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MATHEMATICS EDUCATION AND LANGUAGE: LESSONS AND DIRECTIONS FROM TWO DECADES OF RESEARCH

Núria Planas, Candia Morgan, & Marcus Schütte

1 Introduction

Several decades after the conceptualisation of language as a system of signs provided by the linguistic paradigm in language research, a range of questions about language and ways of tackling them have evolved inside and outside the field of mathematics education. In the midst of a diversity of premises of language, we know that language is a system of linguistic rules and texts, but also and importantly, an array of contexts of use for many kinds of rules and texts. In this chapter, we will argue that the progress of mathematics education and language research is taking place through a complex expansion rather than an overthrow of the linguistic paradigm, with an increase in the scope of the domain and in the spread of cultural and social claims. The questions addressed will be:

- What is the scope of the research on mathematics education and language?
- How can we map and link the newer approaches in the domain of mathematics education and language to classical approaches?
- What has been achieved in the last two decades of research?

Studying the progress of our knowledge of language in mathematics education research across the two decades of the European Society for Research in Mathematics Education (ERME) has led us to uncover classical themes regarding the language of the learner, the language of the teacher/classroom and the language of mathematics. In their contemporary forms, these are complementary themes, intertwined, either individually or in combination, with conceptualisations of *language as system*, *language as culture* and *language as discourse*. At the beginning of ERME in the working group entitled 'Social Interaction in Mathematical Learning Situations' and in the present Thematic Working Group (TWG), such themes and conceptualisations have been addressed primarily through *classroom-based research*. The study of language, inside and outside ERME, has mostly involved the study of mathematics classrooms as dynamic environments of interaction between students and teachers and between students and peers. The

dominance of classroom studies and of the three themes regarding whose language is in focus suggests some continuity. Nonetheless, continuity is accompanied by a phenomenon of increasing complexity in the ways of understanding language. The postulation of the inseparability of language from cultural and social contexts (Morgan, 2013) has gained ground, along with interpretations of mathematics classrooms as communities of practice and configurations of discursive activity. The study of the language domain in ERME, therefore, points to a relationship between continuity and complexity. The sophistication in the ways of conceptualising language across major themes in classroom-based research inspires our overall characterisation of the ERME domain as *a continuum of complexity*.

After this introduction, in Section 2 we discuss what is involved in international research on mathematics education and language. In Section 3, we survey research reported at CERME since 1998 as a benchmark for assessing the phenomenon of increasing complexity as well as the relationship between continuity and complexity in the ERME domain. In Section 4, we map some gaps and directions for future research.

2 What does it mean (to) research on mathematics and language?

The review by Austin and Howson (1979) cited research into mathematics and language dating back to the 1940s, with a body of research beginning to establish itself in the early 1970s. Nearly 40 years later, the research developed within ERME and beyond still addresses the broad themes identified by these authors:

The language of the learner (i.e. the language or languages and linguistic skills brought to the mathematics classroom by learners);

The language of the teacher and the classroom (i.e. the language or languages and linguistic skills brought to the mathematics classroom by teachers);

The language of mathematics (i.e. the language or languages and linguistic features of the texts that arise within the practice of mathematics).

We can see substantial development in the sophistication of these themes. There is more widespread and systematic engagement with theories of language and communication from psychology, sociology, linguistics, ethnology, semiotics and anthropology, as well as with specialised frames addressing the role of language in mathematics education. There has also been a growth in the diversity and complexity of the domain as researchers draw on a wider range of theoretical resources combined in new ways. One source of diversity is the fact that research on mathematics and language encompasses three main possible foci. The first takes language itself as the object of study, the second uses language as a vehicle for studying other phenomena and the third views participating in mathematical communication as learning mathematics itself. All three foci formulate descriptions of language-in-use in a mathematical context but analyse the descriptions in different ways. The

description of language in some cases has been formed from 'common sense' knowledge about mathematics without a systematic theory of language or has drawn on tools from linguistics that do not fully serve the purpose of distinguishing characteristics of language use of interest to mathematics education. A major contribution toward more adequate description was the publication of Pimm's (1987) book, but there remains a need to develop greater rigour in the ways in which we define and distinguish between mathematical and 'everyday' language.

