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In the workplace mathematics does not announce itself: towards overcoming the hiatus between mathematics education and work

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Abstract Preparing students for their lives beyond schooling appears to be a universal goal of formal education. Much has been done to make mathematics education more "realistic," but such activities nevertheless generally remain within the institutional norms of education. In this article, we assume that pedagogic relations are also an integral part of working life and draw on Bernstein's work to address their significant features in this context. However, unlike participation in formal mathematics education, where the discipline is central, workers are likely to be confronted by, and need to reconcile, a range of other valued workplace discourses, both epistemic and social/cultural in nature. How might mathematics education work towards overcoming the hiatus between these two very different institutional settings? This article will argue that the skills of recontextualisation, central to teachers' work, should be integral to the mathematics education of all future workers. It will consider theoretical perspectives on pedagogic discourse and the consequences of diverse knowledge structures at work, with implications for general and vocational mathematics education.

Keywords Mathematics at work · Bernstein · Pedagogic discourse · Recontextualisation · Knowledge structures

1 Introduction

Across a wide range of industries and occupations, people are required to use, develop and communicate mathematical ideas and techniques in a diversity of ways with others who have

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differing expertise, experience and interests, including in mathematics itself. Workplace problems requiring mathematical reasoning and calculations are usually embedded in physical or intellectual tasks, rich in context, with a range of constraints that are oftentimes mutually contradictory, but always need a workable answer, and usually within a short space of time. Following the work of Ellström (2010), some problems arise within a *logic of production*, where speed, accuracy and consistency are needed and valued in routine procedures and processes. Other problems arise within a *logic of development*, where questioning, creativity and innovation are more valued. Such problems could include breakdowns in communication or a lack of appropriate physical or human resources. However, in a great many jobs, problem solving is an expected and routine part of the day's work: Every new request or order requires an original or customised solution within given parameters. Whether using mathematics explicitly or implicitly in these processes, no matter how trivial, the worker must also take into account all of the relevant contextual knowledge in their decision making.

1.1 What is mathematical knowledge?

Tall's (2013) three worlds of mathematics framework begins with *practical* mathematics, based on experiences with shape and space leading to their conceptual embodiment, and experiences with number leading to operational symbolism in arithmetic and algebra. Focusing on their properties leads to *theoretical* mathematics of Euclidean proof in geometry, symbolic proof in algebra and a blending of embodied and symbolic reasoning using language. Properties of these, in turn, lead to axiomatic formalism and *formal* proof at the highest level of mathematical thinking. Practical, theoretical and formal mathematics develop with maturation, but, once developed, all forms continue to interact and lead to different forms at ever more sophisticated levels. Cognitive development is based on "human perception, action and the use of language and symbolism that enables us to develop increasingly sophisticated knowledge structures. It is based on … blending together perception, operation and reason" (Tall, 2014, p. 224). The conceptual embodiment of shape and space, and the operational symbolism of arithmetic, with ubiquitous forms of measurement (formal and informal), are the most easily visible forms of practical mathematics at work and elsewhere. The more easily visible aspects of theoretical mathematics, with its symbolic algebra and geometry, can be seen in the functioning and design of spreadsheets, three-dimensional machining and quality control statistics, for example. Although invisible in most workplaces, the logic and mathematical power of formal mathematics underpin technologies of management and production: for example, extending human capabilities in the realms of micro- and nanotechnology, extreme speeds, temperatures, distances, etc. It enables, among other things, predictive modelling and analysis in business analytics, financial mathematics, communications and transportation, complex systems and computer systems (SIAM, 2012).

The above observations mean that people doing mathematics at work at any level are likely to draw on and move between the range of forms of mathematical knowledge available to them, often implicitly but sometimes also explicitly. Doing mathematics at work involves the seamless integration within a specific workplace context of disciplinary mathematics knowledge in any or all of the forms described above, manifested as conceptual or propositional knowledge, and/or practical or praxis knowledge, along with tacit rationality identified by Gustafsson and Mouwitz (2010) as intentional and often unconscious acting constituted by experience but outside of formal, articulable knowledge and therefore not easily recognised in formal assessment tasks. Within this context, Wedege (2013) describes human competence as workers' capacities (cognitive, affective and social) for acting effectively, critically and constructively in the workplace.

1.2 What is vocational mathematics education and how are theories related to practice?

Research in mathematics education is already a complex field with many competing and coexisting theoretical frameworks responding to a diverse range of policy positions on what mathematics to teach and how to teach it, how much, to whom, for how long, etc. For the most part, there is a general assumption of linear progress according to the age of the students concerned. Inserting the qualifier of "vocational" immediately intensifies this complexity through the inclusion of dominant industry voices at policy level, and a vast range of possible vocational/professional outcomes for graduates together with increasing uncertainty about gaining secure and dignified employment in the face of a rapidly changing demand for labour in response to technology innovation and globalisation. Frequently, learners are educationally disadvantaged in relation to their compulsory education peers and may be on nontraditional educational pathways. In brief, the existing intersection of the discourses of mathematics and education is expanded by the inclusion of a third sector, the discourses of industry in general and specific occupations in particular.

The complex relations between formal mathematics education and how people actually do mathematics at work have been addressed extensively. See, for example, Bakker and FitzSimons (2014); Bessot and Ridgway (2000); Damlamian, Rodrigues, and Sträßer (2013); and Straesser (2015) who identified four prototypical forms of case study on how mathematics is used at work: mathematical knowledge, tools, dispositions and modelling. In vocational mathematics teaching practices, there are multiple operational frameworks anchored in different theoretical perspectives with different consequences for developing mathematical knowledge (Hahn, 2016). Ultimately, professional and skilled workers are able to integrate relevant disciplinary domain knowledge, mathematical and vocational, as well as knowledge of the professional or vocational contexts developed through formal and informal learning, including social and cultural knowledges. This is in addition to the knowledge that each worker brings as a result of their personally lived experiences.

A variety of theoretical perspectives have, individually or in combination, informed workplace mathematical studies and subsequent recommendations for vocational mathematics education. These include Cultural-Historical Activity Theory (CHAT), boundary crossing, situated abstractions, mathematical modelling, competency development, the work of Bourdieu on habitus and use/exchange value and also the work of Bernstein on the principles of transforming knowledges into pedagogic communication, the focus of this article. Of particular interest, not the least in this article, is the need to overcome the problematic nature of the concept of transfer, particularly in the immediate epistemological and sociocultural contexts of work (e.g., Beach, 1999; Evans, 1999; Evans, Guile, Harris, & Allan, 2010).

Cultural-Historical Activity Theory integrates epistemological and sociocultural contexts and traces its origins to the cultural psychology of Vygotsky and Leontiev. Several articles recently published in *Educational Studies in Mathematics* have drawn upon CHAT for researching mathematical activity at work and connected this with other theories in order to develop educational implications. For example, LaCroix (2014) also used Radford's Theory of Knowledge Objectification, Roth (2014) also used boundary crossing, Triantafillou and Potari's (2014) study of the mathematical meanings that emerged from their study of telecommunication technicians led them to emphasise the importance of semiotic analysis and Williams and Wake (2007a) drew on boundary crossing, while Williams and Wake (2007b) focused on different genres of mathematical language acknowledging the importance of metaphor on mathematical modelling. Subjectification or identity work was also a common feature of many studies.

