Thematic Analysis of Students' Talk While Solving a Real-World Problem in Geometry

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Abstract

From a social semiotic perspective, students' use of language is fundamental to mathematical meaning making. We applied thematic analysis to examine students' use of geometric and contextual ideas while solving a geometry problem that required them to determine the optimal location for a new grocery store on a map of their local community. Students established semantic patterns to connect the problem context to geometry. Groups differed in how they used geometry in their discussion of the solution, in particular with how students used distance to describe the location of a new grocery store. Overall, students' knowledge of the problem context served as a resource for them to establish geometric meanings. Thematic analysis, which describes the connections in students' talk between out-of-school and discipline-specific knowledge, highlights ways in which instruction can build upon students' prior experiences for the purpose of learning in school.

Keywords: Systemic functional linguistics; Mathematics learning; Thematic analysis; Semantic relations

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1. Introduction

There are many examples in research of the importance of discourse for learning and doing mathematics. Spoken language is one of a number of semiotic systems, which also includes symbolic notation, written language, and visual representations, through which mathematical knowledge is created and communicated (O'Halloran, 2014; Schleppegrell, 2007). Language, and in particular spoken language, may be considered the primary medium through which mathematics is taught in schools (Lemke, 1988). One particularly important aspect of classroom discourse is student talk, especially when students work in groups for doing mathematics. Because language is a fundamental means of doing mathematics, students' work in groups should create opportunities for them to engage in mathematical meaning making. At the same time, students bring a variety of experiences and knowledge to mathematical discussions (Moschkovich, 2008; Turner, Gutiérrez, & Sutton, 2011; Zahner, 2012), and students are not always familiar with the accepted ways of formulating arguments or explanations in mathematics (Forman, McCormick, & Donato, 1997). Groups of students are likely to construct different meanings, even while working on the same task. These differences may be compounded when students work on problems that have real-world connections and draw on students' prior experiences outside of school.

Real-world contexts can be useful for students to engage with mathematics concepts and procedures. We use the term *real-world problem* to refer to a mathematics problem grounded in a real-world context, for example a problem about sharing popcorn among friends (Lubienski, 2000) or evaluating a house plan (Boaler, 1998). Students can learn to appreciate mathematics as relevant through solving real-world problems (Boaler, 1998; Frankenstein, 1987; National

Council of Teachers of Mathematics [NCTM], 2000). There is a strong international tradition of teaching and learning mathematics with real-world problems, as evidenced by a focus on teaching through problem solving (Lampert, 2001), project-based learning (Boaler, 1998, 2008), mathematical modeling (English, 2010), and the development of mathematics curricula through the framework of Realistic Mathematics Education (e.g., Gravemeijer & Doorman, 1999). Given the benefits of real-world problems to students' mathematical learning, there is opportunity to better understand how students construct mathematical meaning through discussions of problems (Herbel-Eisenmann & Otten, 2011). Real-world problems often intend for students to focus primarily on mathematical relationships, but it can be difficult in practice to sustain this focus on mathematics (Otten & Soria, 2014). To support students' mathematical learning, it is helpful to examine how students engage in mathematical meaning making through their talk while solving a real-world problem.

We conducted a study in which students needed to use a map of their local community to determine the optimal location for a new grocery store, with the goal of understanding how students would use and discuss mathematical ideas to solve a real-world problem set in their local community. We asked two research questions: (1) What were the central ideas of students' discussions that shifted between contextual and geometric meaning? (2) How did students give geometric meaning to the task of locating a new grocery store on a map? With these questions we contribute to two interrelated objectives, understanding how students use real-world problems as opportunities to talk about mathematics and how students differ in the mathematical meanings that surface through their discussions.

2. Conceptual Framework

We take a social semiotic perspective, which assumes that meaning is constructed through social practices and through choices of representation within specific settings (Kress & van Leeuwen, 2006; O'Halloran, 2014). In order to communicate, individuals must select a representation that best expresses a particular concept or idea (Kress & van Leeuwen, 2006). Spoken language is a primary means of expressing ideas (Lemke, 1988, 1990; Morgan, 2006), along with other semiotic systems. In mathematics education, visual representations serve a critical role for expressing mathematical relationships and supporting communication (Alshwaikh, 2011; Chapman, 2003; Dimmel & Herbst, 2015; O'Halloran, 2005). Communication in mathematics can also be achieved through the use of symbolic notation (O'Halloran, 2003) and through gesturing (Arzarello & Edwards, 2009; Radford, 2009). Teachers and students in mathematics classes typically use a variety of semiotic resources to convey ideas (O'Halloran, 2005, 2014; Schleppegrell, 2007). Because analysis of discourse can provide a way to understand which features of interaction are significant to creating mathematical meanings (Morgan, 2006), we focus specifically on students' conversations. We also recognize that there are instances in which students' use of semiotic resources such as visual representations support those conversations.

There are three important aspects of a social semiotic perspective—social practices, context, and language—that contribute to our examination of students' discourse while working on a real-world mathematics problem. First, to say that meaning is constructed through social practices suggests that mastering a discipline, for example mathematics or physics, requires correct use of the spoken or written language of that discipline (Lemke, 1988). To learn science requires speaking "according to the accepted ways of talking science" (Lemke, 1990, p. 12). Mathematics requires, in addition to knowledge of grammar and vocabulary, particular "styles of meaning and modes of argument" (Halliday, 1978, p. 195). A logical argument in geometry, or a description of a point on a 2-dimensional plane, requires a style that may not be present in other, everyday language. Although the discipline of mathematics requires a particular style of making meaning, students and teachers in different classrooms establish idiosyncratic practices (Chapman, 1993; Herbel-Eisenmann, Wagner, and Cortes, 2010). The meaning of a particular term, or the ways in which mathematical ideas become connected through a web of relationships, is likely to vary across different classroom communities.

Given the differences in how individuals may talk about mathematics within different settings, it is important to recognize how social practices are situated within contexts. We take a dynamic view of context, which is to say that the contexts in which individuals interact is constituted through those interactions. This view is aligned with a social semiotic perspective of interaction, as described by Halliday and Matthiessen (2014) as a two-way interaction between the ways that language is used and the context defined by those uses (p. 34). Language, in addition to other semiotic resources, is used to construe meaning, which contributes to the context in which that meaning is built. Morgan (2006) defined two different, and both necessary, ways of considering the context in which meaning is made. The *context of situation* includes "the goals of the current activity, the other participants, the tools available and other aspects of the immediate environment," and the *context of culture* encompasses "broader goals, values, history, and organizing concepts that the participants hold in common" (p. 221). We consider another aspect of context in students' discussions, the *problem context*, which refers to the elements included in a particular real-world problem. Although the problem context as it is presented to students is static, students contribute to that context in a dynamic way by drawing upon their knowledge of mathematics and of the world. In doing so, students create and draw upon the context of situation as well as the context of culture. Students in a mathematics class construct meaning according to the typical practices of the class, the goals of the activity, and the available resources. Additionally, students help define the context in which they interact by drawing on their prior experiences.

Language can be viewed as the primary means through which the fundamental ideas and concepts of an academic discipline are taught and learned (Lemke, 1988). There is evidence from prior research of the importance of studying the different ways that teachers and students in mathematics classrooms use language (e.g., Morgan, 2006; Schleppegrell, 2007). For example, teachers' use of the terms "base" and "height" can alternately reference a specific segment of a geometric figure (e.g., the base of a triangle) or the measurement of that segment (e.g., multiply base times height) when teaching about area formulas in geometry (Herbel-Eisenmann & Otten, 2011). Because either use of the term is appropriate in some settings, the example illustrates the importance of language for communicating the ideas of a discipline, as well as the potential ambiguities for students who are new to mathematical discourse. When students work together in groups, they incorporate mathematical language through their discussions with one another. One area of interest is how students construct meaning through their discussions of mathematics, and whether students across different groups establish the same, or different, meanings.

