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# REVISITING MATHEMATICS IN AND OUT OF SCHOOL[1]

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## ABSTRACT

When we first published our study with street vendors (Carraher, Carraher, & Schliemann, 1982, 1985, 1988), we were surprised by the "bimodal distribution of reactions". Some people dismissed the mathematics of street vendors as limited and unimpressive. Others extolled the virtues of their computation routines, going so far as to recommend that a tidy part of early mathematics curriculum be allotted to self-invented algorithms. Apparently our findings were like Rorschach ink blots onto which readers projected their beliefs about social class, economic stratification, self-determination, and nature versus nurture. If so, findings from Everyday Mathematics were likely to be used to promote ideologies rather than to better understand how mathematics is learned, taught and employed in and out of school. In this presentation we will discuss the relevance of Everyday Mathematics, keeping in view what mathematics as a formal discipline is all about

**Palavras-chave: everyday and school mathematics, concepts, invariants, symbols, situations.**

## 1 Everyday mathematics findings and contribution to education

Everyday mathematics research has shown that specific cultural activities such as buying and selling promote mathematical learning that was thought to be only acquired through formal instruction. Individuals with restricted schooling can come to master arithmetical operations, properties of integers and of the decimal system (Nunes,

[1] Most of the ideas discussed in this paper appear in papers previously published by the authors.

Schliemann, & Carraher, 1993; T. N. Carraher, Carraher, & Schliemann, 1982, 1985, 1988; Saxe, 1991; Lave, 1977, 1988), proportional relations (Schliemann & Magalhães, 1990; Schliemann & Nunes, 1990), concepts and procedures related to measurement (T. N. Carraher, 1986; Gay & Cole, 1967; Saraswathi, 1988, 1989; Saxe & Moylan, 1982; Ueno & Saito, 1994), geometry (Abreu & Carraher, 1989; Acioly, 1993; Gerdes, 1986, 1988; Harris, 1987, 1988; Millroy, 1992; Schliemann, 1985; Zaslavsky, 1973), permutations (Schliemann, 1988), and probability (Acioly & Schliemann, 1989; Schliemann & Acioly, 1989).

In retrospect, the finding that mathematical learning occurs out of school may seem obvious. Indeed one might wonder how anyone could have ever thought otherwise! After all, commerce and crafts requiring rudimentary measurement skills have often flourished in societies where schooling has been infrequent or even nonexistent. Furthermore, developmental psychological studies, particularly those of the Piagetian tradition, have long since documented that young children discover, for example, the commutative nature of addition well before entering school.

However, when we initially investigated the mathematics of young street vendors in Brazil (Carraher, Carraher, & Schliemann, 1982), half of the students who entered first grade were not attending grade 2 one year later. At the time, the fact that children in public schools tended to fail in mathematics and drop out altogether came as no surprise to most Brazilian parents and educators of that period. The children were considered undernourished and prone to disease and cognitive-developmental lags. Their parents were not strong supporters of education. They lived in communities where there the struggles of everyday life required a different set of priorities. Most elementary school teachers were under-prepared. And the school day itself lasted only three hours. When we found that street vendor youths could solve arithmetic problems at work, displaying significantly better performance than when "comparable" arithmetic tasks were administered as a school assignment, we realized that wide-spread beliefs about school failure required re-examination. This was probably the main contribution of everyday mathematics studies to education: children who were considered incapable of learning mathematics were in fact capable of mathematical reasoning using their own strategies to solve arithmetic problems.

But how were they solving mathematical problems? If children learning mathematics out of school were not following school-prescribed routines, but nonetheless producing correct answers, they must have alternative ways of representing and systematically solving problems. Much of our work in everyday mathematics pursued this question. It now seems fairly clear that many of the street vendors did not use a place value notational system when mentally solving problems. Furthermore, they seemed to operate on measured quantities (such as 3 coconuts, 35 Brazilian cruzeiros) as opposed to pure numbers (3, 35). In this manner they did not have to perform calculations on numbers and introduce the result of the computation back into the meaningful problem context. Rather, they would always be working directly with countable quantities.