In a similar manner, a number of authors have turned to the concept of boundary crossing (see Akkerman & Bakker, 2011), including Kent, Noss, Guile, Hoyles, and Bakker (2007), Bakker and Akkerman (2014) and Hoyles, Noss, Kent, and Bakker (2010) who developed the concept of *techno-mathematical literacies* and introduced a pedagogic device, TEBOs or technology-enhanced boundary objects. Hahn (2014) developed an intervention based on the work of Vergnaud on conceptual fields and Bachelard on regional epistemologies. Mathematical modelling in formal education, intended in part to help students prepare for the workplace, has also been addressed by researchers (e.g., Blum, Galbraith, Henn, & Niss, 2007; Stillman, Blum, & Salett Biembengut, 2015). However, Frejd and Bergsten (2016) highlight the major differences between doing modelling at school and in the professional workplace and also between the two contexts in terms of objectives and accountability. They relate their description of school mathematics—as an outcome of Chevallard's *didactic transposition process* where mathematical knowledge pre-existing from the world beyond school is adapted to become teachable knowledge in school—to the work of Bernstein (2000) and Dowling (2014) on the recontextualisation process, a major focus of this article.

Based on accumulated research into adult and vocational mathematics education, recent years have seen the development of teaching and assessment materials designed to be supportive of everyday or work-related *numeracy* or *mathematical literacies* (Hoyles, Wolf, Molyneux-Hodson, & Kent, 2002); (see, e.g., the now-defunct National Research and Development Centre for Adult Literacy and Numeracy (NRDC) http://www.nrdc.org.uk); also Coben, 2006; Geiger, Goos, & Forgasz, 2015). However, as with mathematical modelling that remains within the mathematics classroom (real or virtual), the constraints of most education systems prevent or seriously diminish the possibilities of replicating the complexities of much contemporary working life in school mathematics curricula or even in adult numeracy assessments such as PIAAC[1] (Boistrup & Henningsen, 2016; Tsatsaroni & Evans, 2014).

One consequence of these limitations is that most mathematics teachers and students who visit work sites, and even workers themselves, find it very difficult to recognise any activity they are able to judge as being mathematical beyond number and measurement (e.g., FitzSimons, 2014b; Nicol, 2002; Wedege, 1999; Williams & Wake, 2007a, b). Unskilled workplace observers are generally looking for, and making comparisons with, the teaching and learning experiences imprinted in their minds after so many years spent within the walls of school mathematics classrooms. Crucially, the kinds and complexity of problems that occur at work contrast sharply with those found in formal mathematics education texts and assessment tasks. Doing mathematics at school and doing mathematics at work are two very different activities (FitzSimons, 2013), epistemologically and socioculturally.

Not only are the concepts found in typical school mathematics curricula difficult to recognise in most occupational activities, but the idea of education itself is generally considered as being limited to the institutions of school and university. While learning via nonformal

[1] PIAAC is an OECD study, similar to PISA, and stands for Project for the International Assessment of Adult Competencies (OECD, 2013)

education may occur in specific work-related training activities, for example in learning new technical or procedural skills, informal learning at work is an ongoing process that encompasses personal, social and cultural knowledges and skills (Eraut, 2004), often specific to a particular job or worksite. Many people do not consciously recognise that learning takes place at work because it is subsumed in routine activities. Importantly, in workplace discourse, there is an ongoing need for communication of an educative kind between stakeholders, where information is sought and shared, and mathematics-related knowledge is, or can be, created or relocated and transformed from the academic discipline of mathematics into the specific context of the problem at hand. In Bernstein's (2000) terminology, there are *pedagogic relations* at work as well as in education.

Most theories adopted in the field of mathematics education and workplaces do not offer the means of interrogating the complexity of vocational mathematics education to encompass the range of explicit and implicit mathematical thinking required at work from both epistemological and sociocultural perspectives in the way that utilising a Bernstein theoretical framework allows. Nor is workplace mathematics formally recognised as a pedagogic activity, and this is the crucial distinction: Students in formal education need to learn the skills of recontextualisation of mathematics, both for themselves to use *and* to communicate effectively with other stakeholders in the workplace. In order for this to happen, mathematics teachers and students alike need to appreciate and understand the importance of both epistemological and sociocultural knowledges at work. In FitzSimons (2014a), the first author of this article briefly introduced Bernstein's (2000) concepts of vertical discourse and horizontal discourse as a means of providing a theoretical background to distinguish between those vocational curricula which focus mainly on conceptual coherence and those which focus mainly on contextual coherence. This distinction enables critique of vocational mathematics curricula which could be said to disempower students, however inadvertently, by restricting content to collections of supposedly useful examples, inevitably based upon past work practices. Given the rapidly changing nature of work in a globalised and technologised world, and the likelihood of ongoing reskilling or upskilling, vocational students in particular need access to coherent conceptual development in mathematics, even if it is not immediately apparent in entry-level employment or in their currently accredited curricula which are generally linked to tradition and a static worldview. This article extends the Commentary to include: (a) an emphasis on identifying pedagogic relations at work and the complexities of the social division of labour, together with their recognition and preparation for realisation, in vocational mathematics education; (b) the existence of multiple workplace (vertical discourse) knowledge structures and their unstable and fluid value hierarchies with respect to mathematics, according to the particular occupation or even the task at hand, which must be reconciled by workers in practical situations; and (c) further development and refinement of both vertical discourse (i.e., strong/weak grammars within the epistemic dimension) and horizontal discourse underlining the crucial importance of social and cultural knowledges, especially with respect to workplace communication in its many and varied forms.

The aim of this article is to address how school and vocational mathematics education can move towards overcoming the hiatus that exists between these two contrasting institutional settings. As noted above, previous assumptions about "transfer" and "application" of school mathematics are generally inadequate when school leavers or graduates are confronted by the realities of work. In order to address this problem in all its complexity, we draw upon the comprehensive theoretical framework of Basil Bernstein (2000) to analytically explore pedagogic relations involving mathematics and other workplace discourses. This article will draw on studies that have investigated mathematics as an integral component of different kinds of workplace activity (e.g., the project described by Wedege, 2013; see also Boistrup, 2016), in order to offer examples of how workers use and create or develop (locally) mathematics in a variety of explicit and implicit ways. Finally, we will draw some practical implications for school and vocational mathematics education.

2 Theoretical perspectives on pedagogy in different contexts

Drawing on Bernstein (2000), in this section, we address pedagogic and social relations at work, various forms of workplace communication and recontextualising rules in education and at work. In Section 3, we discuss the impact of having to work mathematically with different knowledge structures and the crucial differentiation between vertical and horizontal discourses.

2.1 Pedagogic relations at work

Bernstein's work on pedagogic relations is widely known and used in mathematics education research (e.g., Dowling, 1998, 2014; FitzSimons, 2002; Kanes, Morgan, & Tsatsaroni, 2014; Straehler-Pohl & Gellert, 2013; Tsatsaroni & Evans, 2014). His work (e.g., Bernstein, 2000) extends beyond formal education to encompass informal intentional educative activities and cultural practices which take place at work and elsewhere. In such settings, there is a "purposeful intention to initiate, modify, develop or change knowledge, conduct or practice by someone or something which already possesses, or has access to, the necessary resources *and* the means of evaluating the acquisition" (pp. 199–200). From this, it follows that workplaces are almost certain to be sites of pedagogic relations, even if they involve only one self-employed worker and their interactions with clients or customers, suppliers of materials of production, government agencies and so on, physically and/or electronically. A pedagogic relation requires the evaluation of the *acquirer's* response by the *transmitter* to use Bernstein's terminology, but at work, unlike school generally, there can be ongoing role reversals between transmitter and evaluator depending upon the distribution of the knowledge at stake.