2.1 Thematic Analysis to Examine Students' Talk

We examined the ways that students constructed meaning through a *thematic analysis* of students' talk. Within the framework of Systemic Functional Linguistics (SFL), thematic analysis is a method to describe how ideas in a text are related to one another (Lemke, 1990; see also Chapman, 1993; O'Halloran, 2005). This use of the term "thematic analysis" is specific to the theory of SFL and distinct from other uses of thematic analysis in educational research (e.g., Braun & Clarke, 2006; Voigt, 1995). We use the method of thematic analysis described by Lemke (1990), which has also been applied in mathematics education research (Chapman, 1993; Herbel-Eisenmann & Otten, 2011; O'Halloran, 2005). By describing how students make connections between multiple ideas in their talk, thematic analysis allowed us to consider the question of how students construed meaning through their conversations.

Within the theory of Systemic Functional Linguistics (SFL), thematic analysis addresses the *ideational* metafunction of language.[1] The ideational metafunction refers to the way that language is used to construct meaning (Halliday & Matthiessen, 2014).[2] Because mathematics is comprised of more than a collection of isolated terms and vocabulary, an understanding of mathematical discourse requires examination of how ideas are related through semantic patterns. *Semantic patterns* refer to "the patterns of meaning and meaning relations among what we usually think of as the characteristic *words* or *concepts* of the subject" (Lemke, 1988, p. 82, emphasis in original). Semantic patterns describe the possible ways that particular terms within a discipline may be related to one another. For example, a discussion relating the concepts of *area* and *triangle* in a mathematics class would follow a limited number of semantic patterns: the *area* of a *triangle* typically refers to a measure of space enclosed by the triangle. In this way, the semantic patterns within a discipline such as mathematics provide a set of stable, predictable ways in which terms may be related.

[1] SFL describes the "metafunctions" of language to emphasize the importance of these ideas for the analysis of language as a whole. This is distinct from discussions of the "function" of language, which may describe the direct purpose of language within a specific setting (Halliday & Matthiessen, 2014, p. 31).

[2] SFL posits that language simultaneously serves three metafunctions: an *ideational* metafunction, to construct meaning; a *textual* metafunction, to organize ideas; and an *interpersonal* metafunction, to enact interpersonal relationships between interactants (Halliday & Matthiessen, 2014; Martin & Rose, 2007).

Semantic patterns create a web of relationships among the key ideas or concepts of a discipline (Lemke, 1990). We use the phrase *thematic items* to describe ideas or terms that surface through discourse and that become semantically related to one another through these webs (Lemke, 1988, 1990). Semantic patterns, which are created through language, are what define and give meaning to the thematic items of a conversation. We use italics to indicate our use of thematic items (as in *area* and *triangle* in the previous paragraph) and to distinguish thematic items from other uses of a particular term.[3] For example, a term such as area could be used generally in many aspects of discourse, or could alternately be replaced by synonyms. When we refer to thematic items, we refer to uses of a term that are connected through specific semantic patterns to other terms in a particular setting.

Although the semantic patterns of a discipline such as mathematics or science are generally stable and predictable, speakers often establish unexpected or non-standard semantic patterns in their talk about a particular topic. For instance, although *area* is generally accepted as a measurement of space within a *triangle*, teachers and students may occasionally refer to an *area* as the set of points enclosed by the sides of a triangle (Herbel-Eisenmann & Otten, 2011). There is a difference here in whether *area* is a specific measurement, or whether *area* refers to a region. Unfamiliar semantic patterns can be compounded when words common in everyday language are used in discipline-specific ways (e.g., Morgan & Alshwaikh, 2012; Pimm, 1987). The term *weight*, for example, which has a variety of uses, is semantically related in very specific ways to atoms, gases, and liquids in chemistry (Viechnicki, 2008). The idea that individuals sometimes construct non-standard semantic patterns helps to explain why students often build different meanings through working on a single task. It is reasonable to expect that

[3] There are multiple possible conventions for indicating thematic items in analysis, including the use of italics (Herbel-Eisenmann & Otten, 2011) or the use of capital letters (Lemke, 1988, 1990).

different groups of students, working on a common task, may establish idiosyncratic semantic patterns through their talk, which can reveal the different ways students make sense of mathematics.

Thematic analysis provides a way to describe and represent the web of semantic relationships among the thematic items of a text, illustrating the "type of meaning relations constructed in a text" (Lemke, 1988, p. 85). There are typical semantic relationships that are common in specific areas of mathematics or science (e.g., Herbel-Eisenmann & Otten, 2011; Lemke, 1990; Viechnicki, 2008), such as part-whole relationships (e.g., the hypotenuse of a right triangle is one part of the triangle). We apply thematic analysis for observing shifts in students' use of particular terms according to how those terms can be related to geometric meanings as well as the context in which students solve a problem.

3. Data Sources and Methods

This study was part of a project investigating students' conceptions when working on real-world problems in geometry. We aimed to examine how students would use three different sources of prior knowledge—school mathematics, informal mathematical practices, and context—when working in groups to solve a problem (González, G., DeJarnette, & Deal, 2014). This work is aligned with research examining ways for instruction to connect with students' multiple mathematical knowledge bases, including students' mathematical thinking and community-based funds of knowledge (Drake et al., 2015; Turner et al., 2012).

3.1 Setting of the Study

We conducted a 2-hour after-school session at Tyrian High School[4] during the spring of 2014. Tyrian High School is an urban, high-need school in the Midwestern United States. The student population at Tyrian High School is around 30% Black and 10% Hispanic, and over 45% of students qualify for free or reduced-price lunch. We advertised the study to five different class periods and invited all students from those classes to participate. We gave students a cash stipend for their participation, and we provided pizza during the after-school session.

Fourteen students participated in one after-school session, from the classes of three different geometry teachers at the school. The participants of the study were racially diverse, and all participants were currently enrolled in an Accelerated Geometry course. Accelerated Geometry is offered at Tyrian High School as a more advanced option than the standard geometry course. Students in the accelerated course take geometry during their 9^{th}-grade year, so the participants of the study were 14-15 years old. Students at Tyrian High School used the textbook *Core Connections Geometry* (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013). As per the curriculum, at the time of the study students had studied shapes and transformations, angles, similarity, trigonometry, right triangles, triangle congruence, quadrilaterals and other polygons, and constructions in their geometry courses. Students also had experience with justification and proof throughout the course.

3.2 The Grocery Store Problem

During the after-school session, students worked on a problem that we termed the "grocery store problem." We provided students with a printed map of the community in which students attended school. The map included three adjacent towns: Chesterton (the town in which

[4] In compliance with IRB protocols, we use pseudonyms for all people, institutions, towns, and street names.

the school was located), Union City, and Sycamore. Students needed to determine the optimal location for a new grocery store (Figure 1). The map we provided to students included eight points indicating the locations of various grocery stores in town.[5] Students were instructed to locate the new grocery store so that people in the community would not have to travel as far to get groceries.

The City Council is trying to make the town friendlier for people to walk and bicycle. As part of this effort, the Council is trying to bring in a new grocery store, so that people in the community don't have to travel as far to get groceries.

The map below includes the locations of the main grocery stores in the area. With your group, decide on a location to propose to the City Council for the new grocery store. You can make a few of assumptions in your work:

1. The grocery store should be East of I-271 and West of Hickory Road. It should be South of Oakmeade Drive and North of Old Country Road.
2. For now, the City Council is only recruiting *one* grocery store. So you should decide where to place a single store in order to provide the most benefit.

Figure 1. The context of the grocery store problem.

