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Are Different Students Expected to Learn Norms Differently in the Mathematics Classroom?

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We analyse social interactions during the first days of class in a secondary mathematics classroom (15 and 16-year-olds) with a high percentage of immigrant students. Our analyses show the co-existence of different models for both the interpretation and the use of classroom social norms and socio-mathematical norms. Valorising some behaviours over others appears as part of the discursive practices of mathematics classrooms. Local and immigrant students are not expected to behave in the same way, nor are they treated in the same way. The teacher and some students, who are familiar with the prevailing norms, cancel certain norms for a while in such a way that some immigrant students are excluded from fully participating in the mathematical discussion.

In modern societies, immigrant students tend to accumulate the highest rates of failure at school. This reality of failure is even worse in the case of immigrant students and school mathematics. At present, there is an increasing number of studies in mathematics education research concerned with the low performance of certain groups of students. Recent work (Nkhoma, 2002; Zevenbergen, 2003) has documented that many students from minority groups experience difficulties when trying to participate in contexts of mathematical practices where they do not feel themselves represented, when others do not recognise them, or when they have to cope with actions and behaviours that are different from those they would expect. Research has shown that cultural/ethnic identity is an essential construct to consider when interpreting relationships as well as differential treatments in the mathematics classroom (Abreu, 2002; Khisty & Chval, 2002; Morgan & Watson, 2002).

Many studies (see, for instance, Dowling, 1998; Walkerdine, 1998) have dealt quite explicitly with diversity and inequality in mathematics education, and have suggested the need for analysing discursive practices when looking at differences. These studies have been decisive in opening new ways of thinking, not only in our research but also in the area. They have raised important questions: does every student have a voice in the mathematics classroom?; how are the identities of mathematics learners constructed in socially-situated environments?; and, more generally, what is the relevance of considering social inequality in research on mathematics education?

In our particular context, Barcelona, Spain, the immigrant population from Africa, Asia, and South America is increasing significantly at the same time that mathematics failure within these groups is on the rise. The profound changes that currently affect and will continue to affect our society

in the near future require an examination of the most frequent interpretations given for failure at school, so often exclusively related to the growing diversity within classrooms. From our point of view, failure is not caused by characteristics of certain groups of students. The school in general, and the mathematics classroom in particular, are the ones that fail when attending to minority students. After having reviewed previous literature on multi-ethnic mathematics classrooms with the intention to illuminate possible causes of failure that do not blame the student, we find it necessary to explore how discourse is constructed within these classrooms. For us, discourse has important consequences in the students' school performance and in the distribution of learning opportunities. We focus on the analysis of specific discourse contents: the classroom norms, the personal values and the social valorisations given to the students, and their mathematical practices.

It is not clear to what extent research about norms in the mathematics classroom has addressed social questions (Boaler, 2002; Lerman, 2001). Some of our investigations (Planas, 2001, 2004; Planas & Civil, 2002; Gorgorió & Planas, in press) confirm that immigrant students tend to have difficulties understanding and using classroom norms that the mathematics teacher and most local students consider as shared. In these investigations, we have examined how immigrant students identify and interpret expressions of approval and disapproval given to them and to their mathematical practices, and how they react differently to these expressions. The notion of norm, as it has evolved in our discussion, has profound social implications; not only does it include definitions of what is acceptable, but it also encompasses the values and valorisations within the classroom. How the students adjust their meanings and behaviours to the legitimate interpretations of classroom norms has an influence on which personal values and social valorisations are expressed. The three notions – norms, values, and valorisations – have been instrumental in highlighting the processes through which students construct their identities as mathematical learners.

In previous articles about this study, an extensive micro-ethnographic perspective was adopted in the analysis of (non-) participation trajectories of local and immigrant students in various sessions of different mathematics classrooms. Now concrete moments of the discourse in a mathematics classroom are examined. Interactions centred on the interpretation and use of social norms (e.g., forms of participation) and socio-mathematical norms (e.g., contexts of reference in a problem solving process) are considered using Yackel and Cobb's (1996) terminology. The first days of a new school year in a regular integrated mathematics classroom with fifteen and sixteen-year-old students are the focus. Students know each other but they do not know the teacher. In general, the first days of class are a time in which participants, mainly the teacher, focus on making norms explicit. As a consequence, crucial aspects of classroom discourse are established: who decides the validity of an argument; what the role of the textbook is, if any; how much time is dedicated to each task; etc. The aim of this paper is to explore how the

students' identities as mathematical learners are constructed through discursive practices in classroom interactions. To accomplish this two short classroom episodes that took place on the second day of class are analysed. They are episodes that have important similarities with episodes from the other four observed sessions.

Theoretical Framework

The theoretical framework that informs our research comes from the field of socio-cultural theories (Lerman, 2000; Moll, 1992; Zevenbergen, 1996). Socio-cultural theories emerged as an alternative to dichotomies of the social and the psychological / individual. In the psychological / individual approach, the notion of participation is centred on the learner and pays little attention to the characteristics of the learning context. Here, through the mediating tools of mathematical symbols, the students are able to reorganize new concepts and to participate in the classroom discourse. In the socio-cultural approach, the key notion of participation is viewed as a kind of socialization into the mathematical practices. Here, the basic mediating tools, such as interactions among peers, are what facilitate the mastering of new skills and the students' participation. The participation model, as understood in the socio-cultural approach, focuses on the use of discourse and some of its contents (norms, values, valorisations) as crucial mediating tools in order to interpret the mathematical learner in context. The acquisition of concepts and skills is not essential in the process of becoming a mathematical learner, but the active participation in the reconstruction of a specific kind of discourse is necessary.