The significance of understanding what is specific in mathematical language appears stronger in the light of the development of theoretical understanding of mathematics itself as discursive activity. Recognising the distinctive nature of mathematical communication is a necessary element of any study of mathematical activity, whether one adopts the 'strong' discursive position that mathematical objects have no existence independent of the discursive means of communicating about them, or a less absolutist position that there is no direct material access to mathematical objects but the experience of them through some form of 'representation' or 'realisation'. These two terms reflect distinct ontological positions: speaking of representation of a mathematical object suggests that there exists an independent object, whereas speaking of realisation proposes that the communication about an object is what gives the object existence. In either of these positions, mathematical activity implies engagement in a form of discourse about real or discursive objects. Understanding such activity involves studying that discourse and its features.

Where language is the object of study, description of the language might be an end in itself, addressing the nature of *the language of mathematics*. Understanding the features of mathematical language enables us to describe and evaluate the mathematical discourse of teachers and students in classrooms, while principled description of mathematical language opens up many questions: What are the features of the mathematical discourse in which students are expected to participate? How do classroom activities induct students into (what kinds of) specialised mathematical discourse? To what extent are students engaging in specialised mathematical discourse? From a 'strong' discursive position, any study of mathematical knowledge and learning entails asking questions about *the language of the learner* and of *the teacher/classroom*, and how they change. However, the significance of language in mathematics education includes the use or function of language as well as its form. Paying attention to how language functions suggests questions about reasoning, argumentation, proof, mathematical objects and relationships. Communication in the classroom and in other contexts, including curriculum, assessment and policy, also has an interpersonal function, constructing positions for students and teachers and framing relationships between them and to the mathematics. Studying the interpersonal functioning of language, drawing on theoretical resources developed in fields such as pragmatics, social semiotics and conversation analysis, can contribute to understanding social aspects of mathematics education such as how teachers manage classroom interaction and how students from various groups experience mathematics education.

Drawing on theories that conceptualise language as constitutive, constructive or functional enables researchers to analyse what is achieved in a given context through language use, addressing *the language of the teacher and the classroom*. We see the use of linguistic data as a means of gaining insight into understanding and learning of mathematics. In the first meetings of the TWG, research drawing on social constructivist and social interactionist perspectives is strongly represented, starting from interactional approaches of interpretive classroom research. This research focuses on studying classroom interactions using interactional analysis (Krummheuer, 1999) in order to observe learners' collective negotiation of mathematical meaning. Likewise located in interpretive classroom research but with an additional focus on the special nature of mathematical knowledge, the work of Steinbring (2005) focuses on the interactive construction of mathematical knowledge through classroom interaction and signification. This line has been present at each CERME since the first, building, applying and adapting Steinbring's (2005) epistemological perspective on class interaction.

Although we have tended to refer to the focus of the TWG as 'language', it is relevant to recognise that mathematical communication uses a variety of modes, of which the linguistic is only one. There are specialised modes, especially suited to mathematical activity, including algebraic notation, Cartesian graphs, geometric diagrams and other symbolic and diagrammatic forms used in specific areas of mathematics. In addition to these, studies of face-to-face communication indicate the roles played by gesture and non-verbal language in doing mathematics. The study of multimodal communication has developed in recent years, stimulated in part by the transformations effected by the growth of new forms of communication technology. This development is reflected in the TWG, incorporating multimodal analyses of classroom communication and an as yet small number of studies looking at communication mediated by technologies.

3 What have we learned from mathematics and language research?

While complexity is relatively low near the origins of ERME with language viewed as a system of symbolic structures and a focus on classroom interactions, the complexity rises when researchers take account of the cultural and historical conditions of the researched environment, and it becomes higher when they attend to the social foundations of language and mathematical activity. Along the continuum, the social becomes less subordinated to the study of culture and cultural patterns. The ERME domain has thus experienced progress in parallel with the expansion of the social turn in the field and the understanding of mathematics classrooms as cultural and historical configurations. Drawing on the distinction between the language of the learner, the language of the teacher/classroom and the language of mathematics, we put each theme in relation to major conceptualisations of language as system, as culture and as discourse (see Table 14.1). This organisation allows us to articulate the complexity of the ERME domain in terms of the relationship between complexity and continuity over time. Each

theme involves some continuum of complexity relative to the linguistic, cultural and social components progressively addressed.