We designed the grocery store problem to elicit students' ideas about how to find a point that is equidistant from a set of other points. Given a collection of 3 points, *A*, *B*, and *C*, that are not on the same line (Figure 2a), it is possible to find a point that is equidistant from those three points. To do so, one would construct the perpendicular bisectors of each line segment comprising the triangle *ABC* (Figure 2b). In Euclidean geometry, those three perpendicular bisectors intersect at a single point, and that point of intersection is equidistant from each of the three given points. To locate a new grocery store on the map, students could identify a region in which the existing stores were farther apart than in other regions of the map. We expected that students could reduce the distance residents would need to travel by locating a new store in between a subset of the existing stores. The grocery store problem is an example of a typical

[5] To protect the confidentiality of participants we do not include the map, which included an approximately 40 square mile region surrounding the students' school.

geometric modeling problem requiring the use of perpendicular bisector to divide a map into regions (e.g., Perham & Perham, 2011; Zbiek & Conner, 2006). We did not identify a unique solution to the grocery store problem, and we expected students to use their knowledge of the community to determine regions of the map where the new store could potentially be located.

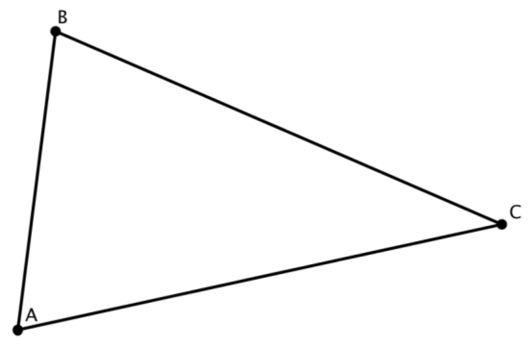	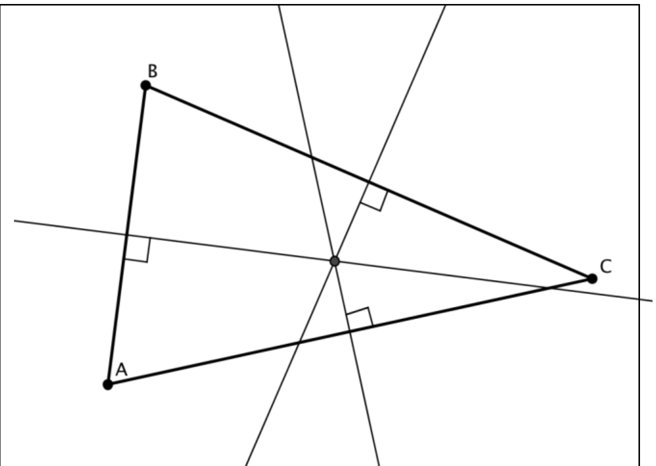
Figure 2a. A collection of 3 points forming a triangle.	*Figure 2b.* The intersection of three perpendicular bisectors of the triangle is equidistant from the three given points.

Figure 2. A center of a triangle at the intersection of perpendicular bisectors of the three sides.

The first author acted as the teacher for the session. During the introduction of the problem, the teacher told students to think of themselves as joining a team of urban planners trying to determine the optimal place for a new grocery store. The teacher encouraged students to consider that they might be working with engineers, architects, and residents of the local community, and that they should bring the geometric perspective to the team. We intended to elicit students' geometric ideas about how they could divide the map to reduce the distances between grocery stores. We did not provide any information about the community other than the streets and landmarks represented in the map. However, given that the map included the neighborhood surrounding the school, and the neighborhoods where many of the students lived, we expected that students would bring a great deal of knowledge about the community to their work on the problem. We gave each student a copy of the map. Additionally, we provided tracing paper, rulers, compasses, and colored pencils. Although we did not require students to use any particular resources to solve the problem, we encouraged students to use the available materials.

3.3 Data Collection

The data for this study come from 14 students in four groups of 3-4 students each working on the grocery store problem (Table 1).[6] We used video and audio recorders to capture students' work. While students were working in groups, we positioned one video camera and one audio recorder at each group. During whole-class discussions, we positioned one camera at a wide angle in the back of the classroom facing the front of the room. After the teacher introduced the problem, students worked in groups for approximately 25 minutes. At that time, the teacher led a brief whole-class discussion, during which time several students shared their group's considerations for deciding where to locate the grocery store. After this discussion the teacher told students, "Remember you're trying to bring a geometric perspective to this." The teacher reminded students that they would need to describe the location of the new store relative to the other points on the map. Following the whole-class discussion, students continued working for approximately 10 minutes. In total, students spent approximately 35 minutes working in groups. This was followed by a longer class discussion in which each group presented a potential location for the new store, and students debated the merits of different solutions.

Table 1

[6] In total, there were five groups of students that participated in the after-school session. Due to a technical failure we did not collect video data of one of the groups, so we only report on the four groups we videotaped.

Students Participating in the Study

Group 1	Group 2	Group 3	Group 4
Brian	Ellen	Eli	Adam
Edgar	Joshua	Camilo	Dominic
Pankaja	Selina	Logan	Nina
Ruth		Zhang	

The first author used the video and audio records to produce a transcript for each of the four groups of students. For each group we transcribed the entire session, including the teacher's introduction to the problem and the whole-class discussion at the end.

3.4 Analysis

We applied the method of thematic analysis introduced by Lemke (1988, 1990), which informed our identification of how students established connections between ideas in their discussions about the grocery store problem. To describe how ideas were related to one another in students' talk, we identified the *semantic relations* (Halliday & Matthiessen, 2014; Lemke, 1990) in students' discussions. Semantic relations are the relationships between words or phrases used in a text. For example, a student could state, *people can walk one mile to the grocery store*. In this statement, "people" are agents who participate in a process, specifically walking. By specifying the distance people can walk to get to the store, the statement provides the scope to which people engage with the process of walking. We include a partial list of semantic relations that were most relevant to our analysis of students' discussions during group work (Table 2). These semantic relations come from the work of Halliday and Matthiessen (2014) and have been expanded and applied for the use of thematic analysis by Lemke (1990).

Table 2

A Partial List of Semantic Relations (Halliday & Matthiessen, 2014; Lemke, 1990)

Relation	Abbreviation	Example
Attribute/Carrier	ATT/CA	The area is not safe for walking.
Epithet/Thing	EP/TH	The open area.
Subset/Set	SUB/SET	Two stores of the existing grocery stores.
Meronym/Holonym	MER/HOL	The middle of the area.
Synonym/Synonym	SYN	The map is the area.
Agent/Process	AG/PR	People can walk to the store. (people, walk)
Process/Target	PR/TA	People can walk to the store. (walk, to the store)
Process/Scope	PR/SC	People can walk 2 miles. (walk, 2 miles)
Location/Located	LOC	The grocery store is in the center of the circle.
Existential	EXISTENT	There is no room by my house.
Extent/Entity	EXT/ENT	The distance is 2 miles. (2 miles, distance)

Our unit of analysis in identifying semantic relations in students talk was the clause. We define a clause as a segment of text in which some participants (i.e., people or things) participate in a process (Martin & Rose, 2007).[7] We described the semantic relations at this level for two purposes. First, by considering students' talk at the level of the clause, we could identify shifts in the ways that students gave meaning to thematic items related to the problem context and according to their mathematical knowledge. By focusing on individual clauses, we were able to see how the context of students' work on the grocery store problem—both the context of situation as well as the context of culture—surfaced and shaped students' discussions. Students

[7] Although not the focus of this study, the social semiotic perspective and analysis with SFL also allow for an examination of meaning making through sound and intonation of speech (e.g., Halliday & Greaves, 2008).

shifted between ideas related to their knowledge of the community (part of the context of culture) and the geometric requirements of the task (part of the context of situation). The importance of context for students' meaning making became apparent through the semantic shifts in the ways that students related ideas to one another. By focusing on semantic relations within individual clauses, we could see how those relations developed and shifted within single turns of speech and as speakers interacted with one another.