Socio-cultural Approach to the Notion of Discourse

There are many perspectives in the study of the notion of discourse due largely to the extremely diverse definitions of discourse. By re-elaborating Potter's (1996) definition, we understand discourse as a set of actions and interactions that take place in a context of social practices and affect the construction of both personal and social meanings. The use of the term 'social practices' in this definition implies a broader social dimension of discourse than the dimension given by the construction of meanings in interpersonal interactions. An interaction between a teacher and a student is more than a dialogue about, for instance, proportional reasoning; it is also part of the social practices that comprise teaching and learning. From the socio-cultural point of view, classroom discourse is constituted by communicative practices that generate the production and transaction of intentions and meanings in socially and culturally situated interactions. The classroom is then a culture with shared models for the interpretation of norms, actions and expectations that are (re)constructed by discourse through social practices (Forman & McCormick, 1995).

Classroom discourse has to do with sharing meanings and ways of interpreting how to behave in each moment. But it has also to do with social relationships such as who is supposed to ask for advice, who is supposed to

ask for or give suggestions, who must be at the disposal of whom, who needs to ask for permission, who gives it and who can refuse it, who says what someone is (not) obliged to do, who determines when an apology is needed, or who expresses what someone is (un)able to do. By answering all of these questions, the culture of the classroom is constructed as an accumulation of social situations where specific contents such as proportional reasoning are in play. Participants are not always agents when contributing to these social situations. When participants discursively establish who must be at the disposal of whom, they are not necessarily aware of all the implications of their actions. They may be denying that their actions contribute to inequalities and, at the same time, they may be asking indirectly for other participants to be at their disposal in a non-reciprocal way.

The question is what kind of data represents classroom discourse: texts, actions, words, gestures, intentions, etc? To develop a complete definition of discourse requires us to consider both conceptual and methodological issues. When conceptualising classroom discourse as a social practice, its analysis becomes the analysis of the actions and intentions of the participants. Actions and intentions form the process of discourse through which participants determine what they say and how they say it in their interaction with the others. When doing so, participants act not only as teachers or students, but also as members of many other social categories (a teacher born into a poor family, a slow student, a good soccer player, an immigrant student, a student whose parents are teachers of mathematics, etc.). The analysis of the classroom discourse, therefore, needs to be interpreted taking into account many other simultaneous discourses. Those who give permission outside the classroom, for instance, are more prone to be categorised as those who give permission within it.

In the case of the mathematics classroom, discourse models what mathematics knowledge and doing mathematics are about, as well as the students' identities as mathematical learners (Klein, 2002). Students position themselves in relation to social and academic roles in the mathematics classroom as a reaction to the ways that the teacher and their classmates position them. They may be seen as good at arithmetic, smart, lazy, persistent when solving problems, etc. and they may behave by reacting to or fulfilling others' expectations. In one classroom, for instance, being good at geometry may not have the same importance as being good at arithmetic. Some students may then tend to hide their geometrical abilities and try hard to solve most of the problems by using arithmetical reasoning, even when they could apply geometrical reasoning. Subsequently they are identified as not good at arithmetic instead of being identified as good at geometry. By making public their meanings and reconstructing others' meanings, students show parts of their intended identities as mathematical learners, their (mathematical) knowledge, and their system of values.

In classroom discourse, the teacher is the main socio-cultural mediator of the participation and learning processes (Forman & Ansell, 2001). To act as

required by the teacher is the main condition for being considered a mathematical learner. Not to act or to act differently is an *outsider* option that makes it difficult to access the teaching and learning discourse. When controlling who can talk about each topic and when assessing the value of the interventions, the teacher mediates the process of (not) integrating students whose interpretations of the norms, actions, and expectations do (not) adjust to legitimate meanings. The ways in which the teacher addresses the students suggest the ways in which these students participate in the construction of knowledge referred to as social and socio-mathematical norms. In turn, when interpreting the classroom norms, students model the classroom culture and are modelled by it through interactions where personal values and social valorisations are expressed. Actions and intentions are then structured through social and socio-mathematical norms, personal values, and social valorisations.

Norms, Values, and Valorisations

Taking into account the symbolic interactionism theories, Cobb and his colleagues have introduced the idea of social and socio-mathematical norms in order to analyse the regularities of the mathematics classroom. Cobb and Yackel (1998) discuss the necessity of a certain mathematical disposition in the student as a starting point in the development of an identity as a mathematical learner that conforms with the teacher's expectations. This mathematical disposition refers to developing fluency in two types of behaviours that affect the interpretation of the various moments of the mathematical practice: the social norms and the socio-mathematical norms. Despite the emphasis on communication processes and the characterization of mathematics as a social activity, there are important discursive issues lacking in Cobb's analyses of the norms of the mathematics classroom.

All interpretations of norms are discursively produced, that is, they are constructed by an individual under the influence of multiple contexts of practice and their discourses. Being discursively produced means, among other things, that a particular interpretation of one norm may be considered as valid or appropriate in a specific context although there can be no universal understanding about how this norm should be interpreted. The socio-mathematical norm regarding the use of real context when solving a problem, for instance, may be interpreted in vastly different ways. However, in a particular mathematics classroom, the teacher may suggest that the real data provided in a problem serves only as an opportunity to explore *authentic* mathematical contents such as proportional reasoning, functions or estimation. The teacher's interpretation may be taken-for-granted, even if there are one or more students who make frequent references to the real context and use alternative interpretations. Many teachers avoid, intentionally or not, making their interpretations of socio-mathematical norms explicit and negotiable (Planas & Civil, 2002). This is mainly due to the invisibility of the discursive nature of socio-mathematical norms.