The dramatic contrasts we encountered among Brazilian street vendors predisposed us to view informal mathematics as inherently more natural and more meaningful than school mathematics. We showed that people attempt what appear to be nonsensical solutions in a school-context while they search for meaningful solutions when the problems are part of their work in everyday contexts (see Carraher, Carraher, & Schliemann, 1985, 1987; Grando, 1988; Lave, 1977; Reed and Lave, 1979; Schliemann, 1985; and Schliemann and Nunes, 1990). The analysis of problem solving solutions in and out of schools suggests that students commonly learn algorithms for manipulating numerical values without reference to physical quantities, only reestablishing clear links to the problem context in the end when the units of measure are finally attached to the numbers. By contrast, individuals solve problems in the workplace using mathematics as tools to achieve goals that are kept present throughout the solution processes, with continuous reference to the situation and the physical quantities involved. As such, the problem solvers in the workplace are normally aware of how the quantities generated in the course of the computations are related to the problem at hand. We also stressed that schools encourage memorization and repetitive practice, whereas at work street sellers solve problems through mental computation, using flexible strategies they develop and efficiently apply to achieve their goals as street sellers.

As we came to document more and more instances of everyday mathematics—among carpenters, cooks, farmers, lottery bookies and construction foremen—the more we realized, spurred by suggestions from Resnick's (1986) work, that alternative mathematics, to be useful at all, would have to pay heed to some basic properties of arithmetic as additive composition and the commutative and associative laws of addition. Once placed in this framework, we began to see informal mathematics and the mathematics of the school as more closely related than we had originally thought. They both had to respect the many of same basic properties of arithmetic, such as the associative law, but they often did so through distinct routes. For example, when subtracting 135 from 200, a street vendor might take away 100, then 30, then 5. This strategy relies on the decomposition of 200 into 100 + (70 + 20 + 10); likewise, 135 is implicitly treated as the sum, 100 + 30 + 5. Although the street vendors did not know how to explicitly express the associativity of addition they revealed their implicit use of the property through the transformations they made on the values given. The standard school algorithm for column subtraction invokes the same general property but decomposes the givens in a somewhat different manner.

If everyday mathematics is based on the same logico-mathematical relations implicit in the school procedures children should be learning in school, how could mathematics education benefit from the mathematics children have learned outside of schools?

Observations from everyday mathematics do not provide straightforward answers to this question. Further analysis of the characteristics, strengths, and weaknesses of mathematical knowledge may help gaining insights into the relevance of everyday mathematics to mathematics education.

Here, Vergnaud's theory of concepts and conceptual fields (Vergnaud, 1979, 1985) may help us understand the similarities and differences between everyday and school mathematics.

## 2 Concepts: Invariants, Symbols and Situations

Vergnaud (1979, 1985) views concepts as consisting of three components: invariants, symbols, and situations.

Invariants refer to mathematical objects, properties, and relations. As an invariant, a number is not a physical thing, but rather an idea connected to other ideas through its properties, relations, and operations. On the other hand, symbols are closer to what is referred to in semiotics as "signifiers". The distinction between invariant & symbol (roughly, signified and signifier) is very important in mathematics education research. The symbol "8" is not a number but rather, a particular kind of signifier—a numeral—that stands for a signified, the idea eight. We have made similar points for the invariants *function and equation* (Carraher et. al. 2007). The same point could be made with regard to any mathematical object. Imagine a line drawn on a blackboard with numbers increasing in value from left to right. The chalk line is not the number line mathematicians talk about: it has a thickness and a fixed length, whereas the real number line has no thickness and it extends to infinity in both directions. And, given any two points chosen on the real number line, there is always an infinite number of points (and corresponding numbers) in between.

Even in elementary mathematics, it is important that students shift their attention toward ideas, relations and structures not available to direct perception. Otherwise they run the risk of confusing that which is drawn, written, or uttered with the things they are meant to stand for, namely, mathematical objects.

Vergnaud employs the term *symbol* in the broad sense of semiotics. Symbols are signifiers that take on a variety of forms within and outside of mathematics. Symbol(ic) systems are structures that allow individual symbols to be composed, operated upon and interpreted within a set of conventions. Just as it is naïve to equate invariants with concepts, it is wrong to equate symbols with concepts. And symbolization is only part of (although an important part of) conceptualization.