TABLE 14.1 Elements in the expansion of the ERME domain

<i>ERME domain</i>	<i>Objects of study/themes</i>	<i>Conceptualisations of language</i>
Mathematics and language research	The language of the learner The language of the teacher/class The language of mathematics	System /Culture /Discourse

Language as system refers to the focus on the semantic and therefore grammatical potential of pre-given linguistic systems brought into play in the interaction (e.g. Rowland, 2002). Language as culture challenges the attention to formal aspects and considers the relations between language and forms of action produced in a context (e.g. Edwards, 2007). Language as discourse further challenges the idea of locality to consider the relations between what we do with

language in a context, our interpretation of that context and our reading of the social activity of the people in it (e.g. Morgan & Alshwaikh, 2010).

3.1 The language of the learner

We identify two lines of interest that have emerged through developing theoretical understanding of language as socially founded and of learning as discourse change. Complexity arises alongside discussion of the social and cultural conditions of mathematics learning in the classroom, and of how understanding these can contribute to understanding mathematics learning. Some of the papers pay attention to the diversity of languages involved in the learning process and negotiation of meaning, while others pay attention to the language-in-context of the learner. All, however, share an emphasis on the contextual conditions needed for mathematics learning to take place (Krummheuer, 1999). The learner is someone who needs to learn ‘the language of mathematics’, which requires access to and use of other languages and discourses of the classroom.

Discourse of the learner

By discourse of the learner, we mean the multiple uses of language that coincide in the learning process and through which the learner communicates realisations of this process. Within this frame, ERME studies differ not only in the notion of discourse they adopt, but also in the level of explicitness about their theoretical tools and how these are used to produce methods for analysing discourse and discourse change. Some studies relate the idea of language-in-use to the interaction of the learner with the material world. Fetzer and Tiedemann (2015) examine how the discourse of the learner is made of discursive interactions with people and with objects. This implies redefinition of the social nature of the discourse of the learner to include objects as actors affecting the use of language for mathematics learning. Thus,

discourse is more than what occurs between people in the form of verbal, written and other forms of symbolic communication. Mathematics learning emerges in the possibility of interacting with objects and abstracting from empirical realities. Although much research into language use still relies on analyses of written transcripts of recorded talk, these authors provide multimodal ways of transcribing video data for analyses of the interaction with objects.

Adopting an interactionist viewpoint, the critical correspondence between explicitness and implicitness in processes of developing conceptual understanding in the mathematics classroom has been investigated. Erath and Prediger (2015) address the question of how students learn to participate adequately in classroom mathematical practices through interaction regulated by explicit and implicit norms. Analysis of verbal interaction in the culture of the mathematics classroom reveals students who are involved in the performance of implicit norms about mathematical explanations. The discourse of the learner develops by participation in discursive practices, including ways of explaining, proving or defining mathematical concepts. It is interesting to note how most of the discursive practices in which the learner is expected to participate take place without detectable occurrence in spoken discourse. Implicitness thus appears as a condition of learning. Nevertheless, the learning opportunities vary depending on how and how much these discursive practices are communicated in visible ways in the discourse of the learner.

The non-verbal dimensions of language and the confluence of space and language in signed communication have been the focus in Krause (2017) regarding the discourse of the deaf learner in the mathematics classroom. The embodiment framework illustrates the interest in the analysis of non-verbal discourse and movement between verbal and non-verbal communication. Considering the discourse of the deaf learner, with signs and gestures produced in social interaction, opens up questions about the multimodal nature of the discourse of all learners. More generally, by understanding the learning processes of deaf learners, we may be in a better position to understand mathematics learning.

Although theories of orality and spoken languages in classroom-based research have dominated ERME research, the study of signs, gestures and particularly signed languages in the discourse of the learner has begun to come into focus. However, we find fewer papers centred on the written discourse of the learner and theoretical aspects of 'writtenness' in the mathematics classroom. One example is Schreiber's (2006) research into an internet-chat-based dialogue, which attends to differences between written data in the chat and spoken data collected during small group work. This experimental work suggests a way of interpreting the relationship between orality and writtenness as a social relationship with impact on mathematics learning. One finding is precisely that the concepts, theories, habits and competences of the participants are decisive for the emergent problem-solving and learning process.

