-1)^{3+2} \begin{vmatrix} 4 & 2 \\ 3 & 2 \end{vmatrix} + 1 \cdot (-1)^{3+3} \begin{vmatrix} 4 & -3 \\ 3 & 1 \end{vmatrix} \Big) =$$

$$\Big(2(-3 \cdot 2 - 2 \cdot 1) + (4 \cdot 1 - (-3) \cdot 3) \Big) =$$

$$\Big(\cdot (-8) + 13 \Big) = -(-16 + 13) = -(-3) = 3$$

-2

Αντίστροφος ενός πίνακα

$A \qquad A^{-1} \leftarrow$ αντίστροφος

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A) \quad , \det(A) = |A| \neq 0$$

$|A| = 0 \Rightarrow A$ ιδιάζων (μη αντιστρέψιμος)

$$A \cdot \operatorname{adj}(A) = \operatorname{adj}(A) \cdot A = \det(A) \cdot I$$

$$\operatorname{adj} A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{pmatrix}^T = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}$$

$$A_{ij} = (-1)^{i+j} \cdot D_{ij}$$

Αν $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det A$

$\begin{pmatrix} d & -b \end{pmatrix}$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{pmatrix} \qquad A^{-1}$$

$\det A$

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1\cdot(-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 0\cdot(-1)^{1+2}\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$+\,1\cdot(-1)^{1+3}\begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = 1\cdot(2\cdot 1 - 1\cdot 1) + 0 + 1\,(\overset{-1}{(-1)\cdot 1} - \overset{+2}{(-1)\cdot 2}) =$$

$$= 1 + 1 = 2 \neq 0 \quad \exists\, A^{-1}$$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \qquad A_{12} = (-1)^{1+2}\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} = -\left((-1)\cdot 1 - (-1)\cdot 1\right) = 0$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = \left((-1)\cdot 1 - (-1)\cdot 2\right) = 1 \qquad A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -\left(0\cdot 1 - 1\cdot 1\right) = 1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = \left(1\cdot 1 - (-1)\cdot 1\right) = 2 \qquad A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = -\left(1\cdot 1 - (-1)\cdot 0\right) = -1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = \left(0\cdot 1 - 2\cdot 1\right) = -2 \qquad A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -\left(1\cdot 1 - (-1)\cdot 1\right) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = \left(1\cdot 2 - (-1)\cdot 0\right) = 2$$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & -1 \\ -2 & -2 & 2 \end{pmatrix}^T = \frac{1}{2}\begin{pmatrix} 1 & 1 & -2 \\ 0 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & -1 \\ 0 & 1 & -1 \\ 1/2 & -1/2 & 1 \end{pmatrix}$$

$\frac{\ }{2}\begin{vmatrix} \ \\ -2 & -2 & 2\end{vmatrix}$ ... $1 \quad -1 \quad 2$ ...

$A=\begin{pmatrix}1&4\\3&2\end{pmatrix} \quad A^{-1}=; \qquad A=\begin{pmatrix}a&b\\c&d\end{pmatrix} \quad A^{-1}=\frac{1}{\det A}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$

$\det A=\begin{vmatrix}1&4\\3&2\end{vmatrix}=(\overset{2}{2\cdot 1}-\overset{-12}{3\cdot 4})=-10\neq 0 \quad \exists A^{-1}$

$A^{-1}=\frac{1}{-10}\begin{pmatrix}2&-4\\-3&1\end{pmatrix}=\begin{pmatrix}-\frac{2}{10}&+\frac{4}{10}\\+\frac{3}{10}&-\frac{1}{10}\end{pmatrix}$

Άσκηση

Ν.δ.ο A^{-1} ενός διαγώνιου A είναι επίσης διαγώνιος και να βρεθεί ποιος είναι.

