REVOICING IN PROCESSES OF COLLECTIVE MATHEMATICAL ARGUMENTATION AMONG STUDENTS

Núria Planas, and Laura Morera

Universitat Autònoma de Barcelona, Spain

In this report we draw on interactionist theories in (mathematics) education to better understand classroom processes of collective mathematical argumentation. We discuss students' uses of revoicing in different situations of mathematical learning taken from two recent micro-ethnographic studies in Barcelona and Tarragona, Spain. We document two examples that shortly illustrate two "positive" uses of revoicing in peer interaction: i) to ensure mutual understanding; and ii) to foster more explanations. We finish with comments on what is new in our research and how it needs to go in new directions to explore other uses of revoicing that appear when considering a more critical perspective in the analysis of classroom data.

INTRODUCTION

Language and discursive practices shape the concepts and processes that organize much of the everyday situations in the mathematics classroom. In their work, Enyedy et al. (2008) refer to revoicing as a discursive practice to promote a deeper conceptual understanding of school mathematics by positioning students in relation to one another, facilitating debate and fostering mathematical argumentation. The study we present here draws on this broad notion of revoicing to examine communication and mathematical argumentation in peer interaction. We discuss students' uses of revoicing in different situations of mathematical learning taken from two recent micro-ethnographic studies in Barcelona and Tarragona, Spain. This approach is part of our more general focus on the role of language as a social resource in the construction of collective mathematical argumentation in classroom settings.

Various works have examined teachers' uses of revoicing and interpreted this practice as an essential part of what the teacher does during the process of instruction (see Krussel, Edwards & Springer, 2004; or O'Connor & Michaels, 1996, among others). So far, there has been much more empirical literature developed on teachers' revoicing than on students' revoicing in peer interaction. Our study is a contribution to the more reduced group of works on students' revoicing, specifically for the area of mathematics education. We claim that the construction of the students' mathematical discourses is highly orchestrated by what other students say and how. Our data from small groups and pairs reinforces evidence to support the importance of knowing the students' reactions to the ways in which their peers "re-tell" their words while being engaged in mathematical tasks.

Before exemplifying data on some of the uses of revoicing in students' interaction, we start with theoretical considerations around the notions of revoicing and collective mathematical argumentation. We then move on to a brief summary of our methods in the analysis of classroom data, and discuss preliminary findings centered on “positive” uses of revoicing. We finish by suggesting some of the problems regarding the exclusive interpretation of revoicing as a facilitator. Our current analysis needs further examination from a more critical perspective that signals practices of revoicing also as markers of legal talk and talkers in the mathematics classroom.

THEORETICAL FRAMEWORK

This section introduces how the notions of revoicing and collective mathematical argumentation are conceptualized in our work. We point to inspiring literature in the effort to establish empirical connections between these two interactional accomplishments. We claim that revoicing is an important part of the processes that lead to argumentation, although we recognize that the relationships between these two practices are problematical: revoicing may be used for different (social) purposes and may have different implications, some of them with no clear orientation towards mathematical learning.

Revoicing

An assumption of the interactionist theories in (mathematics) education (see, for instance, Voigt, 1996; or Krummheuer, 2010) is that talk among students (and between students and teachers) needs to be analyzed as discursive practices through which (mathematical) knowledge is constructed. Some examples of these discursive practices are revoicing, questioning, requesting, telling, or managing. The practice of revoicing essentially tries to repeat some or all of what has been said in a preceding turn as the basis for a move in the interaction. This repetition can be manifested in two forms, either as a linguistically “exact” copy or as a reformulation. Despite the linguistic possibility to exactly repeat a sentence, from a social point of view and taking into account the recursivity thesis by Giddens (1979), we understand that every instance of the use of language is a potential modification of that language at the same time as it acts to reproduce it. Thus we find more adequate to associate revoicing to conceptual reformulation rather than linguistic repetition.