We illustrate our coding process with an example from group 3's discussion (Table 3). We use the conversation from group 3 as it provides a concise example of the types of discussions that were common across several of the group in determining how far would be a reasonable distance to walk. Prior to the excerpt, the students had been discussing the possibility of drawing circles around existing grocery stores on the map to exclude regions of the map that did not need another store. To decide how large to make the circles, Logan initiated a discussion about how far people could bike to get to the store. Students first discussed how far would be an unreasonable distance for people to ride their bikes (Turns 95-97 in Table 3, below), and Eli interjected with a comment about how long it would take him and his dad to ride one mile (Turn 98). Our analysis of the semantic relations illustrates how students constructed meaning through their discussion of the map.

The two rightmost columns in Table 3 include the semantic relations we identified in each row. Within the turns of speech, we used bold letters and underlining to identify semantic relations. For example, in Turn 95, Logan asked the question, "how far would be like, unreasonable to ask people to bike?" In that question, we identified two sets of semantic relations. First, in Logan's question, "people" are agents who participate in the process "to bike." Moreover, there is some particular distance to which people bike. We described the semantic relation here as agent/process/scope (AG/PR/SCOPE) to indicate the relationship between the people who bike and the distance to which they could bike. Embedded within Logan's question was a clause questioning what would be an unreasonable distance. We described this relationship as attribute/carrier (ATT/CA), because Logan indicated that some particular distance would have the attribute of being unreasonable. In Camilo's response to Logan's question (Turn 96), Camilo quantified the distance people could reasonably bike. Eli then commented, "me and my dad can ride about a mile in a matter of like 3 or 4 minutes" (Turn 98). In Eli's statement, "me and my dad" were agents who engaged in a process of "can ride" to the scope of "about a mile." Moreover, Eli provided a classifier of the process with his comment that he and his dad could ride one mile in 3-4 minutes.

Table 3

An Example of Our Analysis of Semantic Relations in Students' Discourse

Turn #	Speaker	Turn	Semantic Relations	
95	Logan	Well, **how far <u>would be like, unreasonable</u> to ask people to bike?**	AG/PR/SC	<u>ATT/CA</u>
96	Camilo	**Three miles.**	QUANTIFIER	
97	Logan	**Three miles?**	QUANTIFIER	
		Okay, what do you think?		
98	Eli	Uh, **me and my dad can ride about a mile <u>in a matter of like 3 or 4 minutes.</u>**	AG/PR/SC	<u>CLASSIFIER</u>

Note: The leftmost column in Table 3 indicates the turn number, which we counted from the beginning of the transcript. We divided students' turns of speech into rows. We use bolding and underlining to indicate correspondence between the section of text and our identification of the semantic relation.

The second phase of our analysis was to create thematic maps (Lemke, 1990), which are 2-dimensional representations to display the semantic relations between the thematic items of students' conversations. The primary purpose of these thematic maps was to see patterns and differences in the ways that students related ideas to one another. In our analysis, the thematic items of the text were primarily the noun phrases that surfaced as *participants* (i.e., people or things participating in a process) of clauses related to the solution to the grocery store problem. Figure 3 illustrates a portion of a thematic map, based upon the semantic relations in Table 3. The key noun phrases in the students' discussion, referring to people in the community and students within the group, are outlined in rectangles. *People* and *students* both acted as agents participating in the process of riding a bike. Students determined that 3 miles would have the characteristic of being an unreasonable distance to require people to ride to the grocery store. Eli classified his statement that he could ride one mile by adding that he could do so in 3-4 minutes. The thematic map illustrates the ways that students began to build semantic relations between the central ideas of their talk. In students' discussions of the grocery store problem, they considered the activity of walking or riding to the grocery store from the perspective of multiple agents within the community. In this example, students' shared context of culture allowed them to define the considerations and constraints of the problem context.

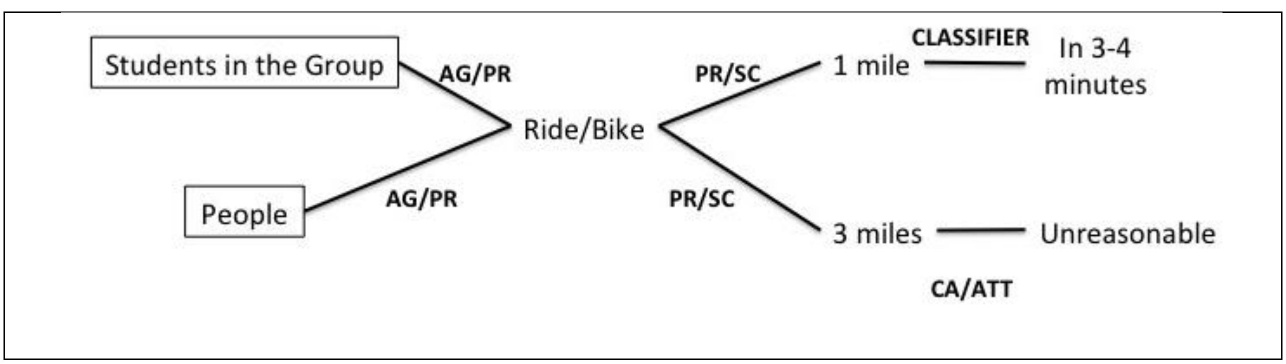

Figure 3. A partial thematic map, based on the semantic relations identified in Table 3.

The map in Figure 3 represents only a portion of the transcript from the conversation among Camilo, Eli, Logan, and Zhang. By translating the semantic relations we coded into thematic maps, we outlined the multiple different connections that students made over the course of their work on the problem. We used the maps to identify how students linked multiple sources of knowledge through their discussions of the problem.

Each author completed the semantic analysis of two groups of students. We exchanged our coding for comparison and to identify disagreements, which we resolved during a sequence of meetings. After completing the semantic analysis, the first author created thematic maps to represent the semantic relations within each group. Finally, we referred to transcripts of the whole-class discussions to inform our understanding of the semantic relations in students' talk. Because students considered many different alternatives during their work in groups, we were interested in which ideas students prioritized in their solutions to the problem. During the whole-class discussion, students' rationales for the location of a new grocery store became explicit. Referring to the whole-class discussion allowed us to better understand how students used, or disregarded, certain ideas that surfaced through the process of giving meaning to the map.

4. Results

We report our results according to our two research questions. First, we describe the thematic items that surfaced in students' talk, and how those thematic items included meaning related to the problem context as well as according to students' knowledge of geometry. Second, we illustrate how groups differed in their use of distance, and how those differences contributed different geometric meanings to students' solutions.

4.1 Thematic Items in Students' Talk

Table 4 lists the thematic items that students contributed in their discussions of the grocery store problem. Many of these ideas were initiated by the presentation of the problem, including the *map*, the *boundaries*, *existing grocery stores*, *location*, and the *new grocery store*. Based on students' understanding of the problem and the community, they introduced ideas such as *walking distance*, *biking distance*, *people* (referring generally to people in the community), and *students* (referring to themselves). Students also introduced geometric ideas including *circle*, *distance*, and *point* (referencing a point on the map), which is of interest because the problem did not specify any particular geometric strategy or solution.

Table 4

A List of Thematic Items in Students' Discussions

Ideas Included in the Problem Prompt	Ideas Introduced by Students
The map	Walking distance
Boundaries	Biking distance
Existing grocery stores	Students (in the group)
New grocery store	Distance
People (in the community)	Circle
Location	Point (on the map)

In students' discussions of the problem, they alternated between their own personal interests, the interests of the community at-large, and how to describe and solve the problem geometrically. Within and across these clauses, the central ideas of students' talk allowed them to bridge their understanding of the problem context with their need to develop a mathematical solution.