In Planas (2001), it is claimed that a study of the norms of the mathematics classroom from a socio-cultural perspective is still in its beginning stages. In particular, the notion of how classroom norms may act as social mediators in the relationships among participants is illustrated. The focus is on processes of valorisation occurring in the classroom environment, especially those related to what counts as valid mathematics and those related to personal characteristics of the students. The diversity of meanings that co-exist in three secondary mathematics classrooms is examined by observing how socio-mathematical norms are interpreted differently by different students. Finally the ways in which students experience the diversity of meanings of the norms and of the valorisations are examined, paying special attention to cases that turn interruptions into participation. Norms and valorisations are analysed in order to better understand social issues of influence on students' (non) participation trajectories.

In the process of discourse, participants can use mere value judgments (opinions) or more complex judgments (valorisations). Valorisations are cultural understandings concerning the value (or lack of value) of a social practice (Abreu & Cline, 2003). When one student does not lend the calculator to one of her/his classmates and s/he is said to be selfish or uncooperative for behaving that way, this is a negative value judgment concerning a concrete episode. But if the judgment is similar to "you'd better let your mates use the calculator" in a situation when some students are computing a certain quantity and it is an immigrant student who volunteers to use the calculator, then this is probably a valorisation – in our context, many immigrant students are not expected to make correct use of calculators because they do not own them. It is not always an easy task to distinguish values and valorisations. The important point is, however, to notice that both types of judgments can have an influence on a student's performance. The impact of this influence depends on the characteristics of the context and those of the individual. When analysing breakdowns in participation, Planas (2001) saw that similar values and valorisations can encourage some students and discourage others by valorising some behaviours over others.

Interruptions in students' participation mean different things in different classroom cultures and from the perspective of different students' histories. Additionally, some interruptions may be seen as a consequence of the teaching discourse. Much of the mathematics taught in classrooms is taught in such a way as to discourage some students from being part of the mathematical practices (Confrey, 2000). Interruptions are then a coherent response to the teaching discourse. Legitimate interpretations of classroom norms are usually presented as unique and negative personal values belonging to those students who use or suggest alternative interpretations. Moreover, social valorisations help to maintain the confusion between the use of legitimate interpretations of classroom norms and the capacity of the students. The efficient use of the established social and socio-mathematical norms is understood as mathematical proficiency, which is described as the

ability to interpret classroom norms in the same way as the teacher and the students from the socially dominant groups. This fact does not facilitate the immigrant students' participation. Students who have attended schools in other countries do not necessarily share the same interpretations of classroom norms.

In particular, Hugh Mehan's (1979, 1992) research on the stratifying practices of schools, the social order that students and teachers co-produce within classrooms, and the consequences of both phenomena on minority students' participation are very useful in our approach to the notion of interruption. As Mehan (1992) states, race, social class and ethnicity are to be taken into account when interpreting the causes of many interruptions. Mehan and Wood's (1975) early writing on ethnomethodology concerning interaction and discourse analysis, has also been very helpful when looking for the meaning of key notions such as norms, values and valorisations.

Gorgorió and Planas (in press) argued for the importance of social valorisations. To do so, norms were characterised as social constructs that act as a filter for certain groups of students. When the teacher calls on a certain socio-mathematical norm and the students tackle it, they all bring their re-interpretations of social understandings about mathematical knowledge ownership and social valorisations of mathematical practices. Moreover, mathematical practices have their existence in communities that hold certain positions in the broader social structure, and that are structured by their institutional contexts. Those broader social structures impact classroom interactions through implicit messages about which are the legitimate socio-mathematical norms within the classroom and which are the groups that own them.

Method

The research was conducted in an urban high school in Barcelona. We collected data in one mathematics classroom with a total of nine students, most of them were immigrants or born into immigrant families, between the ages of 15 and 16 years. Enrolment in this school was lower than usual due to the very low socio-economic backgrounds of the students. Most schools in Barcelona with a high percentage of immigrant students do not have more than 12 students per class. All the immigrant students in the class had sufficient competence in the official languages of Catalan and Spanish, as none of them was a newly arrived immigrant. Three students, two girls (Kholoud, and Ramia) and one boy (Mourad) were from Morocco; one boy (Aftab) was from Pakistan; and five students, three boys (Eduard, Albert, and Roger) and two girls (Maria and Cristina) were local, one of them a gypsy (Maria).

We observed and videotaped the first five days of class. Sessions were transcribed and the transcriptions were supplemented with field notes taken by the first author during the sessions or immediately after them. The transcripts were discussed by the teacher and the two authors at regular

meetings. Long term analyses, based on detailed observations of the videos, helped to distinguish well established routines that shape the students' identities as mathematical learners. The analysis of local episodes, in turn, may highlight the creation of some of these routines.

Two criteria were considered in choosing the participating class. First, we looked for a multi-ethnic class with some local students and with a teacher who was not an immigrant or born into an immigrant family. Second, we looked for a teacher with experience in multi-ethnic mathematics classrooms, used to working in problem solving contexts, and willing to collaborate in the research study by posing problems provided by the researchers to the students. The mathematical tasks were problems chosen for their relevance (they would relate to students' daily lives) and their capacity for generating discussion (they could be solved by a variety of strategies). These tasks were designed in a professional development program the summer prior to the research study described here. We explained to the teacher and the students what the research was about, emphasising the need to observe and analyse forms of interaction and participation in the mathematics classroom.