O’Connor and Michaels (1996) indicate three main uses of revoicing in teachers: 1) to position students in differing alignments and allow them to (dis)claim ownership of their position; 2) to share reformulations in ways that credit students with teachers’ warranted inferences; and 3) to scaffold and recast problem-solution strategies of students whose first language is not the language of teaching. These uses have been documented by these authors as having the effect to focus group discussion and scaffold conversation on the basis of what is said, when, how, with whom… Drawing on these three uses, the work by Forman and Ansell (2001, 2002) is however focused on the examination of the students’ voices. These authors analyze conversational moves in the “follow-up” part of the Initiation-Response-

Feedback sequence in mathematics lessons with frequent practices of revoicing. Although there is a clear emphasis on the social dimension of the IRF sequences and obstacles to the students' voices are recognized, revoicing is primarily seen as a facilitator in the interaction.

Collective mathematical argumentation

By "collective argumentation" we mean the interactional accomplishment given by: 1) representing a task or problem alone; 2) comparing representations within a small group of peers; 3) explaining and justifying the various representations to each other in the group; 4) reaching agreement within the group; and 5) presenting the group's ideas and representations to other participants in the class to test their acceptance (see Brandt & Schütte, 2010, for a similar interpretation that expands the idea of argumentation from an individual to a collective notion). Like Cobb (2008), we understand that situations of collective argumentation are mathematical if they are organized around specific ways in which tools and procedures are used to achieve mathematical goals. This is still a very general conceptualization of argumentation if we pay attention to the mathematics, but it becomes useful because it puts the emphasis on the processes of teaching and learning.

Sfard and Kieran (2001) have also discussed the role of the students' interaction in processes of collective mathematical argumentation. These authors interpret collective mathematical argumentations as interactive processes in the learning of how and when to participate in school mathematics discourses. They analyze the way in which students express themselves throughout their mathematical talk by means of discursive tools that help advance towards the construction of shared meanings. In particular, certain practices of revoicing in the resolution of mathematical tasks are interpreted as a social tool in the students' exploration of what counts as an accepted and "repeatable" reasoning in the mathematics classroom.

In the situated context of our work, revoicing becomes a reformulation of language to achieve new possibilities for further mathematical argumentation. The attempt is collective and entails a complex system of voices (Planas, 2007). It makes sense then to consider what it means for a student to participate in the "legal" process of constituting a culture of argumentation in the classroom. In this report, we look at the "unanimous" voice of the interaction, instead of pointing to individual voices trying to grasp what becomes necessary for them to gain membership while moving from one state of participation to another. At future stages, however, our study pretends to contribute to the much reduced group of works on the act of socially using revoicing in the delimitation of voices in the mathematics classroom.

EMPIRICAL CONTEXT

Our two research projects [1] share an interest in the qualitative analysis of narrative classroom data. Since 2005 we collaborate with a group of mathematics teachers [2] to develop inquiry-oriented tasks that are to be proved as facilitators of collective mathematical argumentation in small group and pair discussion. The implementation of tasks together with the characterization of situations of collective argumentation by means of the Toulmin's scheme (2007) has provided considerable information regarding students' reasoning in peer interaction. We have recently started to analyze broader discursive practices to include language and social issues in our analysis of how students help each other through talk in their joint construction of mathematical argumentations. The integration of mathematical, language and social issues guides our current process of gaining theoretical and empirical understanding.

In the context of the group of mathematics teachers and researchers, we examine data from secondary mathematics classrooms that were first chosen to validate the implementation of tasks. Up to now, we have searched for examples of students' revoicing in two main sets of data coming from two classrooms in two schools. We have had various meetings –some of them with the teachers– to comment on classroom lessons represented through video data. The meetings have been oriented by three main questions: 1) what is the evidence of revoicing in this lesson (if any)? 2) in what sense two examples of revoicing are similar/different?, and 3) what are the explicit uses to which different practices of revoicing are put? In what follows, we introduce two examples of peer interaction that hold the potential to investigate more deeply other episodes with a variety of practices of revoicing involved.

EXAMPLES OF FINDINGS

In this section, we describe processes of using revoicing as a resource. We document two types of "positive" revoicing. They are thought of as positive because they contribute to the continuity of peer interaction and mathematical argumentation. The two examples that follow illustrate two uses that prompt: i) mutual understanding, and ii) more explanations. These uses of revoicing seem to appeal forms of communication, argumentation, and interaction that would otherwise be difficult to achieve. Furthermore, these uses help interpret how the interaction is to be read: with what degree of respect or resistance towards students who are involved, whether with an eye to the mathematical contents or to the lack of them, etc.