$\begin{pmatrix}a_{11}&0&0\\0&a_{22}&0\\0&0&a_{33}\end{pmatrix} \Rightarrow \begin{vmatrix}a_{11}&0&0\\0&a_{22}&0\\0&0&a_{33}\end{vmatrix}=a_{11}\cdot a_{22}\cdot a_{33}=\det A$

$a_{11}a_{22}\cdot a_{33}\neq 0$

$A_{11}=(-1)^{1+1}\begin{vmatrix}a_{22}&0\\0&a_{33}\end{vmatrix}=(a_{22}\cdot a_{33}-\overset{0}{\cancel{0}\cdot 0})=a_{22}a_{33}$

$A_{12}=(-1)^{1+2}\begin{vmatrix}0&0\\0&a_{33}\end{vmatrix}=0 \quad A_{13}=(-1)^{1+3}\begin{vmatrix}0&a_{22}\\0&0\end{vmatrix}=0 \quad A_{21}=(-1)^{2+1}\begin{vmatrix}0&0\\0&a_{33}\end{vmatrix}=0$

$A_{22}=(-1)^{2+2}\begin{vmatrix}a_{11}&0\\0&a_{33}\end{vmatrix}=(a_{11}\cdot a_{33}-0\cdot 0)=a_{11}\cdot a_{33} \qquad A_{32}=(-1)^{3+2}\begin{vmatrix}a_{11}&0\\0&0\end{vmatrix}=0$

$A_{23}=(-1)^{2+3}\begin{vmatrix}a_{11}&0\\0&0\end{vmatrix}=0 \quad A_{31}=(-1)^{3+1}\begin{vmatrix}0&0\\a_{22}&0\end{vmatrix}=0$

$A_{33}=(-1)^{3+3}\begin{vmatrix}a_{11}&0\\0&a_{22}\end{vmatrix}=(a_{11}\cdot a_{22}-0\cdot 0)=a_{11}\cdot a_{22}$

$A^{-1}=\frac{1}{a_{11}\cdot a_{22}\cdot a_{33}}\begin{pmatrix}a_{22}a_{33}&0&0\\0&a_{11}a_{33}&0\\0&0&a_{11}a_{22}\end{pmatrix}^T=\begin{pmatrix}a_{22}a_{33}&0&0\\0&a_{11}a_{33}&0\\0&0&a_{11}a_{22}\end{pmatrix}\frac{1}{a_{11}a_{22}a_{33}}=$

$=\begin{pmatrix}\frac{1}{a_{11}}&0&0\\0&\frac{1}{a_{22}}&0\\0&0&\frac{1}{a_{33}}\end{pmatrix}$

... $a \quad 0 \,|$ ... $(-1)^{1+1}\,|a_{22} \quad 0|$ ... $(a_{22}\cdot a_{33}-0\cdot\overset{0}{\cancel{0}})=$

$0 \quad 0 \quad a_{33}$

$$\begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}\cdot(-1)^{1+1}\begin{vmatrix} a_{22} & 0 \\ 0 & a_{33}\end{vmatrix} = a_{11}\cdot(a_{22}\cdot a_{33} - \cancel{0\cdot 0}) =$$

$$= a_{11}\cdot a_{22} a_{33}$$

$$+\ 0\cdot(-1)^{1+2}\begin{vmatrix} 0 & 0 \\ 0 & a_{33}\end{vmatrix} = 0 \quad + \quad 0\cdot(-1)^{1+3}\begin{vmatrix} 0 & a_{22} \\ 0 & 0\end{vmatrix} = 0$$

Ασκηση

Να βρεθεί ο A^{-1} του $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$$\det A = 1\cdot 5\cdot 2 = 10 \neq 0$$

$$A^{-1} = \frac{1}{10}\begin{pmatrix} 5\cdot 2 & 0 & 0 \\ 0 & 1\cdot 2 & 0 \\ 0 & 0 & 1\cdot 5 \end{pmatrix} = \begin{pmatrix} 1/1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