The analyses undertaken were informed by the perspectives of several researchers such as Coulthard (1992), Gee (1999), Lerman (2001), Pomerantz and Fehr (1997), and Wood and Kroger (2000). By referring to all of these authors we do not mean that we attempted a full discourse analysis on classroom conversations, but an analysis based on shared theoretical approaches. When a theoretical approach to discourse analysis is confronted with real data, there is a need to concentrate more on some analytic tools than others. Using Gee's terminology, we concentrated on two specific tools: socioculturally-situated identity and relationship building, that is, narratives concerning attitudes, values, ways of feeling, ways of knowing and believing, as well as ways of acting and interacting; and political building, that is, narratives concerning issues of valorisation, status, and power, coming from the macro-context. The connections between the data obtained from the application of these two tools to one piece of transcript facilitate assumptions about how certain norms, values and valorisations influence the construction of students' identities as mathematical learners.

We isolated several episodes as the basis for our analysis of the classroom social interactions. We focused on fragments of transcripts where different interpretations and uses of classroom norms, both general social norms and specific socio-mathematical norms, were in play, in a direct or indirect way. Within each fragment of transcript, we outlined and identified narratives concerning the mentioned analytic tools, and wrote down gestures and actions that we saw in the videos. Narratives were selected according to their significance (or not) in the interruption situation. That is, we discussed the interpretations of norms, values and valorisations expressed during the interaction that seemed to be relevant in promoting/obstructing students' participation. In this paper, we only present the final stage of the analysis,

where connections between data have already been done. In Planas (2004), a detailed description of the methodology used and a justification for the analytic tools selected are provided.

Connections between data coming from different pieces of transcript were also considered (some of these connections are discussed in the conclusion section). However, we looked at each piece of transcript as a complete unit. According to interactional sociolinguistics (Cook-Gumperz, 1986), each interaction itself constitutes a communicative act and can be looked at as a whole, and therefore can be analysed as a complete discourse. In turn, each interaction has to do with previous and simultaneous interactions, and anticipates those that have not yet happened. This approach allows us to develop the analysis of the process of discourse in a particular interaction. The participants who did not intervene in a particular interaction were assumed to be part of the audience.

Different Students Are Expected to Learn Some Classroom Norms Differently

In this section, two partial examples of the analysis that was developed within one of the observed sessions are presented. Attention is paid to the mathematical activity in which the teacher and the students are engaged and to the way that different forms of relationships and participation are made public. Both examples are interesting due to the references to various classroom norms, the different ways of interpreting the use of these norms, and the diverse ways in which different students are expected to learn them. When introducing the second example, similarities with the first example are discussed and common discursive features with the other four sessions are briefly explored.

Forms and Spaces of Participation

The nine students involved in this session were organised in groups of three. In alphabetical order by given names they were: Eduard, Kholoud and Maria; Albert, Mourad and Roger; and Aftab, Cristina and Ramia. Each group had a worksheet with the problem on it and a calculator. The problem involves the ingredients and quantities to cook an apple cake for three people being provided, and asks for the quantities needed for ten people. The day before, the teacher had provided students with general information about the subject, had talked about the evaluation, and had explained that they would be working in small groups for some weeks on the mathematical topic of *proportion* in a problem-solving environment. The following transcript shows a conversation during whole group discussion which took place after the students had been working in small groups. The conversation has to do with the problem solving task and the classroom dynamics.

Teacher: Let's see… do you need more time?
Mourad: I think the problem may be thought about in very different ways.

Teacher: Have you thought about it at least in one?
Mourad: Well, I haven't finished it yet, I only need to go over it.
Teacher: Mourad, if you need to, ask for help *(pointing at Ramia and Kholoud)*. And, please, remember you must write everything down in your notebook.
Mourad: Can I go to Kholoud's place?
Teacher: Of course! The work in small groups does not mean that you cannot collaborate with other people in the class.
Eduard: So, what shall we do now? Shall we wait for a bit?
Cristina: We've already finished it.
Albert: We also have it nearly finished. Roger and I have been working on it.
Teacher: OK *(to Albert)*, you finish it.
Cristina: Do I explain it now?
Teacher: OK. Let's see what we'll do. There's not much time left, so we'd better start. Her group *(pointing at Cristina)* starts with the explanation and then his group *(pointing at Albert)* adds anything that may be missing. It could be that she *(Cristina)* gets distracted and we need him *(Albert)* to complete her explanation.
Cristina: *(smiling)* I won't get distracted! And I've made no errors!
Eduard: If you've made an error, we'll find out what has happened, won't we?
Teacher: OK *(to Eduard)*, we'll find out what has happened. In how many different ways have you solved the problem (to Cristina)?
Cristina: In two. I'll begin by explaining the first way and then I'll explain the second one.
Eduard: How have you done it?
Teacher: OK, let's start with the first approach.
Cristina: First, we've thought out the problem as if it was a real problem, as if we had been told to cook a real apple cake.
Ramia: I got the idea!
Teacher: As if it was a real problem?
Ramia: Yes, being careful with the decimal numbers.
Teacher: What does it mean here being careful?
Cristina: It means to avoid certain types of decimal numbers.
Ramia: It means not to make errors.
Teacher: Ummm… if you both want to speak, we'd better organise ourselves. You *(to Ramia)* explain step by step what you've done, give us the result for each ingredient, without making errors, and then you *(to Cristina)* tell us in detail why you've done it in this or that way. All right?
Ramia: All right.