Situations are the third component of concepts in Vergnaud's theory. This is the most difficult component to understand, and Vergnaud has provided no more than a fleeting sketch. Nonetheless they are critical to the present discussion.  One often discusses situations places or occasions where mathematics is applied or *deployed in situations. In Vergnaud's theory,* situations are actually an *integral component of concepts*. This is humorously evident in the early stages of learning, where irrelevant characteristics of situations are wedded to the concepts—for example, when young students believe that fractions are about pizzas or density is fundamentally about floating and sinking in water.

## 3 Concepts in Everyday Mathematics

When we first observed street vendors solving arithmetic problems in work settings we were tempted to conclude that we were witnessing a "different mathematics". But it soon became apparent that such a coarsely worded statement sheds little light. The street vendors were not operating directly on written symbols as pupils are taught to do in school. Maybe they were imagining actions involving currency and items purchased. But such a system of representation would be useless if they were not able to keep track of precise values or amounts—something very unlikely if they relied on mental images of physical objects.

When we looked closely at the intermediate values involved in their mental computations, it became clear that the mental algorithms were not the same ones taught in school. In addition and subtraction, for example, one performs column-wise computations proceeding from right to left.  School addition required one to "carry" values from one column to the next if the total in a column surpassed 9; school subtraction required one to "borrow" from the column at the left in order to proceed.  Our street vendors did not use such algorithms.  They did compose and decompose amounts, but they did so in ways that did not quite match standard procedures taught in school.  And they often broke apart amounts opportunistically, in ways that made good use of the particular values at hand. For example, in subtracting 58 from 253, a vendor might appear to first decompose 58 into 53+5. He might then subtract 53 from 253, obtaining 200. Next, he might subtract the 5 (the remaining part of the subtrahend) from 200, reaching an answer of 195. The way chosen allowed the problem-solver to pass, on the way to a solution, through the number 200.

For street vendors "round numbers" such as 200 involved less cognitive overhead. Their mental representations of 200 did not contain three digits (Carraher, 1984a, b). Mentally, one only needs to represent the hundreds, of which there was a single amount, namely, two; there was no need to keep track of tens and units. It seemed that the number system of the street vendors was not a place value system at all!  So their mental arithmetic involved somewhat *different symbolic representations* from those taught in school.

How it was possible for two different symbolic systems to consistently produce the same, correct answers to arithmetic problems.  This might have been obvious to those familiar with the history of mathematics. But it took some time for us to realize that each representational system would need to respect the same properties of arithmetic.

Here is where the notion of invariant proved useful. When talking about whole numbers and measures, addition does not depend on the order of the addends. That is, addition is commutative regardless of the values being used: A + B = B + A. This simple law[2], and others like it[3], cannot be violated without wreaking havoc on the results. But there are multiple ways to correctly enact arithmetic operations that respect the law. Historically, there were also various methods for multiplying, including the Egyptian 'halving and doubling method', the Venetian grid methods, abacus-based methods, and of course our own column multiplication.

Research suggests that the street vendors were comfortable with the commutative and associative properties of addition when doing oral mathematics. The difficulties they exhibited with school algorithms seemed to be more tied to the symbolic procedures themselves (or to other invariants).

Concerning the situational aspect of mathematical concepts, much research has shown, in ways consistent with Piaget's description of cognitive development, that children learn mathematics through actions in the physical world and reflections on the results of those actions. Number is introduced through counting (things), rational numbers through the measurement of quantities. Early mathematics instruction often relies on modeling, with a curious twist: instead of simply applying previously learned mathematical methods to represent phenomena in the physical world, children acquire knowledge of mathematics through making sense of worldly phenomena. But because mathematics is drawn to the increasingly abstract (Alexsandrov, 1989), children need to learn to extricate themselves from empirical observation, demonstration, trial and error methods, and the need to explain mathematics, at every turn, through appeal to extra-mathematical phenomena. Mathematics must take on a life of its own, so to speak, and students need to develop an appreciation of validity independent of empirical corroboration. Likewise, they need to be able to derive new symbolic expressions from existing expressions by treating the written forms as syntactical objects, without having to translate the forms into extra-mathematical terms. How students make (or fail to make) such a transition is an important topic for research.