Multilingualism in mathematics teaching and learning

In the early years of ERME, only a few papers addressed the issue of language diversity in mathematics teaching and learning. Where language diversity was an issue, most papers focused on linguistic aspects of mathematics that bilingual learners have to address. In the last decade, several papers have dealt with the experience of language diversity by the learner in more nuanced ways. Although there is not a unified theoretical approach to language diversity, recent work in sociolinguistics is present. Diversity refers to the languages of the learners as they interact with mathematics but also to the languages for communication: official languages of instruction, languages of teaching, and languages of thinking and learning. Learners of mathematics might switch from one language to another for different moments of communication in a lesson and combine aspects of these languages for different purposes. It is thus problematic to perpetuate discourses of monolingualism in the understanding of mathematics teaching and learning. Some studies are located in the transition between deficit perspectives on multilingual learners and views of language diversity as an asset for mathematics teaching and learning. The deficit perspective on multilingual learners is still present, though strongly contested nowadays, with language increasingly seen as an asset rather than a handicap. Far from focusing on obstacles for vocabulary, oral fluency and understanding in the language of instruction, we find studies centred on the resources that the languages of the learners bring to mathematics learning. Chronaki, Mountzouri, Zaharaki and Planas (2015) interrogate implications of the construction of the deficient multilingual mathematics learner. The case of a child whose dominant language differs from the language of the teacher reveals this child's participation in negotiation of numerical meanings. The support for flexible language use facilitates all children's engagement with diverse meanings for numbers. This study challenges taken-for-granted 'truths' about who is the competent learner of mathematics in the multilingual classroom, whose mathematics is valuable, and which discourses sustain language policies, curricular decisions and didactic actions.

Barwell (2015) also addresses the social dimension of language in studying multilingual learners in a way that challenges many common assumptions. This author draws on contemporary sociolinguistics of multilingualism to analyse the bilingual mathematics classroom, particularly on the notions of heteroglossia and orders of indexicality. The diversity of languages and the social diversity of speech types within any language, translated as heteroglossia, are stressed. Barwell suggests that the construction of mathematical learning in multilingual settings is often guided by views about languages and their speakers, rather than views of mathematical competence, performance and achievement. Other authors have developed from focusing on language forms and devices in the multilingual mathematics classroom to considering the social dimension of multilingual mathematics learning. This is the case of Poisard and colleagues, who have expanded their initial psycholinguistic frames of language. In Poisard, Ní Ríordáin and Le Pipec (2015), we find a move toward recognition of the relevance of other

influences, such as the culture of the mathematics classroom and the discourses at large in society. There is reflection on some of the compensatory responses in interpreting the needs of students whose home languages are different from the language of instruction. They note that some research has shown the positive pedagogic effect of using the languages of the learners in the multilingual mathematics classroom. This is in line with views of language as pedagogic resource and language use as cultural and social practice.

3.2 The language of the teacher/classroom

The previous section examined mathematics education and language in relation to learners; this section provides a change in perspective. Mathematics learning takes place in different social settings. Often in the interaction, one or more participants have more advanced skills, for example teachers or parents. In this context, the focus falls upon the language of such individuals (here briefly called teachers, even if including kindergarten teachers and others) and upon the language in the classroom or kindergarten. Studying the interpersonal functioning of language can contribute to understanding social aspects of mathematics education (Steinbring, 2005), including how teachers manage classroom interactions and how students from various social and cultural groups gain opportunities for mathematics learning in the 'learning spaces' structured by teachers and peers.

The standpoint that mathematical activity is socially originated and developed is central to most of the research concerned with classroom language. The earlier expansion of the linguistic paradigm was brought into clearer focus within the discussion of studies using interactionistic approaches of interpretive classroom research. These studies were distanced from the previously dominant view that learning was merely an internal psychological phenomenon. Thereafter, the inclusion of interactionistic aspects of learning and teaching meant a shift of focus from the structure of objects to the structures of learning processes, and from the individual learner to the social interactions between them. The transformed understanding of learning led to the development of theories that regard meaning, thinking and reasoning as cultural products of social activity. Based on the assumption that meaning is negotiated in interactions between individuals and that social interaction is thus to be understood as fundamental for learning processes, language can no longer be understood only as the medium in which meaning is constructed. Rather, speaking about mathematics in collective argumentations is to be seen as the doing of mathematics and the development of meaning. Thus, language acquires central significance in the building of mathematical knowledge and mathematical thought.