2.2 Social division of labour

In education, as in work and society generally, there are mostly unwritten rules about power relationships. Following Bernstein (1990), any *social* division of labour has two dimensions, horizontal and vertical: The *horizontal* dimension refers to specialised categories sharing memberships of a common set, for example workers sharing a common status. The *vertical* dimension refers to the rank position of a category *within* a set, for example supervisors and subordinates within an area of operation, and the ranking relation *between* sets, for example between technical staff and operators or labourers. Although power is immanent in any workplace whether recognised or not by all workplace participants, according to Singh (2002), power relations create, legitimise and reproduce boundaries and thus establish legitimate relations of social order. Singh continues:

> Despite legitimating relations of social order, power relations are never static or stable. Rather, they are challenged, contested and negotiated in the relations of pedagogic communication. In addition, power relations are internalised via pedagogic communication or the social relations of control … (p. 578)

These social relations of control refer to *who* exercises control, *where* and *when*; and *what* pedagogic discourses are made possible. From our perspective, this means that power is distributed explicitly and/or implicitly within a particular work group or team, and that in typical workplaces power is hierarchically distributed, even within a work group. From a mathematics perspective, the implications for work are that:

1. Official recognition of workers' mathematical (and other) authority depends on hierarchical power between levels of authority (i.e., classification and boundaries).
2. Within a given level of work function, recognition of mathematical competence can occur through legitimate demonstration of ability within the specific context.
3. Social forces of control (i.e., framing) may prevent a worker from overtly displaying their mathematical abilities (e.g., due to fears of ridicule or isolation within the work group).

Clearly, the social division of labour is much more complex at work than in formal education settings where traditional power relations are strongly delineated, even if transgressed from time to time. At work, the authority to speak and to act can shift according to the specific situation.

2.3 Pedagogic context

In this section, we discuss classification and framing, and recognition and realisation rules, leading to what legitimate meanings may be made and how legitimate texts may be produced. This is in order to illustrate significant differences between school and work with respect to mathematics and interpersonal power relations. The term *text*, according to Bernstein (2000), refers to a broader set of communicational resources than merely words. A legitimate text "is *any* realisation on the part of the acquirer which attracts evaluation" (p. xvi), and this can be as "simple" as a slight gesture or facial expression.

Bernstein (2000) developed a comprehensive theory of symbolic control concerning how the concepts of *power* and *control* translate into principles of *classification* and *framing*, respectively. Classification is used to examine power relations *between* categories (agencies, agents, discourses and practices). The discipline of mathematics is generally a strongly classified discourse within the institution of education: It is usually very clear when mathematics is the subject in focus. However, in most workplaces, where the focus is on the task or the job, the situation is reversed, and, as noted above, mathematics is often difficult to see. Bernstein uses the term *framing* to examine means of control over communication *within* local, interactional, pedagogic relations.

The principles of classification convey power relations and give rise to the determination of *recognition rules* for identifying *what* counts as a legitimate text within a certain context. Familiarity with relevant recognition rules confers more power, both at school and at work. However, Bernstein (2000, p. 105) notes that "although recognition rules are a necessary condition for producing a legitimate context-specific text or practice," they are not sufficient. In addition to distinguishing between contexts, *realisation rules* are also necessary for producing contextually specific texts or practices: that is, *how* rather than *what* meanings are made. These rules function in the sense of evaluations in specific contexts such as at work and in school in general terms, and in particular situations as they are played out in day-to-day interactions. This means, for example, that mathematical texts as produced by students in formal education contexts may have little relevance in a workplace discourse, even if they are deemed complete and correct within the mathematics education context (FitzSimons, 2002, 2015; Tsatsaroni & Evans, 2014). Communicating at work, where there are a multiplicity of discourses, can require being able to move fluently between formal and informal mathematical discourses as needed, in line with the dialogic perspective recommended by Barwell (2016) regarding mathematics classroom talk.

What counts as a legitimate text (in Bernstein's terms) differs markedly between school and workplace environments. Whereas in school, it is usually assumed to be clear what meanings can be made in a mathematics class and what texts are acceptable to an evaluator; in work, this is not necessarily the case. In work contexts, mathematics is often not announced as such, but is nevertheless present as part of normal routines, often highly sophisticated and technologised. Even people highly qualified in mathematics need to learn about the specific context of the work activity in which they are to engage, in order to *recognise* the meaning of the task, what might be expected of them and what they might (or might not) actually be able to contribute legitimately to the situation by drawing on their existing mathematical knowledge, perhaps extending it in the process. This is not to say that workers are necessarily expected to recognise their ideas *as* mathematics, in the way that mathematics teachers—or even researchers—often recognise, by focusing on specific processes, topics or suites of processes and content. Rather, it means that competent workers should be able to synthesise their understanding of the particular workplace situation with their current mathematical ways of knowing and looking at the world, potentially in ways not available to other workers with a more limited repertoire of mathematics. At the same time as recognising that their mathematical capabilities may be salient, workers also need to learn how, why, when and under what circumstances they might be able to contribute to the situation. Without this contextualised understanding of the social structures and power relations in the division of labour, together with the epistemic knowledge structures including vertical and horizontal discourses in operation (discussed below), they are unlikely to produce legitimate text; that is, if they do not have access to the realisation rules.

Workplace text may include, for example, discussing a proposal involving design or costing with a customer or colleague in a way that ensures each party understands the other's intentions. However, as Bernstein (2000) notes, text is not something which is mechanically reproduced, so that under certain conditions it can change the interactional practice: that is, the strength of classification and framing. When contextualised knowing and experience is widely distributed, mutually respectful conversations oriented towards achieving a common goal can lead to locally, and sometimes globally, new mathematical knowledge being developed (e.g., Nakagawa & Yamamoto, 2013). Conversely, if newcomers are not given sufficient opportunity to become familiar with the rich context of work in a particular occupation or profession, and to learn how their current repertoire of mathematical and other knowledges and skills might play a role, they may become frustrated or even alienated. The question arises as to how formal school and vocational mathematics education might help students to prepare for such a transition to the workplace. From our perspective, Bernstein's concept of *recontextualisation* is a crucial factor.

2.4 The relevance of recontextualisation

Bernstein (2000) theorised principles underlying the pedagogising of knowledge and its transformation in different contexts. With reference to education, he identified rules operating at three levels: (a) the macro, political institutional or systemic level of curriculum and qualifications determination; (b) the meso, school level of organisation of teachers' work; and (c) the micro, classroom level of interaction between students and teachers. Similar rules operate in other forms of work. At the macro level are what Bernstein termed *distributive rules*, distributing different forms of knowledge to different groups of people. At the meso level, *recontextualising rules* regulate the formation of pedagogic discourse. This *recontextualising principle* "selectively appropriates, relocates, refocuses and relates other discourses to constitute its own order" (p. 33). To illustrate the distinction between an original discourse and its recontextualisation, Bernstein gave the example of the *real* workplace discourse of carpentry being transformed into the *imaginary* school discourse of woodwork. Similarly, the academic discipline of mathematics, through its recontextualisation, is transformed into the discourse of school mathematics. Finally, at the micro level, *evaluative rules* shape pedagogic practice in the context of acquisition, operating at the individual level. Evaluation determines what counts as a "valid realisation of that knowledge on the part of the taught" (Bernstein, 1973, p. 85). These three levels are inter-related, and in education there are spaces for contestation and control over the selection of mathematics content, the pedagogies employed and the formal and ongoing informal evaluation of students (see, e.g., FitzSimons, 2002), as in work more generally.