## 4 Can Everyday Mathematics be the foundation for School Mathematics?

We believe it would be a fundamental mistake to suggest that schools attempt to emulate out-of-school institutions. After all, the goals and purposes of schools are not the

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[2] Note that this law says nothing about procedures for adding two numbers. A+B=B+A is not a method.

[3] They are the Field Axioms or more correctly, the commutative ring axioms (Bass, 2008).

same as those of other institutions. The specific everyday problems street sellers are asked to solve and the goals of their computations may hide important mathematical properties that should be part of the school curriculum. Consider for instance the scalar approach so pervasive in everyday computation to solve multiplication problems. This approach involves a linking of a unique y-value (price) to each value of x number of items) and, as such, captures the essential idea of a function and reveals an implicit understanding of proportionality. It may therefore constitute a meaningful initial approach to solve multiplication and proportion problems. But this understanding may be limited to mathematical principles that are relevant to the specific goals of the situation while principles that are not relevant to these goals are never considered. The commutative property of multiplication as applied to repeated additions seems to be a case in point (Schliemann, Araujo, Cassundé, Macedo, and Nicéas, 1998).

However we will overlook the most important contributions of life outside of school to mathematical learning if we restrict ourselves to the finished tools of mathematics: particular algorithms, material supports such as tables and graphs, notation systems and explicit mathematical terminology. Some of the most profound ideas in mathematics rest upon concepts learned in the physical and social world in what appear to be mathematics-free settings. Actions on physical objects—slicing modeling clay into several parts, joining multiple instances of elements, setting objects of one type in one to one correspondence with those of another type, nesting objects within others, dismantling toys—provide us with a wide range of experiences that later may prove crucial to understanding arithmetical and algebraic operations and relations among numbers, quantities, and variables. Commercial situations provide us with a wealth of knowledge about trading, profitability, interest, taxes, and so on that will prove necessary for understanding mathematics. The behavior of colliding objects, the exertion required to lift objects in different ways, judgments of the relative quickness of two automobiles and experimentation with how our eyes work provide us with elaborate knowledge and intuitions about dynamics and statics, velocity, acceleration and a host of other scientific concepts that ultimately play a major role in our making sense of advanced concepts in calculus, geometry, topology, and analysis. We repeat: they do not provide finished knowledge. However they provide rich repertoires of experience, data, and schematized understandings of how things work without which advanced mathematical understanding would be inconceivable.

Everyday situations provide a foundation for constructing mathematical knowledge, but not a rock solid one onto which students can quickly erect, with scaffolding supplied by teachers and parents, mathematical skyscrapers. When construction proceeds at a rapid pace, as it typically does, school mathematics will occasionally wobble on its intuitive foundations. For example, students may become puzzled when they discover that multiplying does not always make quantities grow bigger—a view long supported by their growing intuitions in elementary mathematics instruction. This fault can be superficially patched by telling the students that the old rules no longer apply ("rational numbers are different from integers"). But a satisfactory fix of the problem requires examining the foundations and seeing how they can be accommodated to support the weight of new knowledge. For example, they may need to understand that fractions have both a multiplication- and division-like quality. The numerator of a fractional operator acts like a natural multiplier; the denominator acts like a natural divisor. Their relative magnitude determines whether the result will be greater, less than or the equal to the original quantity.

The construction site metaphor perhaps suggests that the upper floors will develop well once the foundations are solidly established. However, the relationship between intuition and new mathematical ideas is one of constant tension and readjustment. The Greeks of antiquity had to adjust their intuitions about number when they realized that the diagonal of a unit square could not be expressed as an integer ratio of the side. Similar tensions have arisen in the history of mathematics in the cases of Zeno's paradoxes ("it takes a finite amount of time but an infinite number of steps to reach the tortoise"), negative quantities ("how can there be less than nothing?") and Cantor's infinities ("how can one infinite set be greater than another?").