Ασκηση

Ν.δ.ο

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} =$$

$$= (a+b+c)\cdot\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$$

$\rightarrow 0$

$$| \ a_{11} \quad a_{12} \quad a_{13} \ | \qquad | \ ka_{11} \quad ka_{12} \quad ka_{13} \ |$$

$$k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} k\,a_{11} & k\,a_{12} & k\,a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= \begin{vmatrix} k\,a_{11} & a_{12} & a_{13} \\ k\,a_{21} & a_{22} & a_{23} \\ k\,a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & k\,a_{13} \\ a_{21} & a_{22} & k\,a_{23} \\ a_{31} & a_{32} & k\,a_{33} \end{vmatrix}$$

$$\left.\begin{matrix} \det(cA) = c^{4}\det(A) \\ A_{4\times 4} \end{matrix}\right\}$$

Άσκηση Ν.δ.ο. $= 0$

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ x+4a & x+5a & x+6a & x+7a \\ x+8a & x+9a & x+10a & x+11a \\ x+12a & x+13a & x+14a & x+15a \end{vmatrix} =$$

πολλαπλασιάζω την 1η γραμμή με (-1) και την προσθέτω στην 2η γραμμή

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ -x+x+4a & -x+x-a+5a & -x+x-2a+6a & -x+x-3a+7a \\ x+8a & x+9a & x+10a & x+11a \\ x+12a & x+13a & x+14a & x+15a \end{vmatrix} =$$

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ 4a & 4a & 4a & 4a \\ x+8a & x+9a & x+10a & x+11a \\ x+12a & x+13a & x+14a & x+15a \end{vmatrix} =$$

αντίστοιχα 3η με 4η γραμμή

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ 4a & 4a & 4a & 4a \end{vmatrix} =$$

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ 4a & 4a & 4a & 4a \\ x+8a & x+9a & x+10a & x+11a \\ \cancel{x}-\cancel{x}-8a+12a & x-x-9a+13a & x-x-10a+14a & x-x-11a+15a \end{vmatrix} =$$

$$\begin{vmatrix} x & x+a & x+2a & x+3a \\ 4a & 4a & 4a & 4a \\ x+8a & x+9a & x+10a & x+11a \\ 4a & 4a & 4a & 4a \end{vmatrix} = 0$$

ίδιες γραμμές

$$\begin{vmatrix} x & x+a & x+2 & x+3a \\ 4a & 4a & 4a & 4a \\ x+8a & x+9a & x+10a & x+11a \\ \cancel{4a}-4a & \cancel{4a}-4a & \cancel{4a}-4a & \cancel{4a}-4a \end{vmatrix} = 0$$

Ν.δ.ο χωρίς ανάπτυξη,

$$\begin{vmatrix} 3 & 2 & 3a+2b+1 \\ 2 & 4 & 2a+4b+6 \\ 1 & 3 & a+3b+2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & 6 \\ 1 & 3 & 2 \end{vmatrix}$$

$\leftarrow (-a)$

$$\begin{vmatrix} 3 & 2 & 3a+2b+1 \\ 2 & 4 & 2a+4b+6 \\ 1 & 3 & a+3b+2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & -3a+3a+2b+1 \\ 2 & 4 & -2a+2a+4b+6 \\ 1 & 3 & -a+a+3b+2 \end{vmatrix} =$$

$\swarrow (-b)$

$$\begin{vmatrix} 3 & 2 & 2b+1 \\ 2 & 4 & 4b+6 \\ 1 & 3 & 3b+2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & -2b+2b+1 \\ 2 & 4 & -4b+4b+6 \\ 1 & 3 & -3b+3b+2 \end{vmatrix} =$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & 6 \\ 1 & 3 & 2 \end{vmatrix}$$

χωρίς ανάπτυξη ν.δ. το κ = ;