Given that recontextualisation of the discipline of mathematics is central in formal mathematics education, teachers generally spend lengthy periods learning how to recontextualise their own mathematical knowledge in order to communicate effectively with intended learners, and this is an ongoing process. However, our central claim is that recontextualisation of mathematics is a skill that also needs to be developed as part of *all* future workers' repertoire of skills, including the vast majority of students who will be employed in non-academic fields across a range of industries and occupations. From our perspective, it is the pedagogic discourse of workers, constructed according to recontextualising principles, that enables them, when needed, to transform potentially relevant aspects of their formal disciplinary ways of knowing mathematics in respectful dialogical communication with other significant stakeholders—including customers, clients, patients or coworkers involved in the same problem or task. Such communications must take into account not only salient features of the specific context—such as parameters of time, money, ethical and legal requirements and available human and physical resources—but, just as importantly, the knowledge claims of other stakeholders who may have critical scientific, technical, cultural and other expertise to contribute. Such knowledge claims need to be addressed along with mathematical claims for truth so that, although final decisions may not be the recognised as the *ideal* mathematical solutions (formal, theoretical, or practical)—so highly valued in the education system—, they must *not* contain mathematical errors. To explore the complex activity of recontextualisation at work, further we turn to Bernstein's (2000) work on knowledge structures.

3 Knowledge structures at work

In this section, we draw on Bernstein's (2000) work to distinguish between vertical and horizontal discourses in order to capture the different structures of knowledge that (potentially) interact at work and which need to be taken into account in relation to workplace mathematics and mathematics education.

3.1 Vertical and horizontal discourses

Bernstein's (2000) analysis of the structure of knowledge takes two forms: *vertical discourse* and *horizontal discourse*. Vertical discourse refers to disciplinary knowledge, such as formal academic mathematics or the natural sciences, and is described as being theoretical, conceptual and generalisable knowledge and coherent, explicit and systematically principled, with strong boundaries between itself and other disciplines. The procedures of vertical discourse are linked to other procedures hierarchically and thus allow the integration of meanings beyond relevance to specific contexts. Context specificity is achieved in vertical discourse through recontextualisation in the way that mathematics teachers, for example, offer context to theoretical mathematics concepts using a variety of discursive strategies, techniques and artefacts. Once understood, and integrated into workers' repertoires, these concepts can apply to a range of contexts beyond those available in the classroom. This is in contrast to learning an everyday skill which is best achieved in the context of use, but may not necessarily be applicable anywhere else; known as horizontal discourse in Bernstein's terminology.

Horizontal discourse, according to Bernstein (2000, p. 157), "entails a set of strategies which are local, segmentally organised, content specific and dependent, for maximising encounters with persons and habitats." It refers to contextual knowledge, which is likely to be "oral, … tacit, multi-layered, and contradictory across but not within contexts" (p. 157). The *segmental* differentiation of knowledge is the crucial feature of horizontal discourse: Learning to tie one's own shoe laces, to brush one's teeth or to count change are examples of horizontal discourse, with no necessary relation between these different forms of learning. Knowledges are related not by integration, as is the case with vertical discourse, but through the functional relations of the segments or contexts to everyday life, and pedagogic practice may well vary with each segment. Such knowledges "are culturally localised, and evoked by *contexts whose reading is unproblematic*" (p. 159). A person may build up an extensive *repertoire* of strategies which may vary according to the context, and the group may likewise build up a *reservoir* of strategies of operational knowledges. In horizontal discourse, there is not necessarily one best strategy relevant to any particular context.

It may be useful to distinguish the conceptual differences between doing mathematics at school and doing mathematics at work, indicating the depth of the hiatus between the two pedagogical settings. Doing mathematics at work, as in school, explicitly or implicitly, needs to remain within the principles of vertical discourse; that is, it should follow logical mathematical reasoning based on a sound understanding of relevant theoretical mathematics concepts. However, the social relations of the workplace are strongly tied to horizontal discourse which, unlike school, determines, to a large extent, the organisation of mathematical and other meanings and their expression within any given context (e.g., accuracy and precision may be negotiable within their specific context of use; even the decision whether to ultimately use a mathematical approach or not). It is likely that both the horizontal discourse and the largely implicit social relations at work contribute to the difficulties in recognising the mathematics, which both structures the work being done and is, in turn, constrained by workplace discourses and practices.

From a workplace perspective, horizontal discourse could be described as doing something in the course of working in a way which is idiosyncratic to that particular context (task, setting, etc.), ideally within ethical, legal and health and safety requirements, and which is not generalisable in formal qualifications. At work, explicit mathematical or "mathematics-containing" activities "where mathematics is an integrated but identifiable part" (Wedege, 2000, p. 129), there are often "rules of thumb" which could qualify as horizontal discourse. For example, within the space of an individual company worksite, experienced lorry loaders who are qualified with respect to national legal and safety requirements for loading and driving lorries, and who as drivers face potentially fatal consequences as a result of incorrect procedures, are able to use their "local" knowledge (or horizontal discourse) to position the pallets either lengthwise or widthwise, while also using the pallets as informal measuring units to optimise the loading for the benefit of both their employer and the consumers (Boistrup & Gustafsson, 2014). Critically underpinning such decision making is an understanding of the vertical discourse mathematical concepts of size, shape, location and balance. Wedege (2000) discusses how experienced workers use "practical" knowledge which legitimises their decisions to override office- or computer-generated instructions at the local level. While official qualifications draw on the vertical discourse of mathematics and are consistent across an industry, the locally generated knowledges of the particular workers based on their own experience of "what works" are examples of horizontal discourses which are also dependent on the individual worker's legitimacy of status, and hence authority or permission, in each instance.

The crucial importance of both vertical and horizontal discourses at work cannot be overstated. Workers need not only to be secure in their relevant formal mathematical knowledge and understanding (i.e., vertical discourse) but also to have access to relevant workplace horizontal discourses in order to be able to participate meaningfully. The different epistemological contexts at school and at work underline the fact that what counts as knowing mathematics at work is likely to be significantly different from what counts as knowing mathematics in school. Not only are there different ways of social and cultural knowing relevant to given industries, occupations, or even work sites, but also a range of different types of knowledge structure exist within vertical and horizontal discourses with relevance to the range of vocational and professional occupations. Bernstein (2000) offers a way to analyse these knowledge structures to help identify where mathematical knowing could confront other valid and valued ways of knowing. Such confrontations between mathematics and other discourses are rarely, if ever, addressed in the realm of mathematics education, where mathematical approaches and solutions are almost always regarded as superior and incontestable.

3.2 Bernstein's knowledge structures

Here, we describe Bernstein's knowledge structures, while elaborating on vertical and horizontal discourses. We also further develop the concept of recontextualisation, beginning with a brief reference to research in vocational education more broadly.

Recontextualisation offers a powerful means of "bridging the gap" between theoretical and experiential knowledge. This is a crucial notion for vocational curricula and pedagogy (Young, 2006, cited in Hordern, 2014), just as it is for mathematics teacher education in general. In this context, Hordern (2014, p. 22) understands recontextualisation "as an epistemic process which is influenced by the interrelation between the distinct structures of different knowledge types and the social dynamics of vocational education infrastructure." Among the critical features of vocational knowledge in general, and hence vocational pedagogy, are the structure and relationships between various forms of knowledge that coexist at work (see Fig. 1).