One can find numerous studies from the early days of ERME that focus on children's participation in classroom interaction. This focus is connected to the aim of these works to primarily understand, rather than change, children's learning processes. Krummheuer (1999) examines the relationship between students' participation in argumentative processes and their individual content-related development. Using transcripts from two research projects to reconstruct aspects of narrativity in interactional processes in the classroom,

he emphasises a 'folk psychology' of learning, where learning is conceptualised as a social process of cultural co-creation. In Price (1999), there is equal emphasis on understanding and change, indicating opportunities for the teacher to support mathematical learning. Price addresses the social nature of learning by analysing a transcript of a simple addition exercise in a group of children aged 4–5. She shows the importance of teachers using examples from everyday experience to promote children's learning, pointing out that although mathematical concepts such as addition are essentially abstract, they should not be taught only in an abstract way. Rowland (2002) also adopts the interactionistic focus on language in mathematics teaching. He examines utterances of two 10-year-old pupils discussing a problem with a teacher and notes that language has an interactional function, expressing both social relationships and inner attitudes. He argues that linguistic means can be used to analyse social and affective factors in mathematics teaching. Edwards (2007) focuses on participation in classroom mathematics learning and places the emphasis on learning in small groups. Reporting on collaborative classroom group work, her findings suggest that groups self-selected by pupils on the basis of friendship and trust produce dialogical reasoning and exploratory talk. This supports the idea of social interaction as a means toward cognitive change.

Jung and Schütte (2015) investigate to what extent the linguistic discourse in kindergarten and primary school gives children the opportunity to achieve mathematics-specific discursive competences that allow them to participate in the discourse of the mathematics classroom. This contribution illustrates an increasing trend to focus on the potential of improving conditions for learning mathematics. The teacher and the teacher's language become increasingly central in studies toward more optimal conditions for mathematics learning. Schütte (2006) analyses the linguistic accomplishment of instruction in a class. His results support a hypothesis of limited learning opportunities for a multilingual pupil body in classes because the linguistic accomplishment of the teacher orients itself toward perceptions of unity of a monolingual 'normal' child and the diversity is barely considered.

With the change of focus from the learner to the language design of classroom interaction of the teacher and the interactive interdependence between all participants, starting in the early 2000s, special emphasis has been given not only to the description of learning processes but also to demonstration of potential change or even initiation of these changes. Tatsis (2011) shifts the focus from the identities of learners supported by teachers to those of the teachers. He looks at the importance of language in the narratives that define teachers' identities, arguing that these identities are useful in understanding teachers' relationship to their actual practice and to the practice that they would expect to perform in the future. Through observation and analysis of teachers' participation in a training course, he finds that their identities and stories emerge from first-, second- and third-person narratives in verbal and written contexts.

Because of the increasing diversity of student populations, all places of learning inside, outside or before school – whether with an individual with

advanced skills in the interaction, as in the classroom conversation, or in small groups without such an individual – will increasingly be characterised by a plurality of interpretations in negotiations of meaning. It is of particular importance to note that mathematical language is itself diverse.

3.3 The language of mathematics

In this section, we review work that has addressed the relationship between language and mathematics, describing both the forms and functions of language-in-use. A source of complexity in this area is the developing breadth and sophistication of the conceptualisation of language itself. It has long been recognised that any consideration of mathematical language needs to take account of the specialised forms of communication distinctive of written mathematics, in particular algebraic notation (Pimm, 1987). So-called 'natural' language has been an object of study throughout the period, both in oral interactions and in written texts. In the early years of ERME, the orientation toward classroom interaction meant that the majority of research focused on spoken language. While transcriptions of classroom episodes sometimes included mention of gestures, artefacts or writing, these tended to be treated as contextual information and their roles in mathematical communication were not analysed. Reflecting the development of fuller theorisation of multimodal communication in the fields of linguistics and semiotics as well as in mathematics education, the scope of the group has come to incorporate a wider range of communicative modes, including gestures, diagrams and the multiple modes offered by new technologies. While forming rigorous descriptions of non-linguistic modes has been an essential part of expanding the conceptualisation of the language of mathematics, the main focus of research has been on how (multimodal) language functions in the construction of mathematical knowledge, and how use of various modes of communication contribute to support mathematical reasoning. Bjuland, Cestari and Borgersen (2007) studied how students and teacher combine their use of gestures and verbal language while interpreting a Cartesian diagram. They distinguished pointing and sliding gestures and identified how students integrated these with verbal language as they reasoned about the mathematical situation, using discursive strategies such as comparison or coordination.