More generally, our examples show that the mathematical learning gains "force" when it is invoked by one student and re-invoked by other students in the context of peer interaction. Data shows different students using correct mathematical reasoning that "does nothing by its own" until it is reconstituted by others through talk. In a big variety of episodes, the argumentation is first introduced by one of the students in the small group, and then becomes an object of negotiation through conversations in which revoicing helps distribute turns among speakers. There are also episodes in which revoicing becomes a strategy to overcome punctual interruptions in some of the students' mathematical implication during the resolution of the task.

Students' use of revoicing to reinforce mutual understanding

In group discussions with three or more students, there is a "positive" use of revoicing that contributes to make one of the student's ideas available to the rest of the members in the group, and helps reinforce mutual mathematical understanding. We have several examples of episodes in which a student partially explains an argumentation, and another student in the group uses revoicing to emphasize particular aspects of that argumentation, provide additional information, and facilitate a more adequate mathematical understanding from her/his peers.

In the example below, documented in Morera (2010), we find four students –Elba, Joan, Carles and Uriel– searching for how to transform one line segment onto another by means of a rotational superposition. The students are using a dynamic geometry package to identify the right place to construct the rotation centre. Although the four students are working together in the same small group, they are distributed in two different computers next to each other. The distribution in two pairs –Elba and Joan, and Carles and Uriel– leads to develop two different initial approaches to the problem (see Figures 1 and 2, with the given line segments printed in black).

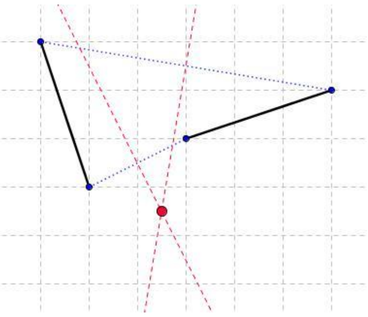

Figure 1. Elba & Joan's approach

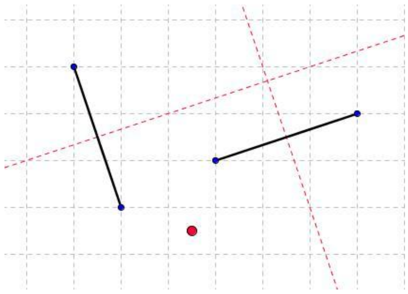

Figure 2. Carles & Uriel's approach

Two approaches to the resolution of the problem can be inferred from the transcript of the group discussion: while Elba and Joan plan to construct the segments by joining the different extremes and then drawing the perpendicular bisectors, Carles and Uriel draw the perpendicular bisectors of the initial line segments. There is a moment when Elba and Joan realize that Carles and Uriel are not considering the extremes of the given line segments. This is the starting point of the following dialogue [3] in which the two pairs bring together their ideas:

Joan: Shall we talk about what we've been doing?

Elba: I was discussing with Joan… Probably, if we drew now the perpendicular bisectors here, they would coincide at the same point [see the red point in Figure 1].

Uriel: It is impossible for the perpendicular bisectors to coincide over there! [see the red point in Figure 2]

Joan: [to Uriel] **If we drew the perpendicular bisectors here [3]**, when the two points come together, not the two line segments, **they would coincide at the same point**.

Carles: Ah, okay! I thought you meant this one!

Uriel: So did I!

Due to our focus on the identification of practices of revoicing in the construction of collective mathematical argumentations, we only represent the part of the transcript in which the use of a particular revoicing –with the exact repetition of a sentence in this case– helps overcome a misunderstanding in the resolution of the problem. In the transcript above, it can be seen how Elba introduces a mathematical argumentation that leads to confusion as it is not clear for Carles and Uriel which are the line segments for the perpendicular bisectors. Following an intervention by Uriel that confirms this confusion, Joan repeats Elba's explanation and includes a short clarification in-between –"... when the two points come together, not the two segments..."– that helps Carles and Joan expand their understanding of the problem. The whole transcript offers more evidence of this interpretation.

This first example illustrates the collaboration among the students in the group. Joan might have completely reconstructed Elba's sentence and started with a new explanation, instead of building into his peer's explanations. More generally, this example points to the deep social dimension in the elaboration of mathematical argumentations in the classroom. Argumentation emerges in conversational contexts and is oriented toward an audience. The context and audience determine how many details and mathematical clarifications are needed to go on with a reasoning, as well as to what extent certain explanations may be publicly considered as "repeatable".