In this exchange, some students are asked to explain their mathematical practice. Both the teacher and Eduard want Cristina to explain her approaches to the problem ("In how many different ways have you solved the problem?", "How have you done it?"). It is also accepted that Albert may intervene, if necessary, to complete Cristina's explanation ("[Albert] adds anything that may be missing"). When Mourad intervenes, however, nobody

asks him to explain his approach to the problem. He is not recognised as a legitimate speaker, at least not in this episode. Mourad's clear attempts to participate in the mathematical conversation ("I think the problem may be thought about in very different ways") are not seriously considered.

Instead of answering the teacher's question related to a social norm ("Do you need more time?"), Mourad provides information that he has not been asked about ("I think the problem may be thought about in very different ways"). Despite receiving an unexpected answer, the teacher goes on talking about the topic introduced by Mourad ("Have you thought about it at least in one?"), although he avoids initiating a discussion about the existence of different valid resolutions. In anticipation of Mourad's words, the teacher assumes that this student may have developed, at best and with difficulty, one valid answer. It would have been reasonable, for instance, to ask the student to explain his approaches or to make an effort and try to develop other possible strategies. The message sent would have been quite different. Not only would the existence of different valid approaches have been recognised, but Mourad's mathematical proficiency would have also been suggested, as well as his capacity to discover different strategies and to explain them. However, the teacher did not accept Mourad's answer and used an indirect form to discredit him.

Both Cristina and Mourad claim that the problem may be solved in very different ways yet they are not encouraged in the same way by the teacher to explain their reasoning. Mourad is asked for neatness and a certain order ("And, please, remember you must write everything down in your notebook"). Cristina is asked for ideas and creativity ("In how many different ways have you solved the problem?"). It is well accepted that Cristina may forget any important issue when explaining her approaches to the problem ("It could be that she gets distracted"), but even so her mathematical proficiency is not seriously questioned. In her case, the possibility of making an error is linked to external influences (getting distracted, not paying enough attention, etc.) and not to her individual capacities. In the case of Mourad, external influences are not suggested. The student's capacities are directly questioned ("Have you thought about it at least in one?"). Moreover, Eduard contributes to obstruct Mourad's participation when taking the teachers' initial question up again ("So, what shall we do now? Shall we wait for a bit?"), and blocking (intentionally or not) his peer's intervention.

Something similar occurs to Ramia. She is not recognised in this episode as a legitimate speaker. The teacher facilitates her pedagogical participation (she is allowed to intervene) but, at the same time, obstructs her mathematical participation (she is not allowed to talk about certain mathematical practices). When Ramia shows her intention to participate and to explain her strategy, she perceives herself as an agent in her mathematical learning process ("I got the idea!"). But the teacher only asks her to enumerate a series of numerical solutions ("You explain step by step what

you've done, give us the result for each ingredient (…"). Cristina, on the contrary, is asked to make her reasoning public ("you tell us in detail why you've done it in this or that way"). This distribution of tasks (Ramia is to enumerate and Cristina is to discuss and argue), with such a different level of mathematical demands, situates both students very differently as mathematical learners.

Asking Ramia only to enumerate numbers suggests that this student is either not prepared enough for more complicated mathematical tasks or is less prepared than Cristina. The teacher has just remembered that there is not much time left and, as he wants to finish the problem in that session, entrusts Cristina with the task of explaining in detail her strategies which is a way of publicly showing confidence in this student's mathematical proficiency. Nevertheless, and despite the highly positive self-concept that Cristina seems to have developed ("I've made no errors!"), while working in small groups Cristina has had to turn to the calculator on many situations when mental arithmetic was much more appropriate and also needed Ramia's help. Ramia has been an active member of her group although she is being given a more passive role in the whole group discussion. Although there is no evidence of these facts in the transcript excerpt above, the videotape provides this information.

The use of explanations and argumentations was interpreted differently depending on which students were involved. In this episode, discourse helps to establish the category of students who can (and must) explain and argue their mathematical practices, and the category of students who are not expected to explain and argue their reasoning, although they are still left some room to participate. Participants regulate their interventions and adjust themselves according to the expectations defined by these categories. Cristina and Albert trust in their possibilities and act as others would expect. Mourad and Ramia, however, do not insist on wanting to explain their strategies, nor do they nominate themselves to discuss others' ideas either. These students appear rather obliging with the tasks they are assigned. Mourad goes to Kholoud and keeps out of future interventions in the mathematical conversation. Ramia agrees to simply enumerate the numerical solutions, while one of her peers copes with a more sophisticated task.

The interpretation and use of the norm 'who is to be asked for help' also shows clear signs of differential treatment given to certain students. During work in small groups the teacher promotes student autonomy. As much as possible, he avoids answering students' questions and redirects their questions to other students ("Mourad, if you need it, ask for help *[pointing at Ramia and Kholoud]*"). The teacher should not be asked for help, and not just any student can be asked for help either. The teacher could have referred Mourad to, for instance, any student or to Eduard and Albert – local students. However, the teacher points at two immigrant Moroccan students, like Mourad, who do not belong to this student's group. By pointing at

Ramia and Kholoud, the teacher gives the right and predisposes Mourad to ask these peers for help and, in turn, does not establish the obligation for other students to help Mourad.