The top line of Bernstein's map highlights the critical inter-relationship between vertical and horizontal discourses in terms of power relations. In terms of *vertical discourse*, shown on the left in Fig. 1, the natural science disciplines have the strongest knowledge structures, continuously developing knowledge into single *hierarchical knowledge structures*. Related to these are the highly scientific professions such as scientific and medical research, various streams of engineering and so on. The mathematics requirements of these formal education courses are often taught by mathematicians, or at least by experienced professionals who have rich contextual knowledge to support their teaching of mathematics, including statistics. The

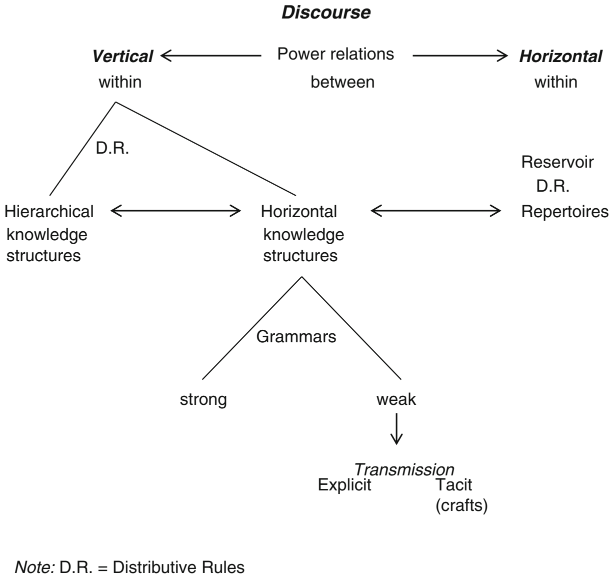

Fig. 1 Bernstein's map of knowledge structures (Bernstein, 2000, p. 168)

theoretical mathematical "truths" are taken seriously, even if the vocational orientation to optimal mathematical solutions is pragmatic. Para-professionals or technicians in these industries also are required to have a similar commitment to mathematical theory, albeit at lower academic levels. As shown by Bakker and Akkerman (2014), medical and other laboratory technicians are required to develop sophisticated understandings of statistics in sub-fields where statistical analysis and interpretation are critical features, for example in scientific or medical technology work in relation to humans or other animals, to subfields of engineering, or to quality control processes in food and pharmaceuticals manufacturing industries. Clearly, the capacity for meaningful communication is essential.

Mathematics, with its strong disciplinary boundaries not only regarding what is accepted as mathematical knowledge and by whom, but also having a series of parallel languages (e.g., algebra, geometry and statistics), is classified by Bernstein (2000) as having a vertical discourse with a *horizontal knowledge structure* (shown in the centre of Fig. 1) and is said to have a *strong* grammar. Other disciplines also classified as having a vertical discourse and horizontal knowledge structures but with weaker boundaries than mathematics are distinguished by what Bernstein calls the strength of their grammar. These include hybrid disciplines such as those found in professions, such as health, education, business studies, transport and logistics, which have weaker grammars than mathematics. These professions very often have practical components to their certification in the form of work placements or simulations of actual work (especially when there are potentially high costs of training in terms of money and/or safety). Blended disciplines with weaker grammars than mathematics are likely to incorporate some mathematics, including statistics, into some of their undergraduate and para-professional technician courses, but mathematics may ultimately compete with other valued ways of making decisions, as will be referred to briefly below in relation to business management.

Bernstein (2000) further subdivides the vertical discourses with weaker grammars into those with either *explicit* or *tacit* forms of what he terms *transmission* (shown at the bottom of Fig. 1). Explicit transmission refers to a pedagogy which attempts to make clear "the principles, procedures, and texts to be acquired" (p. 169) (e.g., trade training), while in tacit transmissions "showing or modelling precedes 'doing'" (p. 169) (e.g., crafts training). Vocational preparation in a trade area (e.g., skilled work in electrical, mechanical, building and construction industries, etc.) requires a mixture of explicit theoretical knowledge and accumulated experience in specific techniques and the tools and materials of the particular trade. It is here that vocational preparation in terms of mathematics tends to focus—in some countries at least—on a so-called competency-based approach (see, e.g., FitzSimons, 2002) where in-depth theoretical development of the vertical discourse of mathematics is largely neglected in favour of assorted skills perceived to be used in the particular trade (Wheelahan, 2009), pedagogically segmented as in the horizontal discourse. Thus, the students are focused on completing collections of practice examples but are not able to develop structural integration of mathematical meanings, and so may not have sufficient mathematical (and other core skills) to retrain in new careers, especially when their trades or occupations change radically or cease to be viable. Finally, Bernstein addresses the crafts domain where most, if not all, skills are learned implicitly, over an extended period of apprenticeship when students also develop increasing familiarity with the specific contexts of their employment. However, it should be acknowledged that any self-employed craft worker also requires explicit mathematical knowledge to operate under a business model within the financial and legal requirements of the state (see, e.g., FitzSimons, 2014b).

In summary, all forms of skilled work contain, to some degree, elements of mathematics (including statistics), used explicitly or implicitly, and sometimes totally embedded within vocational activities. Moreover, within the bigger picture of work, the vertical discourse of mathematics is not the only discourse in operation: The horizontal discourses offering contextual understanding in terms of social and cultural discourses and ways of operating specific to an individual workplace or to a sub-sector of a trade are equally relevant to mathematical meaning making in *all* kinds of vocations. These can only be learned and refined by practical experience in the workplace, but are crucial in determining workable solutions to workplace problems. However, the quantity, variability and complexity of this kind of contextual information to be taken into account makes it difficult to articulate in a typical mathematics textbook or assessment task. Thus, it is all the more important that school leavers and university graduates not only have a secure knowledge of mathematics at whatever level they reach to support their recontextualisation at work, but they should also have experience within their formal education of the processes of recontextualising mathematics in contexts more complex than can be offered by textbooks. In the workplace, each knowledge discourse involved will have its own rules, some much weaker than mathematics, and these differences must be understood, respected and resolved in the multidisciplinary context of work and consequently in vocational pedagogy; also, we argue, in general mathematics education.

4 Implications from work for study

In this final section, we address some implications for research and education in an attempt to help overcome the current hiatus between mathematics at school and at work. In Section 4.1 we illustrate how recontextualisation might take place in different kinds of work, and in Section 4.2 we differentiate the ways that recontextualisation in mathematics might be integrated into forms of vocational education where the mathematics content is able to be identified and a vocational context is offered.

4.1 Recontextualisation and knowledge structures in different professions and vocations

Graduating students are expected to be able to relate theory and practice in a wide variety of mathematical or mathematics-containing situations at work, structured by production logic or development logic or both (Ellström, 2010). They should also be able to participate in meaningful communication with a variety of stakeholders in order to understand, as fully as possible, the nature of any problem or request and to continue until the problem or task is resolved. Hence, from the gaze of the workplace, there is not really any hiatus between mathematics learning and knowing on one hand and work activities and workers' competences on the other. From an analytical point of view, the hiatus is, rather, present in relation to how the mathematics learnt in formal education may be recontextualised and hence made potentially useable in the context of work. The main implication to be drawn is how the activity of recontextualisation of mathematics *in itself* needs to be considered as a major focus, not only across a range of vocational education programs, but also as part of compulsory and post-compulsory professional mathematics education. In the following, we address what this might mean for both research and the teaching and learning of mathematics, particularly in relation to Bernstein's map of knowledge structures (Fig. 1).

4.1.1 Vertical discourses with hierarchical knowledge structures

In scientific professions where the main knowledge structure is characterised by a vertical discourse with *hierarchical* knowledge structures, recontextualisation of mathematics could, for example, occur in theoretical modelling where the mathematics is mainly algebraic, sometimes geometric. Frejd and Bergsten (2016) discussed "model-generated modelling" in situations where "some mathematics or some established mathematical models can be directly applied" (p. 26), involving the interplay of mathematical theoretical considerations, some empirical aspects and some application elements. In one example, a mathematical biologist applied Fourier transformations to a set of data to model the impact on the spread of forest diseases of disturbances such as climate or weather conditions. Once the model had been evaluated and validated statistically, the set of outcomes was then discussed with clients and other experts to identify an acceptable solution based on communications concerning other relevant vertical discourses and, most likely, local contextual information in the form of horizontal discourse. In another study, Gainsburg (2006) discussed the work of a structural engineer in designing a new building, a highly complex project without precedent, and working entirely in the abstract. The parts of the working model were distributed widely and only brought together for the final solution to the problem. Gainsburg observed that the modeller could not afford to forget the "'fragile links' to reality" (p. 32)—in the form of other vertical discourses—and concluded that in such cases mathematical theories, methods, and representations are always subordinated to the engineer's judgement about their use; mathematical justifications alone are insufficient. Nakagawa and Yamamoto (2013) also highlighted the need for respectful interactive communication between mathematicians and engineers. Designing an innovative furnace for the steel industry, able to withstand extreme pressures and temperatures beyond any previous experience while controlling a diverse range of phenomena, required both groups of professionals to fully comprehend the perspectives of one another and to effectively recontextualise their mathematical knowledge in order to collaboratively achieve a successful outcome.