4.1.1 Students' personal interests. There were different ways in which students considered themselves in relation to the grocery store problem. Students located themselves on the map according to where they lived. Also, students described themselves in terms of whether they could walk or bike to an existing grocery store. In these cases, the thematic items of *walking distance* and *biking distance* were semantically related to individual *students* according to their preferences. In addition to where students lived and where students could walk or bike, many students expressed their preferences for where their families did their grocery shopping. In these cases, students described spatial relationships between their homes and the *existing grocery stores* on the map, or they considered the attributes of the *existing grocery stores*.

4.1.2 Students' participation in the local community. Students in all four groups gave most consideration to the infrastructure of the town and needs of the community. We use an excerpt from group 1 to exemplify students' consideration of the local community, and how those considerations contributed to students' meaning making through their work on the problem. The conversation in group 1 is illustrative of the groups' considerations of the different attributes of the map, as well as how groups would eventually abstract these attributes through geometric representations. In the exchange below, the students noticed a particular region of the map that seemed to be sparsely populated with grocery stores, although the students knew the region to be heavily populated with people.

Pankaja: There is a lot of open area there.

Brian: There is a lot. And there's a lot of people.

Pankaja: But how many – Yeah there is. Like Carmel Lakes area.

Ruth Yeah.

Brian: Carmel Lakes is over there.

Ruth: I actually think that's a really good place.

Pankaja: The hospital's right there.

We highlight particular aspects of the above exchange for the purpose of illustrating the semantic relations that surfaced in students' talk. First, Pankaja noted the existence of a particular open region on the map, to which Brian ascribed the attribute of having many people.

The students identified this open region as a named neighborhood in the community, Carmel Lakes. Finally, Ruth noted that the region of map around Carmel Lakes could be "a really good place" to locate a new grocery store. According to Ruth's comment, the region carried the attribute of being a good location given the combination of open space, a high population density, and the location of a hospital near the region.

In students' discussions across all of the groups, the idea of *people* became the most prevalent thematic item in students' talk. Overall, students were concerned with where people lived, attempting to locate people at specific *points* or in regions of the map (Table 5). Students also considered how far people in the community could reasonably walk to get to a grocery store, and the idea of *distance* became central to these conversations. The ideas of *walking distance* and *biking distance* became more abstract, and less directly related to the specific students in the group. While discussing people in the community, without considering specific individuals, students needed to make some generalizations about what would be a reasonable walking or biking distance. Students typically agreed upon a distance of around 1-2 miles, thus assigning specific numeric values to these distances.

Table 5

Students' Considerations of Attributes While Solving the Grocery Store Problem

People in the Community	Regions on the Map	Grocery Stores
Live in neighborhoods. Can walk or bike 1-2 miles. Prefer to take the bus. Do not have a grocery store nearby. Do not need a new store. Are wealthy.	Have space for a new store. Are crowded. Are downtown. Are not safe for walking. Are sparsely populated. Are near a hospital.	Some stores exist that are not included in the given map. Are more expensive than others. Are larger than others.

Do not own cars. May not want a grocery store right by their house. May not be in the best shape. May go to the grocery store straight from work. Cannot walk as far carrying bags. Need lots of groceries. May not want the traffic and debris associated with a grocery store. Like certain grocery stores more than others.	Have more competition for a new store. Unable to support a new store. Are near a church. Are friendlier to walkers and bicyclers. Are residential. Have one-way streets. Are near an interstate. Have lower local sales tax. Have commercial land for sale.

Students' discussions of people in the community also led them to consider specific attributes of the various regions of the *map* and of the *existing grocery stores* (Table 5). Students ascribed information to the map that we had not provided our presentation of the problem, including whether certain neighborhoods were populated enough to support a new store. Several students considered that the most convenient locations might not be the best locations for sustaining a new grocery store. Students also considered possible locations relative to other landmarks, for example hospitals, churches, or an interstate. Students gave attention to which stores were larger, with a greater variety of groceries, and which stores were more expensive than others. They also noted the existence of multiple stores that we had not included on the map, including smaller discount grocery stores, pharmacies, and retail chains that sold groceries in addition to other products.

4.1.3 Establishing a geometric solution. Throughout the session, the teacher reminded students of their responsibilities to use geometry to give a precise location of the new grocery store. At times, the thematic items in students' talk took on meanings that were more abstract and geometric. Once students determined what would be an appropriate walking or biking distance, students needed ways to represent these distances on the map we provided. To do so, students spoke of *distance* not in terms of how far people could walk but as a measurement between two *points* on the *map*. The attributes of various regions and grocery stores, which had been particularly important in students' discussions of the community, became less relevant. Instead, the *existing grocery stores* became synonymous with *points* on the *map*, and *distance* provided a measurement between *points*. Multiple groups introduced the idea of *circles* as a way to represent a fixed distance from the existing grocery stores.

Returning to the example of group 1, who had previously discussed the option of locating the new store near a neighborhood known as Carmel Lakes, the students used circles to determine whether that particular region of the map needed a new grocery store. Getting ready to construct the circles, Pankaja suggested that the Carmel Lakes area might have already been within a 2-mile radius of an existing store:

Pankaja: It looks like the area by *G* and *H* like, west of that, it seems like it'd be covered, two miles.

The points *G* and *H* in Pankaja's statement above referenced the location of two of the existing grocery stores on the map. Pankaja noted that the region west of these points, which was the region including the Carmel Lakes neighborhood, may have been included within a 2-mile radius of those existing stores. Pankaja's comment was significant in its indication of a shift in the way the group considered the Carmel Lakes region of the map. Previously students relied entirely on their knowledge of the community to describe the region, specifically that it was open, populated with residents, and near a hospital. Now, Pankaja and the members of her group began to abstract the regions of the map by considering their locations relative to circles surrounding existing points. By drawing upon the context of situation, and specifically the teacher's request for students to develop a geometric solution to the problem, the students began to make a connection between the problem context and the mathematical ideas relevant to solving the problem. To better determine the regions of the map that needed a new grocery store, the group decided to construct circles with 2-mile radii around existing stores on the map.

Ruth: Measure it [the circle] out so that it's two miles, because bike, I think that biking distance is about, or walking distance is about two miles.

Ruth's comment illustrates an important aspect of the semantic relations among the thematic items in students' talk. Ruth instructed the members of her group to measure the circle so that "it's two miles," indicating that the circles students drew on the map needed to have the attribute of being 2 miles in radius. This semantic relation is illustrated in Figure 4, below.

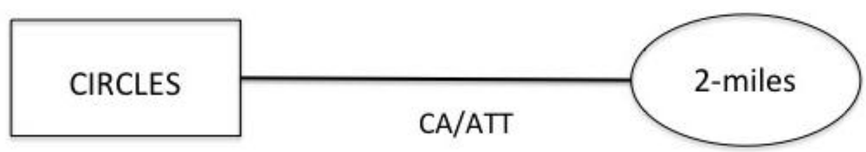

Figure 4. An illustration of the semantic relation indicating 2 miles as an attribute of circles.

Ruth continued by describing the purpose of constructing the circles this way, in particular because 2 miles represented a reasonable walking or biking distance for people in the community. By arguing that biking or walking distance would be 2 miles, Ruth used the 2-mile distance to make a connection between the people in the community and the geometric representation on the map. As illustrated in Figure 5, there was a semantic shift in how students used the phrase "2 miles", which served both as the distance people in the community could walk and as an attribute to describe the circles students would create on the map.