In a context different from that of Elba's group, the sentence "If we drew the perpendicular bisectors here, when the two points come together, not the two segments, they would coincide at the same point", might not be representing a "good" mathematical argumentation: it is not indicated what is meant by the expression 'two points coming together', and it is not said which are the two line segments. That sentence needs to be interpreted at least in relation to what has been said in previous turns, and what is the knowledge that Elba and Joan have of their peers' reasoning. The adequacy of a mathematical argumentation in the social context of the classroom is informed by its mathematical quality, but also by the representations that the students (and the teacher) have of how mutual comprehension is facilitated.

Next, we offer a second example of collective mathematical argumentation in peer interaction with a slightly different use of revoicing that reinforces the occasions for mathematical talk, and fosters further interaction among students.

Students’ use of revoicing to foster more explanations

Students need talk moves that help them deal with lack of clarity of other students’ contributions in the resolution of mathematical tasks. Some students’ use of revoicing has the function to ask for more explanation on what has not been completely argued. In our research, this use tends to happen in pair work situations in which a student wants another student clarify a mathematical position and elaborate more on a specific idea. The example below shows the collaboration between two students, Anna and Ona, to find the quantity of squares in a chessboard.

Teacher:	You work in pair and collaborate with each other, okay?
Ona:	First we reflect on the problem on our own.
Anna:	Yeah, we need to know what to talk about […]
Ona:	[A few minutes later, to Anna] What are you writing here?
Anna:	Just counting all the squares in an easy way.
Ona:	You have the number?
Anna:	It’s one, four, nine, sixteen… they are always square numbers.
Ona:	**So you’re saying that they always are square numbers? ... And easy?**
Anna:	Yeah. You know why? [She points to a page in her notebook plenty of numbers with her written resolution]. You have one big square with sixty-four small squares, that’s eight times eight. Then you have four squares with forty-nine squares, you see, seven times seven [see Figure 3]. You see that?

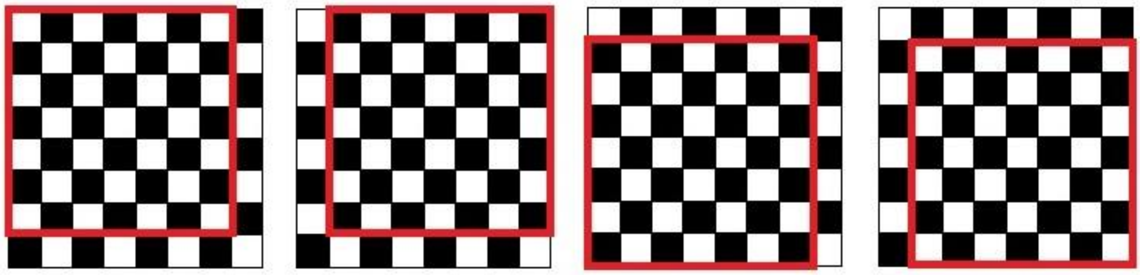

Figure 3. Anna and Ona’s approach

The excerpt above illustrates a classroom situation in which two students have been working separately for a few minutes and then come together to comment on their approaches to the problem of the squares in a chessboard. Anna has developed a complete and mathematically correct written resolution for this problem, but starts explaining it to her peer in a rather synthetic way –“Just counting all the squares in an easy way.” At the end of the conversation (the entire episode is not reproduced here), Ona comes to agree that there is an “easy” way to count all the squares in a chessboard; it is improbable that this agreement has been facilitated by Anna’s interventions in which she seems to expect that her peer mathematically “reads” through her words. This second example is similar to the first one, in what revoicing is used by Ona as an instrument that helps provide a way of testing Anna's claims on both the mathematics and the simplicity of the resolution method.

Anna initiates her explanations as if she was interacting with a mathematically "ideal" peer that was going to share and quickly understand her reasoning. In this context of interaction, Ona's use of revoicing acts as a way of forcing attention to who is the peer and which are her specific needs to gain agency in sharing a particular mathematical argumentation. Here, the use of revoicing facilitates to Ona the role of one who invokes a sort of participation that has a control on the sharing of reasoning. Anna's notebook contains a complete resolution of the problem, and by pointing to it she evokes an "ideal" reader that might feel sufficiently satisfied with the written text. Ona's reaction, with the repetition of a sentence and the emphasis on the idea of simplicity, makes it difficult to avoid further explanations and offers the possibility in practice to develop a discourse based on the resolution of the problem.