4.1.2 Vertical discourses with horizontal knowledge structures

In each of the above cases, mathematical and scientific discourses were brought together in high level pedagogic discourses between mathematics specialists and other experts and/or clients to ensure that the optimal outcome for the given context was achieved. By contrast, there are scientific areas where the qualifications required of workers are much lower, and so the mathematical calculations may appear to be more trivial: for example, people undertaking chemical spraying work, or working as animal technicians calculating and administering medicines or vitamin supplements. Importantly, these are also situations where mathematical or other errors could have potentially serious consequences, such as death or destruction of critically endangered species, food crops or habitats (see, e.g., FitzSimons, Mlcek, Hull, & Wright, 2005). Here, the vertical discourses of life sciences, with horizontal knowledge structures and strong grammars like mathematics, mean that workers need to take into account relevant mathematics-containing information—such as local meteorological data, physiological data or historic records of previous applications along with local contextual knowledge of the target groups of plants or animals—in order to appropriately recontextualise their mathematics.

Guile (2011) gave an example of business professions where vertical discourses with weaker grammars interact with mathematics. Here, decisions may be based implicitly on abductive reasoning, drawing on experience of similar cases, in conjunction with explicit mathematics-based economic modelling. Although there is no necessity for optimal mathematical solutions to prevail over business solutions, any underlying mathematical reasoning and analysis used in the recontextualisation process must be error-free, from initial assumptions through to final interpretation.

Finally, there are vocations where the pedagogic transmission of mathematics can be rather implicit. In the day-to-day work of a nursing aide, doing mathematics is normally but one aspect of a wide range of activities. Johansson (2014) discusses a nursing aide who, in life-threatening situations, may draw implicitly on her mathematical knowledge, among many other salient factors, when deciding on whether or not to call for the presence of a doctor (see also Boistrup, 2016). In this situation, she has the authority to act. However, if questioned directly by people across the social division of labour—senior nurses, doctors, or the patient—she should be able to communicate clearly and accurately any relevant explicit mathematical information at her disposal concerning the patient's well-being and, if necessary, explain her reasoning. In this case, the nursing aide was able to recontextualise her thinking and actions in judging a mathematical indicator together with other non-mathematical indicators of patients' medical conditions for the interviewer, even though these had most likely become tacit knowledge based on her lengthy experience.

4.2 Recontextualisation of mathematics as part of pedagogic discourse in vocational programmes

In Table 1, we summarise possible mathematical content in pedagogic discourse in vocational education and other professional education courses. We then discuss explicit and implicit uses of mathematics in the work performed by construction workers, on the job as well as in their vocational education.

This table highlights that mathematics in vocational education should not only be about decontextualised mathematics (A in Table 1), nor relegated to vocational studies and reduced to the status of horizontal discourse (D). The intersection between these two (B and C) is where the recontextualisation of mathematics at work takes place. Importantly, although in our model we offer two discrete types, in actual vocational education contexts, there may be various combinations of these and blends of explicitly and implicitly recontextualised mathematics. Nevertheless, all mathematical recontextualisation necessarily involves the person doing the mathematics maximising their familiarity with the complexity of relevant contextual knowledges (other relevant vertical discourses and horizontal discourses) in conjunction with appropriate and error-free mathematics.

The explicit mathematical recontextualisation that takes place in construction work, and in authentic projects as part of the education of prospective construction workers, may be characterised as decision making and problem solving where recognisable mathematical concepts and techniques are utilised in specific contexts (Type B in Table 1). Such activities could include the use of measurement formulas or other techniques widely used in the industry. In most if not all cases, the measurements or calculations are recontextualised and communicated with significant others involved in the process for confirmation and possible further action. Construction workers who work alone are highly likely to re-measure or to check their calculations by repeating them or by comparing results with their previous experience or other alternative methods, historic records, codified practices, etc. (cf. FitzSimons et al., 2005). Type B could also include mathematical procedures involved in the planning of a new job, including an estimation of labour costs, as well as the interpretation and practical implementation of the architect's plan, for example. There is also planning involving the purchase and maintenance, or hiring, of necessary tools, as well as materials to be used in the construction process: how much of which materials to purchase and with what costs (see, e.g., FitzSimons, 2014b). Recontextualisation would probably also involve using printed texts and diagrams, as well as conversations, gestures, etc., shared between people such as co-workers and clients. For students in vocational programs, some elements of this kind of planning may also be done in relation to their own construction work projects, on the job or simulated in vocational workshops and/or using software programs (Boistrup, Bellander, & Blaesild, forthcoming). Such work would probably include using a spreadsheet or paper-based work using mathematical and vocational artefacts.

At other times in the building and construction industry, mathematical concepts and methods are interwoven implicitly, and largely invisibly, into mathematics-containing workplace activities (Type C in Table 1). This may be described as a weakly classified discourse in terms of mathematics. Type C then represents the kinds of recontextualisation of mathematics where workers or vocational students on a daily basis, generally onsite, estimate numbers or quantities (measurements and so on), and where the mathematics is not always easy to distinguish from the wholeness of the overall task: for example, deciding how deep to dig when laying bricks or laying tiles with unequal measures and allowing for

Table 1 Mathematics in vocational education

"Context-free" mathematics	Recontextualisation of Mathematics	"Mathematics free"
A. Mathematics **without** vocational context	B. **Explicit** use of mathematical models etc. before, during and following work activities C. Mathematical concepts and methods, etc. **implicitly integrated** into work activities	D. Vocational activities apparently **without** mathematics

breakages. In such cases, workers may also draw on past experience, measuring by eye, trial and error and so on.

Table 1 offers a foundation for an educational context where the pedagogic discourse is characterised by a respectful collaboration between the vocational subject area of, for example, construction work, and the subject area of mathematics (see Boistrup et al., forthcoming), implying that mathematics teachers need to become more familiar with vocational knowledge objects (FitzSimons, 2014a). With this model as a starting point, it should be possible to make clear to the teachers themselves, as well as the students, the different kinds of activities where mathematics may be recontextualised, both in terms of the explicit use of mathematics (Type B) and in terms of mathematics as one essential but implicit aspect of the many decisions to be made (Type C). Students at any level of formal school and vocational education should be given the opportunity to develop the skills to be able to confidently recontextualise their theoretical mathematics work in communications with significant others outside of the classroom in nontrivial contexts where mathematics could be involved.

Mathematical activities suitable for general as well as post-compulsory education would involve students in contexts with which they are familiar and/or likely to be engaged. Following a holistic approach, typical of industrial practice, students would identify an issue or problem of importance to them, then investigate the problem in its various dimensions in order to pose the necessary questions and to decide what a solution might look like. The choice of a problem inherently rich in mathematics would involve students communicating with a range of people and information sources both inside and outside of the classroom, and they would also need to work within realistic parameters of resources such as time, money, space and safety. Problems could be set within the students' classroom, the wider school community or even the broader community. They may be focused on improving on an existing situation, designing something new, making a case for policy change and so on. In this respect, the four types, A, B, C, and D, in Table 1 are not restricted to a specific location, such as a classroom. The critical thing is that students are using and recontextualising mathematics in meaningful contexts, which may include working within various parts of Table 1 at different times during an activity, communicating with a range of other people through a variety of means, verbal and non-verbal. It is also essential that the activity be valued through official assessment processes that take into account both vertical and horizontal discourses, as well as the quality of communication, especially with respect to mathematics: That is, to be able to actively listen, and to reason mathematically including explaining, justifying, and evaluating the arguments of others who may have diverse backgrounds and interests in mathematics.