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 5b_1 & 5b_2 & 5b_3 \\ 2a_1+3b_1-7c_1 & 2a_2+3b_2-7c_2 & 2a_3+3b_3-7c_3 \end{vmatrix} =$$

$$\cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

1η γραμμή (-2) κ' προσθ. 3η γρ.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 5b_1 & 5b_2 & 5b_3 \\ 2a_1+3b_1-7c_1 & 2a_2+3b_2-7c_2 & 2a_3+3b_3-7c_3 \end{vmatrix} =$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 5b_1 & 5b_2 & 5b_3 \\ -2a_1+2a_1+3b_1-7c_1 & -2a_2+2a_2+3b_2-7c_2 & -2a_3+2a_3+3b_3-7c_3 \end{vmatrix} =$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 5b_1 & 5b_2 & 5b_3 \\ 3b_1-7c_1 & 3b_2-7c_2 & 3b_3-7c_3 \end{vmatrix} =$$

$$- \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 3b_1-7c_1 & 3b_2-7c_2 & 3b_3-7c_3 \end{vmatrix} =$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{vmatrix} =$$

$$\begin{vmatrix} a_1 & & \\ b_1 & b_2 & b_3 \\ -3b_1+3b_1-7c_1 & -3b_2+3b_2-7c_2 & -3b_3+3b_3-7c_3 \end{vmatrix} =$$

$$5\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ -7c_1 & -7c_2 & -7c_3 \end{vmatrix} =$$

$$\cdot(-7)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$k = -35$

Αν Α είναι ταυτοδύναμος ν.δ.ο και ο $B = I - A$ είναι επίσης ταυτοδύναμος και $AB = BA = 0$.
Ν.δ.ο επίσης ~~ο~~ $A - I$ δεν είναι ταυτοδύναμος.

Ταυτοδύναμος $\underline{\underline{A^2 = A}}$ ν.δ.ο $B^2 = B$ (Ι Μοναδιαίος)

$B^2 = (I-A)(I-A) = I^2 - I\cdot A - A\cdot I + A^2 = I^2 - A - A + A^2 =$

$= I - A - \cancel{A} + \cancel{A} = I - A = B$

$A\cdot \overset{B=I-A}{B} = A(I-A) = A\cdot I - A^2 = A - A^2 = A - A = 0$

$BA = (I-A)A = I\cdot A - A^2 = A - A^2 = A - A = 0$

$\underline{\text{Αν}}$ ο $A - I$ ταυτοδ. τότε θα έπρεπε

$(A-I)^2 = (A-I)$

$(A-I)^2 = (A-I)(A-I) = A^2 - AI - I\cdot A + I^2 =$

$= \cancel{A} - \cancel{A} - A + I = -(A-I)$ άρα δεν είναι ταυτοδύναμος

$= \cancel{A} - \cancel{A} - A + I = -(A - I)$ άρα δεν είναι ταυτοδύναμος

Γενική σχέση $AB \neq BA$

$(A \pm B)^2 = \cancel{A^2 \pm 2AB + B^2}$ $AB \neq BA$

$(A \pm B)^2 = (A \pm B)(A \pm B) = A^2 \pm AB \pm BA + B^2$

Δίνεται η σχέση $A \cdot X \cdot B = I_3$

$A = \begin{pmatrix} 1 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ $X = ;$

$\det(A)$ κ' $\det(B) \neq 0$ για να $\exists$ A^{-1}, B^{-1}

$A \cdot X \cdot B = I_3 \iff$

$A^{-1} A X B = A^{-1} I_3 \iff$

$I X B = A^{-1} \iff$

$X B = A^{-1} \iff$

$X B \cdot B^{-1} = A^{-1} \cdot B^{-1}$

$X I = (B \cdot A)^{-1}$

$X = (B \cdot A)^{-1} = B^{-1} \cdot A^{-1}$

$\left(\begin{matrix} A \cdot A^{-1} = A^{-1} A = I \\ B B^{-1} = B^{-1} B = I \end{matrix} \right)$

$\left(\begin{matrix} I \cdot A = A \\ A \cdot I = A \end{matrix} \right)$

$\left(\begin{matrix} A^{-1} A X B \cancel{} \\ A A^{-1} X B = \end{matrix} \right)$