Like in the first example, the cooperation among students is required for the achievement of collective argumentation. Ona's use of revoicing contributes to a new conversation with the inclusion of more explicit explanations of the resolution processes that have been followed by Anna. It is necessary that Anna accepts the new basis for this conversation. When revoicing, as an instrument, is put to work it requires all parts to be involved. Ona, Joan or any other student, on their own, do not have agency enough to convert the reformulation of sentences into an instrument with the purpose of serving collective mathematical argumentation.

Although the empirical relationship between revoicing and collective mathematical argumentation still remains unclear in our work and interpretations of the episodes need to be reinforced with complementary perspectives, we can say a few things at this moment. We have chosen two examples of "positive" revoicing for this report, but we do not affirm that revoicing either expressly leads to more argumentation or more collaboration among speakers. We have data with practices of revoicing that do not turn into "more mathematics and/or more collaboration." The status of revoicing as an instrument for the sake of mathematical conversations appears linked to the social nature of this practice. In Planas and Civil (2002), some of the social issues of influence on the interpretation of discursive practices in classroom settings were already documented, with specific attention to recognition among peers. In our second episode, for instance, Ona's revoicing is effective because Anna is willing to explain her reasoning. Nevertheless, what would happen if Anna imagined her peer as an obstacle in her learning? Would she give detailed answers to her questions?

FINAL REMARKS

Together with the interest in examining relationships between revoicing and collective mathematical argumentation, a research focus on revoicing in mathematics classrooms raises many other questions. What is new in our work is the interest in the exploration of some of the critical functions that are carried out by practices of revoicing that are initially linked to "positive" uses only. Much remains to be done in this direction, and we are in fact still at the stage of empirically illustrating "positive" uses and "generating suspicion". It is not clear whether one can critically examine revoicing in the strict context of the micro level of the small group or the whole class with no attention to the multiplicity of voices from the different and various macro levels that have influence on how discourses are re-elaborated in classroom settings. The repetition of a sentence may serve as a strategy to foster mutual understanding and mathematical explanations, and at the same time represent messages of incorrectness, doubt, disapproval… depending on who the speakers are.

It seems unlikely that a one-dimensional view on revoicing or any other discursive practice, based on either mathematical or social issues, helps better understand how mathematical conversations are prompted in classroom settings. On the one hand, from the joint perspective of language and mathematics, we cannot claim that all "significant" mathematical meanings are maintained the same when a sentence is reformulated, neither can we affirm that "repetitions" always stand for evidence of learning. This uncertainty points to serious methodological obstacles, especially when trying to justify processes of individual mathematical learning that are constructed in contexts of conversation with frequent practices of revoicing. On the other hand, from the joint perspective of language and social interaction, even when a sentence is repeated exactly the same, we still cannot guarantee that the context and the interpretation have not varied. The precision of the language of mathematics and the complex social discourses around it (e.g., 'who is considered as mathematically competent', 'what is expected to be included in school mathematics') makes it difficult to answer all these questions without adopting a multi-dimensional view on how everyday situations in the mathematics classroom are organized.

NOTES

[1] The work is part of Projects 'Estudio sobre el desarrollo de competencias discursivas en el aula de matemáticas', EDU2009-07113, and 'Contribución al análisis y mejora de las competencias matemáticas en la enseñanza secundaria con un nuevo entorno tecnológico', EDU2008-01963, both funded by the Spanish Ministry of Science and Innovation. The two authors are members of the Research Group 'Educació i Competència Matemàtica', SGR2009-00354, recognized by the Catalan Department of Universities. The second author owns Grant BES2009-022687.

[2] The Group EMAC –Catalan acronymus for Critical Mathematics Education– is supported by Associació de Mestres de Rosa Sensat, and partially funded by Project 'Diagnosi de necessitats socials i educatives de l'aula multilingüe: aproximació des del cas de matemàtiques', ARFI-1-2009-00052, Catalan Government.

[3] All dialogues have been translated from Catalan to English by the first author.

[4] The bold format is used in our transcripts to mark the exact moment in which revoicing appears.

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