5 Concluding discussion

In this article, we have attempted to argue that the skills of recontextualisation of mathematics are necessary to develop in students, young and old, in order to overcome the hiatus between the spheres of formal mathematics education and the world beyond. In other words, it is to ensure that mathematics education confronts the realities of people's work (FitzSimons, 2014a) and their lives beyond the classroom. Furthermore, we have tried to illuminate how the recontextualisation of mathematics into workplace contexts in education may be viewed as a content area on its own. In this, the focus lies not on mathematics per se, *or* on the workplace context (vocational or professional) per se, but both (see Table 1) in terms of recontextualisation of mathematics into the workplace context. Although our example in 4.2 addresses vocational education, we assume that similar kinds of recontextualisation of mathematics occur also in the work performed by professionals. Hence, the table may offer a point of departure for a discussion on how to overcome the hiatus between mathematics education and work to support research and developmental work across a broad range of professional and vocational education. In such an endeavour, variations and additions to Table 1 may occur. Within such discussions, we also include the mathematical content per se, which, as we have argued, is essential in order to empower students in a changing world and with unpredictable career paths (FitzSimons, 2014a).

Further research opportunities may come through the study of intentional teaching activities involving a conscious focus on recontextualisation enacted through interdisciplinary teaching in secondary school, or as part of normal practice in primary school education through project work. This would require mathematics teachers and their students working respectfully and collaboratively with other disciplines (see, e.g., CIEAEM 66, 2014, *Mathematics and Realities*, WG 1 *Mathematics and its teaching in relation to other disciplines*. Available at http://math.unipa.it/~grim/CIEAEM%2066_Pproceedings_QRDM_Issue%2024,%20Suppl.1_WG1.pdf).

Evaluation is an integral part of Bernstein's (2000) theoretical work, as described above in Sections 2.3 and 2.4. In the setting of formal education, it is clear to participants who are acquirers (students) and transmitters (teachers, textbooks, etc.). In day-to-day workplace settings, these roles can reverse continuously between different instances and work tasks. Nevertheless, this is not arbitrary but recognised as a legitimate way of "doing the job". Hence, there is a need to control the ways in which workplace knowledge is realised as part of the job. For a person who has recently completed vocational or professional education, and who is entering a new position in a workplace, the role as acquirer is rather dominant. As in formal education contexts, it is about purposefully initiating, modifying and developing knowledge (Bernstein, 2000). For formal and informal supervisors (including peers), it is notoriously difficult to assess the knowledge needed in the work tasks under standard test conditions. This is in contrast to formal education, in mathematics for example, where it is common to evaluate students' knowledge through tests. As a consequence, we argue that evaluating the knowledge of recontextualising mathematics in authentic workplace contexts (B and C in Table 1) through "paper-and-pen" tests would not be relevant to any substantial extent at all. While this argument may seem obvious in a workplace, such as the construction site described under 4.2, it also applies in education settings where the focus is on B and C in Table 1. Havnes, Smith, Dysthe and Ludvigsen (2012) show how vocational students experience more feedback (as part of formative evaluation) in vocational subjects than in academic subjects such as mathematics. This illustrates how evaluation in authentic workplace activities in education, to a large extent, may be performed implicitly as part of day-to-day communication. The work of Havnes et al. (2012) is one of very few examples of research on assessment in relation to vocational/professional education *and* mathematics, yet it does not focus on mathematics as part of workplace activities, but remains within the traditional paradigm of comparing assessment in the different subject areas. We contend that in order to take new steps towards overcoming the hiatus between mathematics education and work, more research is needed that pays specific attention to evaluation practices in mathematics education, vocational studies, *and* in workplaces. Clearly, an important aspect here is the specific knowledge of recontextualisation of mathematics as part of vocational and professional practice.

In this article, we have argued that the ability to recontextualise the mathematics that occurs explicitly and implicitly as part of workplace activities is essential for workers in vocational and professional spheres. Developing this kind of knowledge will give students the foundation for more easily acquiring relevant ways of using mathematics as they participate in rapidly changing and evolving workplaces of the future. Drawing on Wedege (2013), it can be said that it will prepare them for acting effectively, critically, constructively and reflectively in the workplace.

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References

Akkerman, S. F., & Bakker, A. (2011). Learning at the boundary: An introduction. *International Journal of Educational Research, 50*, 1–5.

Bakker, A., & Akkerman, S. F. (2014). A boundary-crossing approach to support students' integration of statistical and work-related knowledge. *Educational Studies in Mathematics, 86*, 223–237.

Bakker, A., & FitzSimons, G. (Eds.). (2014). Characterising and developing vocational mathematics knowledge [Special issue]. *Educational Studies in Mathematics, 86*(2).

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics, 92*, 331–345.

Beach, K. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. *Review of Research in Education, 24*, 101–139.

Bernstein, B. (1973). *Class codes and control* (Vol. 2). London: Routledge.

Bernstein, B. (1990). *The structuring of pedagogic discourse*. London: Routledge.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (Rev. ed.). Lanham, MD: Rowman & Littlefield.

Bessot, A., & Ridgway, J. (Eds.). (2000). *Education for mathematics in the workplace*. Dordrecht: Kluwer.

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education: The 14th ICMI study*. New York: Springer.

Boistrup, L. B. (2016 July). *Mathematics in the workplace from different perspectives: The case of Anita, a nursing aide*. Paper presented at 13th International Congress on Mathematical Education, Hamburg, Germany.

Boistrup, L. B., & Gustafsson, L. (2014). Construing mathematics-containing activities in adults' workplace competences: Analysis of institutional and multimodal aspects. *Adults Learning Mathematics: An International Journal, 9*(1), 7–23.

Boistrup, L. B., & Henningsen, I. (2016, in press). What can a re-analysis of PIAAC data tell us about adults and mathematics in working life? *Proceedings of the tenth research seminar of the Swedish Society for Research in Mathematics Education in Karlstad January 26–27, 2016*. Retrieved from http://ncm.gu.se/media/madif10/madif_011_bjorklund_boistrup_henningsen.pdf. Accessed 5 April 2017.

Boistrup, L. B., Bellander, E., & Blaesild, M. (forthcoming). Mathematics in pre-vocational education: Two subject areas' relationships to each other. In A. Rogers, B. Street, K. Yasukawa, & K. Jackson (Eds.), *Numeracy as social practice: Global and local perspectives*. London: Routledge.

Coben, D. (2006). What is specific about research in adult numeracy and mathematics education? *Adults Learning Mathematics—An International Journal, 2*, 18–32.

Damlamian, A., Rodrigues, J. F., & Sträßer, R. (Eds.). (2013). *Educational interfaces between mathematics and industry: Report on an ICMI-ICIAM-study*. New York: Springer.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/ pedagogic texts*. London: Falmer Press.

Dowling, P. (2014). Recontextualisation in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 525–529). Dordrecht: Springer.