Discourse not only contributes to establishing the category of those students who can be asked for help, but it also contributes to the establishment of the category of students who need help. In anticipation of demands for help, the teacher interprets Mourad's words ("Well, I haven't finished it yet, I only need to go over it") putting the emphasis on the idea of the task being unfinished ("If you need it, ask for help"). The teacher's anticipation turns into a real demand for help from Mourad to Kholoud ("Can I go to Kholoud's place?"). Mourad is supposed to need help. On the contrary, Albert, who has made a similar comment ("We also have it nearly finished"), is not reminded to ask for help, nor is he positioned in the category of those needing help ("OK, you finish it"). Later, this student is told to actively participate in the mathematical discussion although he had been recognised as not having finished his approach.

The role of errors in the mathematics classroom and their implications also suggest different forms of participation by different students. Cristina is the one who first introduces a comment on errors ("I've made no errors!"). Eduard, a peer from another group, replies and interprets the meaning of 'making an error' ("If you've made an error, we'll find out what has happened, won't we?"). The teacher repeats the same idea ("OK, we'll find out what has happened"). To make an error is interpreted here as an opportunity to explore Cristina's process of resolution and reasoning. Two minutes later, making an error is interpreted in a very different way. After Ramia has said that it is necessary to be careful in order not to make errors, the teacher points at the possibility of this student making errors ("…explain step by step what you've done, give us the result for each ingredient, without making errors"). To make an error here means to enumerate incorrectly a series of numerical solutions. When talking to Ramia, the error refers to a wrong number. When talking to Cristina, it reflects the need to explore the whole process of resolution.

There are also different interpretations and uses for referring to the forms of working in groups. The students are organised in groups of three and the teacher encourages them to collaborate ("The work in small groups does not mean that you cannot collaborate with other people in the class"). However, and despite talking occasionally amongst themselves, some students do not accept collaborating with particular students, nor do they feel represented by some members of their groups. Albert, for instance, distinguishes two subgroups within his own small group: Roger and himself, on one hand, and Mourad, on the other ("Roger and I have been working on it"). Albert points out that Mourad does not speak on behalf of his group. It is not necessary for Albert to mention Roger. In doing so, he makes especially visible the fact of not having mentioned Mourad. Although the teacher is the main socio-cultural mediator of the forms of participation

within the classroom, the students may also have an influence on how and when other students must participate.

Contexts for the Resolution of a Problem

A few minutes later, another episode confirms the existence of differential treatment of students. Now the conversation is about the use of valid contexts in the mathematics classroom. Participants go beyond the mathematical task and reflect on it. The teacher suggests the use of academic contexts to local students and the use of real contexts to immigrant students.

Teacher: She *(Cristina)* has told us how to solve the problem in round numbers. It's better to cook with round numbers. 350g of butter are much better than 333.333g, aren't they? Kholoud, have you ever cooked an apple cake?

Kholoud: Yes, with my mother.

Teacher: Cooking with your mother, has it been useful in solving the problem?

Kholoud: She is the one who cooks, I only watch her.

Teacher: Tell us more, does she use the calculator to prepare everything?

Kholoud: *(smiling)* No! She knows very well how to do it. She doesn't need a calculator.

Teacher: And you (to Eduard)? Have you ever solved a similar problem?

Eduard: In class?

Teacher: Yes, in class, for instance.

Eduard: Last year we worked on some problems where you had to check the final solution and put it in round numbers because you couldn't have decimal numbers to refer to people.

Teacher: Have these problems been useful to solve the one we are doing now?

Eduard: Yes, a bit, because they are almost alike.

Teacher: Tell us something else about these similar problems.

Eduard: Once the former teacher made us think out a problem with ingredients and quantities. But you only had to multiply and all numbers were exact.

Teacher: That's good *(to Eduard)*. To remember similar problems may help understand this one. *(to Aftab)* Have you also helped to cook an apple cake at home?

Aftab: No.

Teacher: You've never cooked an apple cake?

Aftab: No.

Teacher: And what do you do when you're at home?

Albert: *(laughing)* He's never at home!

Teacher: And you (to Albert)? Has it been very difficult for you to think out the problem? Did you solve similar problems last year?

Albert: This problem is very easy. It's only a question of multiplying with the calculator and rounding the numbers.

Teacher: OK. You may be wondering why is he asking so many questions, right? What I want to comment is the following: when doing mathematics, one can look for examples within the school, but

> one can also look for examples in real life. Kholoud has seen her mother cooking an apple cake, and Eduard and Albert have solved problems that look very similar to them. One learns mathematics both in the school and in real life *(the school's bell rings)*. See you tomorrow.

The teacher addresses the students differently. He refers to what he thinks they are able to do and talk about. Although the teacher's expectations concerning the students' knowledge and abilities have only a tentative character, they orientate the students in their learning of who is who in the classroom. In this episode, discursive categories related to who is allowed to use a certain context when doing mathematics are constructed. As in the previous episode, participants adjust their actions to the expectations defined by these categories. Cristina, for instance, had before introduced a direct reference to the real context ("…we've thought out the problem as if it was a real problem"), but she does not take this reference up again. She realises that only Kholoud and Aftab are asked about real contexts.

In the episode discussed in the last section, the teacher emphasises the need to obtain a correct result in the end. In this episode, the teacher seems more interested in the ideas and procedures that have been developed by the students. The use of the expression "tell us more" on two occasions indicates a change in this sense. However, some of the teacher's previous interventions have not contributed to create in all the students the feeling of developing ideas and procedures being of equal or even more importance than writing down final results. When Kholoud and Aftab are encouraged to "tell more", they only answer with monosyllables. Eduard and Albert, on the contrary, fully intervene and introduce issues of discussion. The different reactions of these students probably have to do with the different spaces and the forms of participation that have been facilitated for them.