The adoption of discourse perspectives on language has introduced further complexity. Within such perspectives, language (including multiple modes) is not conceptualised merely as a means of communication or as a tool for doing mathematics but as constitutive of the mathematics itself. Analysis of language use in a classroom interaction or a written text can thus illuminate the nature of the mathematics that is made available for students to experience. One distinction between types of school mathematics discourse focuses on how students might construe their position with regard to mathematical activity: whether they are invited to engage in creative intellectual activity and to see mathematics as involving making decisions and choices, or whether they are subject to an external authority that presents mathematical knowledge to be received as unquestionable. This distinction is made in Stamou and Chronaki's (2007) analysis of a mathematics magazine for lower secondary students. The

authors note the interdiscursivity of the texts they study – that is, the way that linguistic characteristics typical of one discourse, in this case the ‘traditional’ authoritative discourse, are incorporated into texts that appear to be within another, ‘progressive’, discourse. They identify this as a possible source of confusion rather than providing students with access to mathematics. Interdiscursivity, the mixing of resources from different discursive practices, is the focus of another strand of interest, albeit not always addressed from an explicitly discourse theoretic standpoint: the movement between ‘everyday’ and mathematical forms of language or, to use sociolinguistic terms, between colloquial and literate registers.

Functioning of language in the construction of mathematical knowledge and reasoning

The study of how language functions mathematically has varied from analysis of single signs and their use to studies of the qualities and purposes of whole genres, such as Misfeldt’s (2007) study of the roles of different genres of writing in the practices of mathematicians. The work of Steinbring and researchers influenced by his epistemological perspective on classroom interaction provides insight into the roles specific words, symbols or diagrams play in children’s construction of new mathematical concepts. This perspective emphasises that relations between representations and concepts are mediated by the ‘reference context’, including knowledge and experiences of the children, and hence may vary between individuals with different prior knowledge and may change as the reference context develops to include new knowledge. Nührenböcker and Steinbring (2007) explain how the interpretations by a teacher and two children of the decomposition of 8 into 4+4 varied because of differences in their reference contexts. Although the teacher used mathematical principles to explain why 4+4 should appear only once in a list of decompositions, the younger child persisted in interpreting two occurrences of 4+4 as distinct, referring to differences in the notation used rather than to the mathematical objects. Steinbring’s framework has served various analyses of classroom episodes involving children working on tasks. While this epistemological perspective provides insight into the role that signs play in forming children’s mathematical concepts, other studies have revealed the power of communicational modes to transform mathematical reasoning. Consogno (2006) introduces the notion of the semantic-transformational function of written language to argue that the dynamic process of production and reinterpretation of a text contributes to mathematical reasoning. She shows how, while writing their solutions to a problem, a process of linguistic expansion leads students to associate new words and meanings with the key words of the problem situation, thus shaping the direction of their reasoning. Using a discursive perspective, Morgan and Alshwaikh’s (2010) multimodal analysis of an episode of problem solving in a technologically rich environment demonstrates how students’ use of language and other modes of communication affect their approach to the problem. The variety of

perspectives in the study of how language functions in mathematical activity provides a range of explanatory frameworks but also achieves strong evidence and a powerful consensus that language communicates mathematical activity and influences its trajectory.

A related strand of research, as yet under-represented in ERME, addresses differences between the structures of various 'national' languages and how these might influence the mathematical activity of speakers of these languages. International interest in this topic has tended to focus on non-European languages. The work of Ní Ríordáin (2013) with bilingual students in Ireland begins to address this issue by relating variation in mathematical performance to characteristics of the English and Irish languages. While differences between European languages are generally less than those between the languages of Europe and of Asia, Africa and the indigenous languages of Australasia and the Americas, there is nevertheless scope for further research in this area. This might be of particular importance in light of the increased significance to educational policy of international comparisons based on tests translated into multiple languages.

Distinguishing mathematical language from everyday language

As discussed earlier, mathematics can be considered a discursive activity, using and manipulating specialised discursive resources (language, notations, diagrams, etc.) in distinctive ways. Mathematics education, however, is a hybrid activity, involving pedagogic and mathematical communication. The objective of mathematics education can be seen as induction of students into mathematical activity (and mathematical ways of communicating) rather than as simply doing mathematics. The language used in mathematics education thus inevitably includes non-mathematical and mathematical characteristics. This phenomenon is not unique to mathematics; learning in any specialised practice involves learning to use the specialised language of the practice. Distinguishing mathematical from nonmathematical forms and studying how these function in mathematics classrooms has been a strand of ERME research.