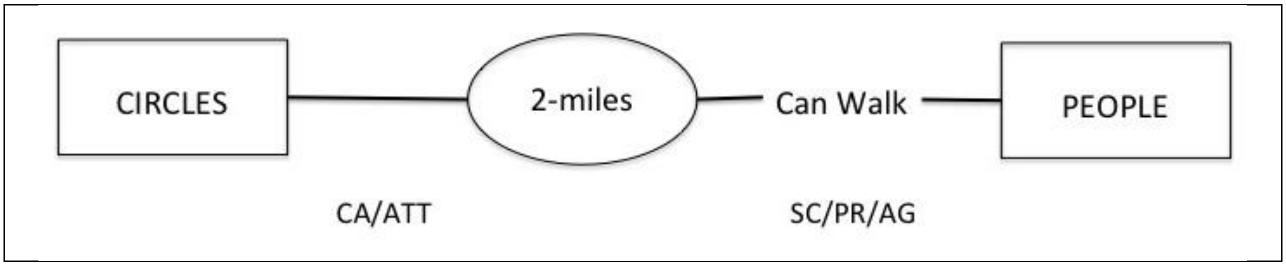

Figure 5. An illustration of the semantic relation indicating 2 miles as an attribute of circles.

Overall, the thematic item of *students*, and the personal interests of students in the group, became less prevalent as students considered the needs of the community and a geometric solution to the problem. *Distance* was an idea that shifted between the specific and the general, for example when considering how far a particular person in the group could ride his bike compared to how to represent a fixed walking distance on the given map. The *existing grocery stores* carried many attributes, which were especially important in students' discussions of the community but which became less important as students established more formal geometric solutions to the problem. Overall, given that we encouraged students to reduce the distance that people would need to travel to get groceries, the central idea of *distance* was prevalent in all aspects of students' discussions. We consider students' uses of distance more closely to understand how students gave geometric meaning to their work on the grocery store problem.

4.2 Differences in Students' Use of Distance to Solve the Problem

All four groups in the study treated the map at times as a geometric representation (Figure 6). Groups 1 and 3 both constructed circles of fixed radius to represent regions of the map that were already within a certain distance of a grocery store. The students in group 1 constructed circles with 2-mile radius, while the students in group 3 constructed circles with 1-mile radius. The two groups selected the two different radii based on their determination of how far people could reasonably walk or bike to the store. The students in group 2 identified two points that seemed farther away from the existing stores, and they used a ruler to find a point that would be equidistant from those two points. The students in group 4 located the new grocery store at the intersection of two streets.

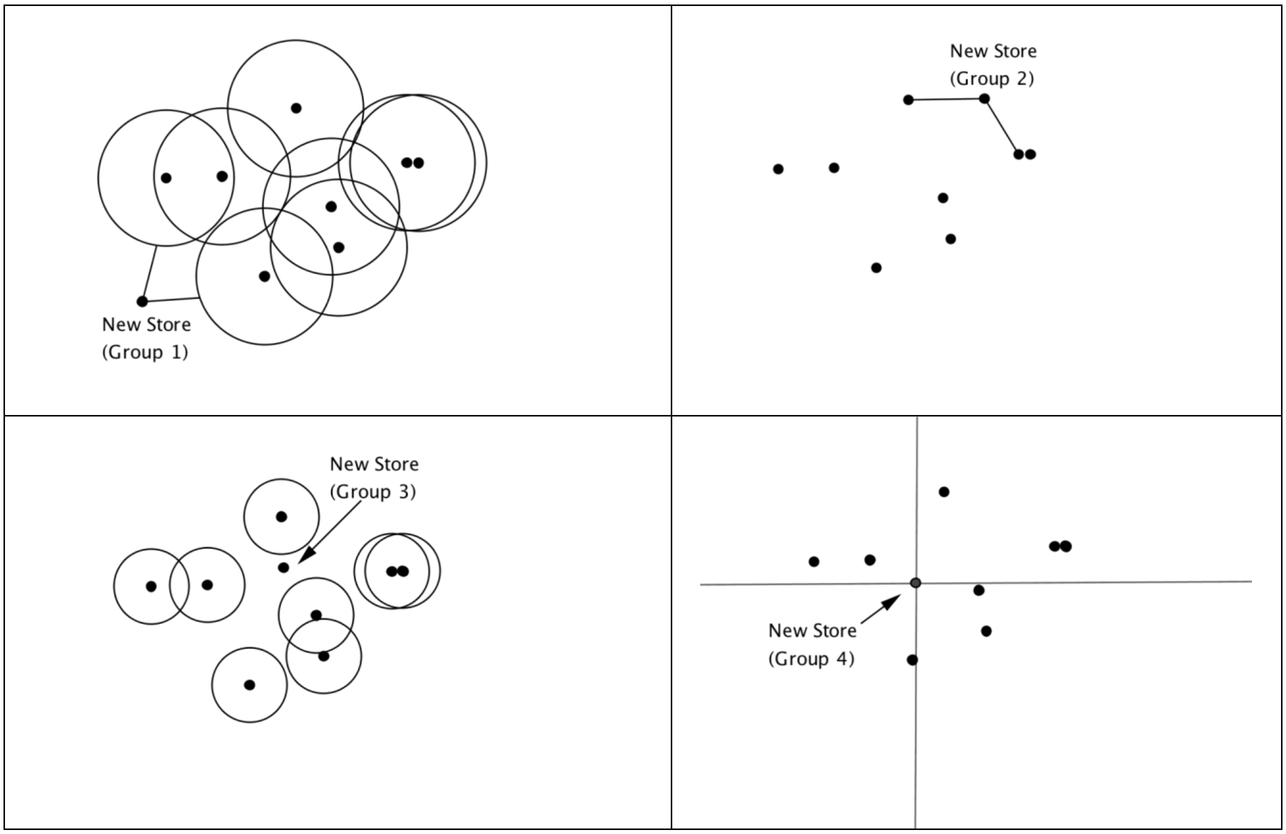

Figure 6. Four geometric representations of the grocery store problem.

Figure 7 provides a partial thematic map for each group of students, focusing on students' discussions of how to locate the new grocery store. These maps illustrate a contrast in the ways that students incorporated geometric strategies to determine the optimal location for the new store. As is illustrated by their diagram in Figure 6, the students in group 1 located the new grocery store at a point that they measured to be equidistant from two circles of 2-mile radius, which they constructed around the existing grocery stores provided on the map. The semantic relation, LOC, between the central ideas *new store* and *distance* indicates that the measurement of distance (and, specifically, equidistance) determined the location of the new store. Although the students in group 2 did not use circles to locate the new grocery store, they were similar to group 1 in how they related *distance* to the *new store*. More specifically, the students in group 2 identified a point that was equidistant from two of the existing grocery stores on the map, and that distance determined the location of the new grocery store. Groups 1 and 2 were similar in that in each group, students used distance to locate the new store.