It is comforting to believe that everyday mathematics is reconcilable with the mathematics of mathematicians. But there are times these approaches will clash and it is instructive for us and for students to become aware of these mismatches. We laugh when we hear that the average family has, say, 2.3 children or that we need 7.3 buses to transport a certain number of people because we know that children and buses come in whole numbers . There is a sense in which even these "artificial" answers are true, and learning mathematics often requires temporarily suppressing common sense and traditional thinking in favor of following a stream of logic along its course.

It is not easy to say how much children should be left to their own devices in solving mathematics problems. Proponents of laissez-faire pedagogy would go to great lengths to favor student inventiveness over the appropriation of conventional knowledge. Some would go so far as to recommend that students create their own notational systems rather than be forced to adopt those created by others. The French approach to the didactics of mathematics (see Laborde, 1989) makes a strong case for a distinctly opposing view. Although they would encourage children to generate their own solutions and choices, and recognize that mathematical knowledge grows around what are personal activities, they are also concerned that children become skilled in using conventional representational tools.

There seems to be relatively little mathematical activity in children's out-of-school activities and, when mathematics comes into play it does not seem to call for a deep understanding of mathematical relations. Cultural and social environments that support the construction of mathematical knowledge may nonetheless constrain and limit the knowledge children and adults will come to develop (Petito & Ginsburg, 1982; Schliemann & Carraher, 1992; Schliemann, Araujo, Cassundé, Macedo, and Nicéas, 1998). Finally, once transposed to the classroom cultural setting the problem is no longer the same.

In schools activities can be organized so that children will experience a wider range of situations and tools for using mathematical concepts and relations, thus allowing them to explicitly focus on mathematical concepts from different perspectives. Schools can also engage children in using a variety of symbolic representations such as written symbols, diagrams, graphs, and explanations, which constitute opportunities to establish explicit links between situations and concepts that would otherwise remain unrelated. Such are the activities that will allow children to understand mathematical concepts as belonging to, in Vergnaud's (1990) terms, conceptual fields.

## 5 Conclusions

Is everyday mathematics really relevant to mathematics education? Yes, but not as directly as many have thought.

The idea that we can improve mathematics education by transporting everyday activities directly to the classroom is simplistic. A buying-and-selling situation set up in a classroom is a stage on which a new drama unfolds, certainly one *based* on daily commercial transactions, but one that, as Burke (1962/1945) might have expressed it, has redefined the acts, settings, agents, tools, and purposes.

Classroom goals are different from but no less complex nor cultural than goals in out of school settings. New situations challenge students to go beyond their everyday experience, to refine their intuitive understanding, and to express it in new ways. In a school setting these situations are always to some extent contrived. When the contrivances lead to playful puzzle-solving inquisitiveness and debate teachers are rightfully pleased. When they fail to engage students, the situations present themselves as artificial. Mathematics teachers cannot totally renounce the use of contrivance or, to use a less charged term, staging, because naturally occurring everyday situations are not sufficiently varied and provocative to capture the spectrum of mathematical inquiry. This leaves teachers with immensely difficult dramaturgical problems, particularly when the students are leery of book knowledge and unfamiliar notational systems.

The outstanding virtue of out of school situations lies not in realism but rather in meaningfulness. Mathematics can and must engage students in situations both realistic and unrealistic from the student's point of view. But meaningfulness would seem to merit a consistently high position on the pedagogical pedestal. One of the ways that everyday mathematics research has helped in this regard has been to document the variety of ways people represent and solve problems through self-invented means or through methods commonly used in special settings. By explicitly recognizing these alternative methods of conceiving and solving problems teachers can hope to understand more clearly how students think and to appreciate the chasms they must sometimes cross to advance their present state of knowledge.

Everyday Mathematics has contributed in important ways to long-standing debates about mathematical concepts, symbolic representation, and the role of contexts in thinking—the latter topic reaching back at least as far as Kant's notion of *scheme*. The descriptive work plays a role, of course. But it is only by *making sense* of the observations that science moves forward. If over time the expression *Everyday Mathematics* drops from usage, I would be neither surprised nor disappointed. Eventually the field needs to become absorbed into the mainstream traditions of research in mathematics education. However I would be disappointed if it is remembered only for its descriptive and proscriptive aspects, without recognizing the contributions to research, theory, and the cultural context of learning and thinking.

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