In his group's discussion, Aftab referred to the same *similar problem* as told by Eduard. But Aftab does not intervene in the whole group discussion when others talk about 'similar problems'. This student has initiated an important activity on comparing ideas and strategies. He has been an agent of his mathematical learning and, despite all this, he only replies "No" to the teacher's two questions. Aftab does not seem to perceive himself as an agent of his mathematical learning. Moreover, on the only occasion that the teacher addresses this student, he asks him about personal experiences and does not insist on going deeply into his answers, as if Aftab was not able to have developed significant ideas for the resolution of the problem. There is even a rather ironical observation from the teacher ("And what do you do when you're at home?"). Another observation from a peer ("He's never at home!") makes almost everybody laugh except Aftab.

The teacher's interest in the students' ideas and procedures seems to be conditioned by a bigger interest. The teacher makes some students intervene in order to facilitate the discussion on a particular topic. When Albert refers to multiplications and round numbers, the teacher keeps talking without

paying attention to the student's answer. The teacher could have explored the comprehension level that Albert has of the mathematical task ("It's only a question of multiplying with the calculator and rounding the numbers"), but this is not his goal. He is more interested in receiving enough information to raise one topic: the diversity of valid contexts of reference for the mathematical practice. The teacher could have raised this topic in a much more spontaneous way a few minutes before when Mourad says that the problem may be solved in very different ways, or when Cristina says that she has solved the problem as if it was a real problem.

Interactions between the teacher and the students in this episode lead all participants to consider two main types of contexts, academic and real. The existence of a diversity of valid contexts when doing mathematics is made explicit. But this claim for the variety of contexts does not mean the use of different contexts by each student. Discourse again establishes two categories concerning who must use each context. Although the teacher does not directly refuse the students' interventions that do not fit into these categories, he controls in subtle ways the use of both academic contexts and real contexts. Students must learn to interpret the teacher's reactions in order to know whether their use of each context is adequate.

Different students are allowed to use different contexts. Kholoud and Aftab are considered competent participants if they use their family contexts in the discussion of the problem. Eduard and Albert are considered competent participants if they use academic contexts and former classroom experiences. Aftab is not prompted to use former classroom experiences, neither is Eduard prompted to use experiences from his family environment. These students are told in indirect and subtle ways what they are expected / allowed to do. For instance, when the teacher asks Eduard "Have you ever solved a similar problem?", and the student answers "In class?", illustrates Eduard's opinion that the reference to an academic context is not so immediate and other possibilities might be considered. However, neither the teacher nor the other students talk to Eduard about the real context in relation to the mathematical problem.

Assigning the use of different contexts to different students has an influence on the distribution of forms and spaces of participation within the classroom. Not all contexts are explicitly related to the mathematical practice or its importance. The teacher talks about both contexts ("One learns mathematics both in school and in real life"), but there are specific discursive elements that distinguish the mathematical relevance of each context. In his last intervention, the teacher uses impersonal formulas ("… one can look for … one learns…") to outline the importance of both contexts. However, when he outlines the importance of the academic context in his dialogue with Eduard, he uses the first person ("[I think] that's good"). Moreover, it is not clearly explained that the use of real contexts may aid in verifying that results are coherent with the initial conditions of the problem. In turn, when talking about the academic context, some advantages in relation to the

mathematical practice are made explicit ("To remember similar problems may help understand this one").

When the teacher talks about the diversity of contexts for mathematical practice, the students learn something about the mathematics itself. On the other hand, the teacher's interactions with the different students and his suggestions for the use of different contexts, allows students to learn about themselves as mathematical learners. Kholoud, for instance, is expected to 'tell [only a bit] more' about how her mother cooks an apple cake. She is encouraged to participate but her participation has important restrictions. When Kholoud answers "She [her mother] knows very well how to do it. She doesn't need a calculator", nobody inquires into her mother's mathematical knowledge. Kholoud's pedagogical participation is facilitated (she is allowed to intervene) while her mathematical participation is obstructed. She is not expected to talk about similar problems, her mother's mathematical knowledge, or her own mathematical knowledge. Eduard's participation also has important restrictions. Eduard is not supposed to mention family episodes.

In general, looking at the five sessions that have been analysed, we can conclude that the model for being a good student is not the same for all of these nine students. In several sessions, students worked on problems with statements somehow linked to real life situations. In all of these sessions, local students were expected to discuss and explain their strategies for solving the problems through the use of academic contexts. They were listened to, they were openly asked to participate, and they were encouraged to introduce references to 'similar problems'. On the contrary, immigrant students' efforts to collaborate in the tasks of explanation and argumentation were systematically refused in subtle ways. They were taught to listen to how other students explain and discuss their ideas, and they were encouraged to use real contexts but not to the extent of fully relating them to their mathematical practices. Immigrant students and local students were expected to learn classroom norms very differently and their obligations as mathematical learners were understood very differently. The difference affected the interpretation and use of both classroom social norms (e.g., the role of those who help / are helped) and specific norms of the mathematical practice (e.g., the role of errors in the mathematics classroom).