1/ $\det(A)$ κ' $\det(B) = ;$

2/ $X = \dots$

Άσκηση

$\dots \begin{pmatrix} 2^n & 3^n - 2^n \end{pmatrix}$

Άσκηση

$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$. Ν.δ.ο $A^n = \begin{pmatrix} 2^n & 3^n - 2^n \\ 0 & 3^n \end{pmatrix}$

$n \geq 1$

1/ Για $n=1$

$A^1 = \begin{pmatrix} 2^1 & 3^1 - 2^1 \\ 0 & 3^1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ ισχύει

2/ Έστω ότι ισχύει για $n=k$ δηλαδή

$A^k = \begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix}$

3/ Αρκεί ν.δ.ο ισχύει για $n=k+1$ δηλαδή

$A^{k+1} = \begin{pmatrix} 2^{k+1} & 3^{k+1} - 2^{k+1} \\ 0 & 3^{k+1} \end{pmatrix}$

$A^{k+1} = A^k \cdot A = \begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} =$

$\begin{pmatrix} \overline{11} & \overline{12} \\ \overline{21} & \overline{22} \end{pmatrix} = \begin{pmatrix} 2^{k+1} & 3^{k+1} - 2^{k+1} \\ 0 & 3^{k+1} \end{pmatrix}$

$1 = 2^k \cdot 2 + (3^k - 2^k) \cdot 0 = 2^k \cdot 2^1 = 2^{k+1}$

$2 = 2^k \cdot 1 + (3^k - 2^k) \cdot 3 = 2^k + 3^k \cdot 3 - 2^k \cdot 3 =$

$3^{k+1} \quad 2^{k+1}$

$2 = 2^k \cdot 1 + (3^k - 2^k)\cdot 3 = (2^k) + 3\cdot 3^k - (3 \cdot 2^k)$

$= 3^k \cdot 3^1 - 2^1 \cdot 2^k = 3^{k+1} - 2^{k+1}$

$\left(2^k - 2^k \cdot 3 = -3\cdot 2^k + 2^k = (-3+1)\cdot 2^k = -2 \cdot 2^k \right)$

$1 = 0 \cdot 2 + 3^k \cdot 0 = 0$

$2 = 0 \cdot 1 + 3^k \cdot 3 = 3^{k+1}$

Άσκηση

Ν.δ.ο για τον $A = \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix}$

ισχύει $A^n = \begin{pmatrix} 2\cdot 3^n - 5^n & 2(5^n - 3^n) \\ 3^n - 5^n & -3^n + 2\cdot 5^n \end{pmatrix}$, $n \geq 1$

Για $n=1$

$A^1 = \begin{pmatrix} 2\cdot 3^1 - 5^1 & 2(5^1 - 3^1) \\ 3^1 - 5^1 & -3^1 + 2\cdot 5^1 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix}$ ισχύει

/Έστω ότι ισχύει για $n=k$ Δηλαδή

$A^k = \begin{pmatrix} 2\cdot 3^k - 5^k & 2(5^k - 3^k) \\ 3^k - 5^k & -3^k + 2\cdot 5^k \end{pmatrix}$

/Αρκεί ν.δ.ο ισχύει για $n=k+1$ Δηλαδή

$A^{k+1} = \begin{pmatrix} 2\cdot 3^{k+1} - 5^{k+1} & 2(5^{k+1} - 3^{k+1}) \\ 3^{k+1} - 5^{k+1} & -3^{k+1} + 2\cdot 5^{k+1} \end{pmatrix}$

$$A^{k+1} = \begin{pmatrix} 2\cdot 3^{k+1} - 5^{k+1} & 2(5^{k+1} - 3^{k+1}) \\ 3^{k+1} - 5^{k+1} & -3^{k+1} + 2\cdot 5^{k+1} \end{pmatrix}$$

$$^{k+1} = A^k \cdot A = \begin{pmatrix} 2\cdot 3^k - 5^k & 2(5^k - 3^k) \\ 3^k - 5^k & -3^k + \cdot 2\cdot 5^k \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix} =$$

$$\begin{pmatrix} \_{11} & \_{12} \\ \_{21} & \_{22} \end{pmatrix}$$