Ellström, P.-E. (2010). Practice-based innovation: A learning perspective. *Journal of Workplace Learning, 22*, 27–40.
Eraut, M. (2004). Informal learning in the workplace. *Studies in Continuing Education, 26*, 247–273.
Evans, J. (1999). Building bridges: Reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics, 39*, 23–44.
Evans, K., Guile, D., Harris, J., & Allan, H. (2010). Putting knowledge to work: A new approach. *Nurse Education Today, 30*, 245–251.
FitzSimons, G. E. (2002). *What counts as mathematics?: Technologies of power in adult and vocational education*. Dordrecht: Kluwer.
FitzSimons, G. E. (2013). Doing mathematics in the workplace: A brief review of selected recent literature. *Adults Learning Mathematics: An International Journal, 8*, 7–19.
FitzSimons, G. E. (2014a). Commentary on vocational mathematics education: Where mathematics education confronts the realities of people's work. *Educational Studies in Mathematics, 86*, 291–305.
FitzSimons, G. E. (2014b). Mathematics in and for work in a globalised environment. In G. Aldon (Ed.), *Actes/Proceedings CIEAEM 66, Lyon, 21–25 juillet/July 2014* (pp. 18–36). Retrieved from http://math.unipa.it/~grim/CIEAEM%2066_Pproceedings_QRDM_Issue%2024,%20Suppl.1_Plenaries.pdf
FitzSimons, G. E. (2015). Learning mathematics in and out of school: A workplace education perspective. In U. Gellert, J. Giménez Rodríguez, C. Hahn, & S. Kafoussi (Eds.), *Educational paths to mathematics. A C.I.E.A.E.M. sourcebook* (pp. 99–115). Cham: Springer.
FitzSimons, G. E., Mlcek, S., Hull, O., & Wright, C. (2005). *Learning numeracy on the job: A case study of chemical handling and spraying*. Adelaide: National Centre for Vocational Education Research. Final Report.
Frejd, P., & Bergsten, C. (2016). Mathematical modelling as a professional task. *Educational Studies in Mathematics, 91*, 11–35.
Gainsburg, J. (2006). The mathematical modeling of structural engineers. *Mathematical Thinking and Learning, 8*, 3–36.
Geiger, V., Goos, M., & Forgasz, H, (Eds.). (2015). *ZDM Mathematics Education, 47*. [Special Issue: Numeracy.]
Guile, D. (2011). *The learning challenge of the knowledge economy*. Rotterdam: Sense Publishers.
Gustafsson, L., & Mouwitz, L. (2010). Mathematical modelling and tacit rationality—Two intertwining kinds of professional knowledge. In A. Araújo, A. Fernandes, A. Azevedo, & J. F. Rodrigues (Eds.), *EIMI 2010 Educational interfaces between mathematics and industry: Conference proceedings* (pp. 253–268). Lisbon: Centro International de Matemática.
Hahn, C. (2014). Linking academic knowledge and professional experience in using statistics: A design experiment for business school students. *Educational Studies in Mathematics, 86*, 239–251.
Hahn, C. (2016, May). *Penser la question didactique pour la formation en alternance dans l'enseignement supérieur. Dispositifs frontières, statistique et management*. Présentée en vue de l'Habilitation à Diriger des Recherches, Université Lumière Lyon 2, May 11.
Havnes, A., Smith, K., Dysthe, O., & Ludvigsen, K. (2012). Formative assessment and feedback: Making learning visible. *Studies In Educational Evaluation, 38*, 21–27.
Hordern, J. (2014). How is vocational knowledge recontextualised? *Journal of Vocational Education & Training, 66*, 22–38.
Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. London: Routledge.
Hoyles, C., Wolf, A., Molyneux-Hodson, S., & Kent, P. (2002). *Mathematical skills in the workplace. Final report to the Science, Technology and Mathematics Council*. London: Institute of Education, University of London: Science, Technology and Mathematics Council. Retrieved from http://eprints.ioe.ac.uk/1565/1/Hoyles2002MathematicalSkills.pdf
Johansson, M. C. (2014). Counting or caring: Examining a nursing aide's third eye using Bourdieu's concept of habitus. *Adults Learning Mathematics, 9*, 69–84.
Kanes, C., Morgan, C., & Tsatsaroni, A. (2014). The PISA mathematics regime: Knowledge structures and practices of the self. *Educational Studies in Mathematics, 87*, 145–165.
Kent, P., Noss, R., Guile, D., Hoyles, C., & Bakker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity, 14*, 64–82.
LaCroix, L. (2014). Learning to see pipes mathematically: Preapprentices' mathematical activity in pipe trades training. *Educational Studies in Mathematics, 86*, 157–176.
Nakagawa, J., & Yamamoto, M. (2013). Cultivating an interface through collaborative research between engineers in Nippon Steel & Sumimoto Metal and mathematicians in university. In A. Damlamian, J. F. Rodrigues, & R. Sträßer (Eds.), *Educational interfaces between mathematics and industry: Report on an ICMI-ICIAM-Study* (pp. 427–434). New York: Springer.
Nicol, C. (2002). Where's the math? Prospective teachers visit the workplace. *Educational Studies in Mathematics, 50*, 289–309.

OECD (Ed.). (2013). *Skills outlook 2013: First results from the survey of adult skills*. OECD Publishing. doi:10.1787/9789264204256-en

Roth, W. -M. (2014). Rules of bending, bending the rules: The geometry of electrical conduit bending in college and workplace. *Educational Studies in Mathematics, 86*, 177–192.

Singh, P. (2002). Pedagogising knowledge: Bernstein's theory of the pedagogic device. *British Journal of Sociology of Education, 23*, 571–582.

Society for Industrial and Applied Mathematics (SIAM). (2012). *Mathematics in industry*. Philadelphia, PA: Author. Retrieved from https://www.siam.org/reports/mii/2012/report.pdf

Stillman, G. A., Blum, W., & Salett Biembengut, M. (Eds.). (2015). *Mathematical modelling in education research and practice: Cultural, social and cognitive influences*. Cham: Springer.

Straehler-Pohl, H., & Gellert, U. (2013). Towards a Bernsteinian language of description for mathematics classroom discourse. *British Journal of Sociology of Education, 34*, 313–332.

Straesser, R. (2015). "Numeracy at work": A discussion of terms and results from empirical studies. *ZDM Mathematics Education, 47*, 665–674.

Tall, D. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics*. New York: Cambridge University Press.

Tall, D. (2014). Making sense of mathematical reasoning and proof. In M. N. Fried & T. Dreyfus (Eds.), *Mathematics & mathematics education: Searching for common ground* (pp. 223–235). Dordrecht: Springer.

Triantafillou, C., & Potari, D. (2014). Revisiting the place value concept in the workplace context: The issue of transfer development. *Educational Studies in Mathematics, 86*, 337–358.

Tsatsaroni, A., & Evans, J. (2014). Adult numeracy and the totally pedagogised society: PIAAC and other international surveys in the context of global educational policy on lifelong learning. *Educational Studies in Mathematics, 87*, 167–186.

Wedege, T. (1999). To know—or not to know—mathematics, that is a question of context. *Educational Studies in Mathematics, 39*, 205–227.

Wedege, T. (2000). Mathematics knowledge as a vocational qualification. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 127–136). Dordrecht: Kluwer.

Wedege, T. (2013). Workers' mathematical competences as a study object: Implications of general and subjective approaches. *Adults' mathematics: Working papers, 2*. Retrieved from https://www.mah.se/Forskning/Sok-pagaende-forskning/Vuxnas-matematik-I-arbetet-och-for-skolan/Vuxnas-matematik-I-arbetet-och-for-skolan/Working-Papers/

Wheelahan, L. (2009). The problem with CBT (and why constructivism makes things worse). *Journal of Education and Work, 22*, 227–242.

Williams, J. S., & Wake, G. D. (2007a). Black boxes in workplace mathematics. *Educational Studies in Mathematics, 64*, 317–343.

Williams, J. S., & Wake, G. D. (2007b). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics, 64*, 345–371.

Young, M. (2006). Conceptualising vocational knowledge: Some theoretical considerations. In M. Young & J. Gamble (Eds.), *Knowledge, curriculum and qualifications for South African further education* (pp. 104–124). Cape Town: HSRC Press.