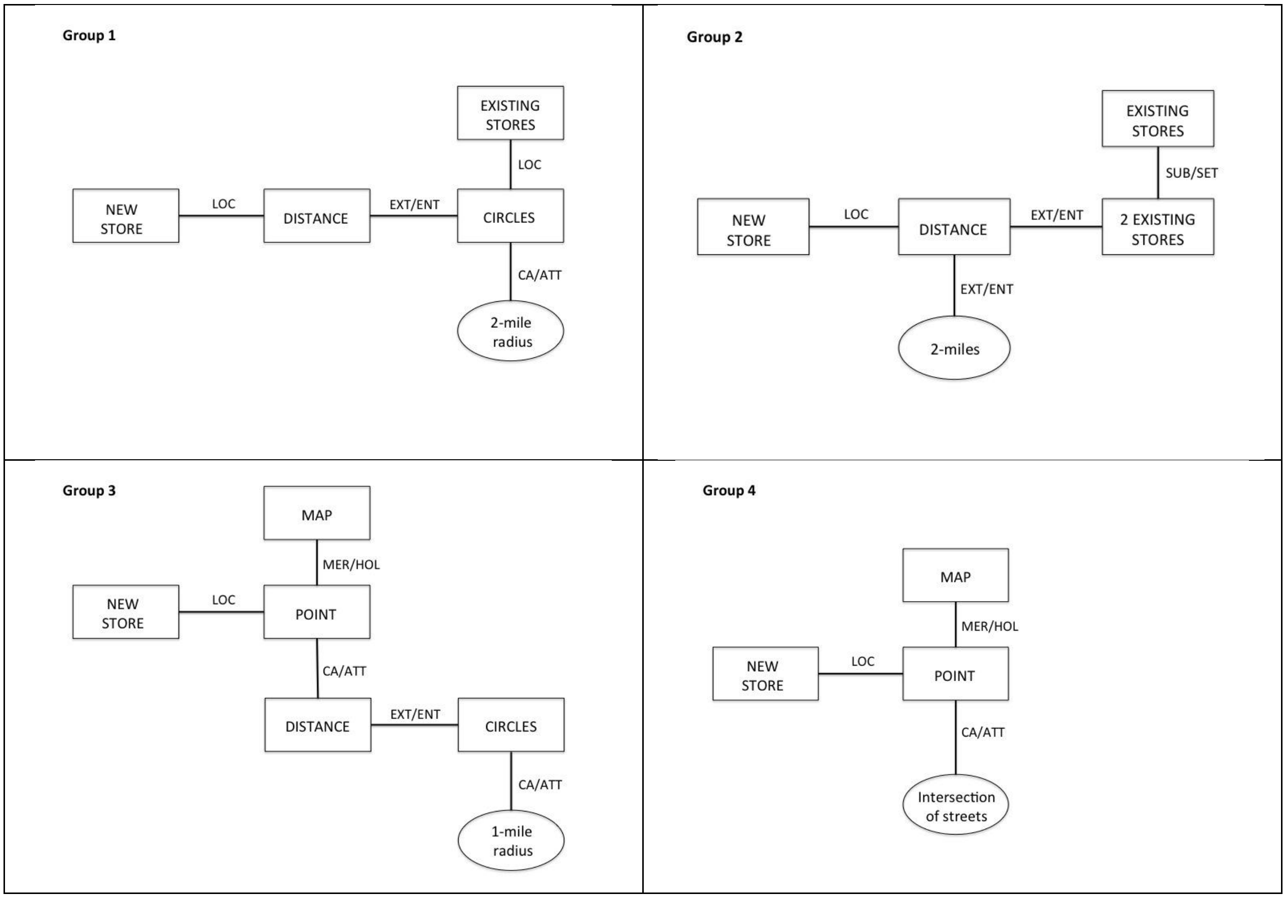

Figure 7. Four thematic maps for locating the new grocery store.

Group 3 used a different set of semantic relations from group 1, even though their written solution looked quite similar. Group 3 used their knowledge of the community to determine what they saw as an ideal location for the new grocery store. By drawing upon their knowledge of streets and intersections, the students in group 3 decided to locate the new grocery store at a particular point on the map. In response to the teacher's request to establish a geometric solution to the problem, the students then constructed circles of 1-mile radius to describe the location of the new store relative to the existing stores. By constructing circles, the students in group 3 were able to say that the new store was farther than 1-mile away from any of the existing stores on the map. Semantically, group 3 used *distance* to describe an attribute of the *point* they decided to locate the *new grocery store*. This is a contrast to groups 1 and 2, both of which used *distance* to determine the location of the *new store*.

Finally, the students in group 4 located the new store without using distance either to define the location of the new store or to describe its location. Like all students in the study, the students in group 4 were concerned with placing the new grocery store in a location that would be appropriate for the needs of the community. Adam advocated against placing the grocery store in a residential area, and Nina argued to her group that they should locate the grocery store in the part of town with lower sales tax. The students decided to locate the new grocery store in the part of town with lower sales tax on a corner that was sufficiently away from residential neighborhoods. They used the intersection of streets to precisely describe the location of the new store. For the students in group 4, the distance between the new grocery store and the existing grocery stores was not a determining factor in locating the new store, as it was in groups 1 and 2. Neither was the distance from the new store to the existing grocery stores an attribute through which they described the new location, as the students in group 3 had done.

5. Discussion

Evidence from this study suggested some of the ways in which students used discourse to construct meaning around a geometric solution to a real-world problem in mathematics. As they discussed the problem context and devised a geometric solution, students accomplished several semantic shifts in their use of terms relevant to the problem. Thematic items such as *walking* *distance* and *existing grocery stores* were at times closely connected to *people* in the community, while at other times *walking distance* and *existing grocery stores* were connected to more abstract mathematical terms. Another finding of this study was the difference in how groups located and described the *point* of a *new store*. While some groups used *distance* to determine the location of a *new store*, other groups located the *new store* by other means and then used *distance* as an attribute to describe its location. We consider the findings of this study in light of the social semiotic framework, and then we discuss the implications of this work for teaching mathematics through real-world problems.

5.1 Considering Students' Discourse From a Social Semiotic Perspective

Students' use of language makes relevant aspects of the contexts of culture and situation through which they participate in a particular social practice (Lemke, 1990; Morgan, 2006). In this study, students participated in the practice of doing mathematics through work on a real-world problem. Students' interpretation of the problem context contributed to how their discussions informed, and were shaped by, the context in which they worked. The map we provided students contained very little information other than the identification of major roads and the locations of existing grocery stores. Through students' shared context of culture, which refers to the shared goals, values, and prior knowledge of the participants of the study (Morgan, 2006), they helped to define the central ideas and constraints of the problem.

The problem context evolved slightly differently for different groups as they emphasized certain aspects of the community. For instance, all groups wanted to locate the new store in a place that would allow it to be successful. However, while groups 1 and 2 emphasized the need to locate the new grocery store far away from existing stores, group 3 prioritized the decision to locate the new store in a more central part of town. To do this, groups considered different attributes, such as how densely populated particular regions of the map were, how much competition a new store would face at different locations, and how far people would walk to get to a grocery store. Group 4, in deciding where to locate their new grocery store, was most interested in the attributes of the neighborhoods and existing stores. Rather than justifying their location through population density or distance, the students in group 4 located the new store in the neighborhood with the lowest local sales tax. Students' considerations of the relevant information for solving the problem illustrate a contrast from what is typically considered appropriate discourse in school versus out-of-school settings. Articulating this contrast, and in particular the important role of the problem context, can be helpful for students to understand how meaning is made in a school setting (Moschkovich, 2007; Schleppegrell, 2004). In this case, students' discussions of ideas that were not explicitly mathematical, for example how far people could walk or which neighborhoods needed a new store, provided an entry point towards giving meaning to the problem. Students' discussions of the problem context informed their mathematical meaning making.

The context of situation, or in other words the shared goals and resources of the after–school environment (Morgan, 2006), became evident in students' talk as they shifted among discussions of the community and discussions of a geometric solution. There were specific elements in the design of the activity that fostered students' geometric meaning making, most explicitly that the teacher of the session implored students to provide a geometric perspective to an imaginary team of urban planners trying to determine the location of a new grocery store. We also did not provide students with information such as population density or economic data, which may have contributed to alternative solutions to the problem. The context of situation required students to use geometry in some way to solve the problem. Because of this, students introduced geometric ideas into their discussions, which allowed them to abstract some of the information included in the problem context. Students' semantic shifts in how they used thematic items such as *people*, *distance*, and *existing grocery stores* on the map enabled them to apply geometric ideas to answer a real-world problem.