Conclusion

Although the notion of learning obstacles has not been directly treated in this paper, it is clearly suggested by our data. The notion of learning obstacles has been interpreted in various ways: from the epistemic interpretation to the socio-political interpretation developed by Skovsmose (2000). Here learning obstacles are not related to students' preconceptions and misconceptions of some mathematical ideas, but to a social phenomenon within the micro-context of the multi-ethnic mathematics classroom. Our data show a multi-ethnic mathematics classroom said to be governed by norms such as: *in this*

class one problem may have different approaches, and the contextualisation of the mathematical task should be considered seriously. Such norms stimulate a practice that is open to explore situated mathematics and to facilitate group work and inquiry cooperation. However, these prevailing norms do not appear to be applied universally. The teacher and some students, who are familiar with the prevailing norms, periodically cancel certain norms in such a way that some immigrant students are excluded from fully participating in the mathematical discussion. The point then is not simply that immigrant students bring different norms to the mathematics classroom, and in this way cause 'conflicts'. Local and immigrant students are not expected to behave in the same way, nor are they treated in the same way.

Even when it is not the intention of the teacher and students to exclude, there are ways of administrating classroom norms that provoke learning obstacles and exclusion. When Ramia decides to become a non-participant, an exclusion has taken place, although she has struggled to become a member of the classroom community for the first period of the session. Causes for exclusion are produced from within the established classroom community, and they need not be sought in the divergent norms of some immigrant students. Causes for exclusion might be found in the stereotyping of immigrant students as socially at risk, which lead to particular ways of administrating recognised norms. Negation of the prevailing norms might be a useful strategy for the immigrant students coping with these situations who, in fact, might have grasped these prevailing norms very well. To prevent exclusion from these practices, the negotiation of norms is essential.

As discussed in Civil and Planas (2004), the accumulation of learning obstacles turns into failure at school. Failure at school, in turn, characterises classrooms as sites for inclusion and exclusion. The impact of the classroom discursive practices on the participation of some immigrant students is rather negative. Immigrant students have to face multiple learning obstacles coming from the values and valorisations expressed by other participants and often internalised by themselves. Our results do not seek to generalise what happens in 'the' multi-ethnic mathematics classroom. However, they permit us to reflect on the way in which the social macro-context and the classroom micro-context are mutually influenced. The subjective criteria used to assess immigrant students affect their performance and increase their initial 'cultural distances' from the school culture. The difficulties experienced by many immigrant students when developing, for instance, the tasks of explanation and argumentation may be understood in terms of the lack of actions that promote these tasks in these students. In our classroom, some immigrant students are being identified as less able than other students in relation to these tasks.

The expression *being identified* is of considerable significance in this paper. It helps to understand how the teacher interprets students' identities as mathematical learners by taking into account their individual and socio-cultural identities. The differential treatment that some students receive

within the classroom needs to be related to social representations from the broader context. During an informal conversation with the teacher at the end of the fifth session, we told him that he asked for tasks with a very different level of mathematical competency depending on which students were being addressed. His answer was: "I make students advance according to their individual possibilities". The teacher had hardly had time to get to know his students in only five sessions yet he talked about the students' individual possibilities. Here, the students' individual possibilities do not refer to a cognitive reality but to a social construction. The teacher constructs each student's possibilities on the basis of certain social representations established by the macro-context. This teacher shares with the dominant social groups social representations of immigrant students that question their mathematical capacities. These social representations shape the teacher's identity, affect the development of classroom practices, and contribute to delimit the use of norms.

To introduce the notion of social representations is an important point in order not to associate *bad practices* with *bad teachers*. The teacher involved in our study described himself as having inclusive practices, although the practices we observed were not always inclusive. The teacher's practices do not necessarily reflect the teacher's intentions; rather, they mirror more general social attitudes. Immigrant students and their behaviours are more prone to be valued negatively due to socially constructed assumptions that go far beyond a teacher's individual positioning. There are some practical questions concerning the teacher's influence on the classroom dynamics. Although classroom discourse is a very complex construction in which many factors can intervene, the teacher has a privileged position when establishing discourse categories (who needs help, who is supposed to help…). Therefore, it makes sense to ask how teachers can learn to observe how they talk to their students and how they can analyse their contributions to classroom discourse. It is quite a complicated task for teachers. To begin with, a positive attitude towards auto-critique/self reflection is needed as well as the technical resources to videotape their teaching. Moreover, mathematics teachers need to integrate into their perspective notions that are often distant from traditional school mathematics, such as power, exclusion, valorisations, and equity.

When interpreting a classroom episode, the students, as well as the teacher, focus on some of its many facets, borrowing from social representations that are part of the collective images of their group's culture. When analysing the meanings that individuals bring to a school situation it has to be taken into account that the meanings are constructed in relation to the socio-cultural context of the school, and to the group to which the each individual belongs. In this article, the influence of valorisations on the orchestration of classroom norms has been illustrated. However we have not provided data concerning the influence of social representations on the orchestration of norms into practice. Future research should provide

evidence of connections between differential treatment in the mathematics classroom and social representations. Some valorisations could then be interpreted as articulating social representations.

The notion of (or lack of) negotiation, suggested by our data, also needs further research. In multi-ethnic classrooms, a lack of negotiation gives rise to obstacles to immigrant students' participation in the mathematical conversation and therefore interferes with the students' learning (Planas, 2001). Some effects of a lack of negotiation on students' learning processes seem quite clear, but the notion of negotiation itself is questioned by our data. If norms, related to what counts as valid mathematical practice and to which participants are positively regarded, are interpreted differently whether they apply to immigrant or local students, the meaning for negotiation should be reconstructed. What exactly is meant by negotiation? Is it a way to overcome obstacles between conflicting ideas? Is it compromised in the sense of achieving common meanings that are less than desirable? Is it simply an argument that ends up with an act of authority? Or is it something much more complex? In order to answer these last questions, it would be helpful to explore to what extent the so-called negotiation processes are prevented by certain valorisations and social representations.

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