Pedagogic strategies frequently involve making connections between mathematics and familiar 'everyday' artefacts or problem situations. Whether these connections are intended as concrete support for developing mathematical concepts and procedures, as motivation for engaging in mathematics or as a form of application of mathematics through modelling and problem solving, the combination and coordination of the everyday and the mathematical also involves using a mixture of everyday and mathematical language. This juxtaposition might appear as a source of confusion and difficulty or as a means by which mathematical knowledge comes to be constructed. During a lesson in which primary school children were measuring and mixing ingredients to make waffles, Rønning's (2010) semiotic analysis of talk about fractions, decimals and measurements of volume suggests that the numbers and measurements given in the written recipe and marked on artefacts such as milk cartons and measuring jugs were interpreted differently by the teacher and by the children. For the teacher, marks such as '1/4 litre' and '15 dl' formed a connected chain of signs, linking the practical activity to

the mathematical activity. The children did not make connections between these signs but instead found practical solutions to the problem of mixing a batter of the right consistency, solutions that did not necessitate use of numerical measurements or calculations.

Connections and disconnections between everyday and specialised mathematical language can also occur when specific words or other communicative elements have potential to be used for making either everyday or mathematical meanings. Some of the authors have discussed differences between teacher and student use of apparently similar words and gestures in the context of the description and construction of mathematical objects in the classroom. Albano, Coppola and Pacelli (2015) use the general distinction between colloquial and literate registers (originating in functional linguistics) as a lens to analyse and discuss errors made by university students on a task involving graphs and analytic properties of functions. The components of the written answers could be said to be elements of specialised mathematical language, but were frequently used in ways characteristic of a colloquial rather than a literate register. The students, for example, evoked the local context of situation rather than general conventions of mathematical notation and treated graphs as iconic rather than symbolic representations. While use of the literate mathematical register is necessary to support mathematical thinking, the colloquial register also plays an essential role in supporting conceptual development. They conclude that the skill of moving between colloquial and literate registers needs to be developed and fostered from an early age by planned teaching activities – a conclusion echoed by other researchers in the field. Studies such as these provide insights into sources of apparent difficulties, misunderstandings and errors, locating these in the structures of mathematical and non-mathematical activities and the properties of language associated with those activities, rather than seeing them as arising from deficiencies in the students. The delineation of lexical, grammatical and structural characteristics of mathematical language developed by researchers involved in the TWG contributes to the knowledge required to underpin teaching that will help students develop skills in distinguishing between everyday and mathematical forms of language and moving between them.

We have exemplified papers situated within the different conceptualisations of language. For example, Krause (2017) and Ní Ríordáin (2013) mostly conceptualise language as system through the respective foci on the potential of a sign language structure and on two oral grammars. In Jung and Schütte (2015) and in Rønning (2010), we see the conceptualisation of language as culture in the respective foci on the relations between language and forms of talking mathematics in kindergarten and early primary school, and between language and forms of talking fractions in the resolution of a problem with everyday artefacts. Finally, in Chronaki et al. (2015) and in Tatsis (2011), we see the conceptualisation of language as discourse in the respective foci on how either learners or teachers view their contexts of language use and the people engaged in the activity there.

4 What more could we learn in the next decades?

We have discussed the progress and vitality of the ERME domain of research in mathematics education and language. Nonetheless, little is still known about many other aspects, e.g. how language is influenced by new technologies that enable new discourse practices (oral, chat, computer-mediated graphics, gestures . . .) and give rise to new questions: Do the newer tools change the ways people speak and write? Do they reflect established patterns of verbal interaction? How do we conceptualise the relationship between conventional forms of verbal interaction and communication and those mediated by new technologies? Little is also known about the ways in which methods and findings from the domain can be applied to mathematics teacher education and professional development. Past research has established the connection between teachers' pedagogical knowledge and experiences of professional development with little attention to issues of language responsiveness in teaching. Working with practitioners who have successfully integrated multilingual and multimodal practices in their classrooms would help.

There is also energy needed to address some practices within our research community. In a domain where language is at the core of the agenda, the ethics and practices of power involved in the use of language by researchers remain surprisingly under-examined. Knowing what we know now, in a period in which global information flows in English, we cannot expect that the question of English does not affect the domain. Researchers from a small number of countries conduct a majority of international work and English is the language with official status for this. The quality of the research experience is framed by how different languages and codes of communication are accepted, represented and acknowledged, particularly those of the participants, which might not even be known by the researchers. Analysis of how this situation influences empirical work is fundamental.

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