Students complied with the teacher's request to establish a geometric solution to the problem, although there were differences in students' meaning making through the question of what counted as a geometric solution to the problem.[8] In particular, the students in group 4 seemed satisfied that they had established a geometric solution by locating the new store at an intersection of streets. The students' use of the term "intersection" illustrates the importance of semantic relations for the ways that students integrate their discussions of a problem context with geometric ideas. Because "intersection" is a typical term in geometry, students may have seen their justification as an appropriate geometric solution. This interpretation is reasonable considering that mathematical discourse often draws upon everyday language in specialized ways for the purpose of establishing mathematical meanings (Pimm, 1987). However, the differences between everyday uses and specialized uses of particular terms are not always transparent from the perspective of students (Morgan & Alshwaikh, 2012). The students' use of "intersection" did not actually carry geometric meaning, as it was not semantically related to other geometric ideas or relationships. The term was related to two particular streets in the community, along with the attributes of those streets and people who lived near that intersection. Although all of the students participated in solving the problem according to the context of

[8] In classroom settings, teachers and students often act according to a *didactical contract* (Brousseau, 1997), or a set of implicit norms that serve as a contract delineating the responsibilities of teachers and students (Aaron & Herbst, 2012; Herbst, 2003). Although we conducted this study in an after-school setting, students knew that they would be participating in a mathematical problem solving session and they seemed to adhere to a similar set of norms.

situation, the ways that students helped define this context were reflected in the ways that they justified the locations they chose.

Similarly to how learning science require learning how to "talk science" (Lemke, 1990), learning mathematics requires students to learn how to speak mathematically (Lemke, 1990; Morgan, 2006; Schleppegrell, 2007). Students used two distinct semantic patterns relating the point representing a new grocery store to distance. Given these different patterns, one could wonder whether there is a more appropriate semantic construction within the context of the grocery store problem, or more generally within a geometry classroom. In geometry, distance or length can sometimes be used to determine the location of a point (as when doing geometric constructions), or to describe an already located point (as is sometimes the case in a geometric proof). For example, to prove that the medians of a triangle intersect at a single point (as in Figure 2b), a typical solution might first locate the point by constructing the intersection of two medians (i.e., locate a point equidistant from two others) and then invoke geometric relationships to describe the lengths of the segments between the intersection and all three of the given points (i.e., compare the relative distances between the points). In other words, the proof would require a combination of semantic patterns, to both locate and describe the point of intersection through the use of length. In the case of points and distance in geometry, as in many other areas of mathematics, there is not a unique correct semantic construction between the terms. However, it is important to recognize and appreciate the nuances of multiple different semantic patterns, which are not always obvious at the moment of having a discussion in a classroom (Moschkovich, 1999). By examining students' discourse through work on the grocery store problem, we expound some of the different ways that students can incorporate different semantic patterns in their discourse.

We expect that the problem context contributed to the variety in students' semantic patterns. In an analogous geometry problem absent of any real-world context, students may be asked to locate a point on a plane equidistant from a collection of given points. In such a case, students would have fewer resources for locating the new point *without* using distance, with the exception of making an initial estimate of where the new point should be located. In the grocery store problem, the 2-dimensional plane represented a great deal of information about students' communities, and students could use that information to determine the location of the new grocery store. A problem situated in a real-world context may expand the resources that students have for solving the problem (Drake et al., 2015; Turner et al., 2012). Because of this, real-world problems may also expand students' opportunities for mathematical meaning making, through a variety of semantic patterns corresponding to different solutions to the problem. At the same time, we recognize that students' semantic constructions may not always align with typical mathematical discourse, as we saw with group 4's use of "intersection." In these cases, the role of the teacher is crucial to provoke students' connections between key mathematical ideas in their talk. Teachers may be able to support students' opportunities for meaning making by establishing a common language with students during the set-up of the task to discuss the relevant contextual and mathematical features (Jackson et al., 2013).

5.2 Implications for Teaching Mathematics Through Real-World Problems

Students have many experiences that contribute to their mathematical meaning making, including prior experiences in mathematics classes as well as knowledge from their homes and communities (Carpenter, Fennema, Franke, Levi, & Empson, 1999; González, N., Moll, & Amanti, 2005; Turner et al., 2012). Because of this, there is opportunity to support students' learning by creating problems that make connections to students' multiple knowledge bases

(Drake et al., 2015). We found that by creating a geometry problem situated in students' local community, we could expand students' meaning making in two ways. First, the thematic items of students' talk carried both contextual and geometric meaning, through the construction of varied semantic patterns. Also, in addition to the ways students incorporated and defined the problem context, students established different geometric meanings in their solutions to the problem. Students' discourse through work on real-world problems can support some of the ideals of mathematics education policy documents (e.g., NCTM, 2000), which emphasize the importance of building upon students' experiences and tasks that can be solved in multiple different ways.

Increasingly, research in mathematics education is attending to how students' mathematical thinking is intertwined with students' social and cultural knowledge (Turner & Drake, 2015). Students' discourse can serve multiple purposes in this effort, both as a resource for students to make connections between these two areas and as a window for researchers and educators to understand these connections. Succinct and efficient explanations and solutions are valued in academic and school mathematics (Moschkovich, 2007), although solutions can become increasingly complex when students must manage the mathematical and contextual aspects of a real-world problem. When students work on real-world problems in groups, they often switch between discussions of mathematics and discussions of context in ways that can be difficult to predict (Otten & Soria, 2014). With evidence of how students connect mathematical meanings to their knowledge of a particular context, teachers can use students' discussions as a resource for introducing and making explicit the ways of speaking mathematically that are typically accepted by the mathematics community. Teachers can build upon students' contributions as opportunities to introduce and make connections with mathematical relationships, which may be especially beneficial for students of culturally and linguistically diverse backgrounds (Zahner et al., 2012). This study provides a concrete example, in a geometry setting, of how students connect mathematical meanings with out-of-school knowledge through discussions of a problem.

6. Conclusion

We examined students' mathematical meaning making through an analysis of students' discourse in an after-school setting where students needed to determine the optimal location for a new grocery store on a map of their local community. Through a thematic analysis of students' talk, we found that several ideas surfaced in students' conversations to which students attached both geometric as well as contextual meaning. We also found differences in the semantic relationships that students established in using distance to locate the new grocery store. Even students who used the same terms, such as distance, in their solution to the problem gave different geometric meaning to the problem through the semantic patterns they established. Overall, students' work on the grocery store problem created opportunities to establish both real-world and mathematical meanings, as well as relationships between the two. Additionally, the problem allowed for groups to make different geometric meanings according to the solutions they proposed.

This study has implications for understanding students' discourse through work on real-world problems in mathematics. Students' *context of situation*, or the particular setting in which students work on a particular problem, can be complemented by the *context of culture*, or the shared knowledge and experiences that students bring to their work (Morgan, 2006). Both of these aspects of context were important for students to engage in mathematical meaning making, the prior for considering and evaluating alternative solution options and the latter for helping students focus on the geometric aspects of the problem. The problem as we provided it, which included a map of the community and required no specific solution strategy, allowed students to draw upon the context of culture and to define the goal of the activity. Through students' discourse, they made connections between their extensive knowledge of the community and the mathematical ideas they were responsible for articulating. Knowledge of how students draw upon contexts of culture and situation can serve as a resource for teachers to help students interpret real-world problems in a mathematical way and to introduce semantic relationships that are mathematically important, in ways that are connected to students' experiences.

Thematic analysis (Lemke, 1988, 1990) allowed us to examine students' conversations through the lens of how they connected words and phrases in order to construct mathematical meanings. In studies of classroom discourse, thematic analysis has illuminated some of the ways that teachers and students make meaning in mathematics and science classes (Chapman, 1993; Herbel-Eisenmann & Otten, 2011; Lemke, 1990; Viechnicki, 2008; Webel & DeLeeuw, 2016), as well as how teachers talk about the mathematics register (Herbel-Eisenmann et al., 2014). We have built upon this prior research by uncovering some of the connections between mathematical and non-mathematical meanings through discussions about a real-world problem. We expect that this analysis can be useful in other settings, in particular given the importance of connecting with students' prior experience across all content areas (Donovan, Bransford, & Pellegrino, 1999). Because differences in semantic relationships are not always apparent at the moment of having a discussion in class (e.g., Moschkovich, 1999, 2007; Lemke, 1990), thematic analysis can expand teachers' awareness of some of the nuances and discrepancies that are crucial for understanding the discourse of a particular discipline.

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