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Contradictions, dialectical oppositions and shifts in teaching mathematics

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Abstract The study reported in this paper concerns the tensions and conflicts that teachers experience while they enact a new set of reform-oriented curricular materials into their classrooms. Our focus is on the interactions developed in two groups of teachers in two schools for a period of a school year. We use Activity Theory to study emerging contradictions and we elaborate on the construct of dialectical opposition to understand the nature of these contradictions and their potential for teacher learning. We provide evidence that discussions about contradictions and their dialectical character in the two groups support teachers to engage differently in mathematics teaching and learning and carry potentials for shifts in the practices that evolve in their classrooms. Our study addresses empirically in the context of mathematics teaching the philosophical claim about the role of contradictions as a driving force for any dynamic system.

Keywords Contradictions · Activity theory · Teachers' learning · Dialectical opposition

1 Introduction

Mathematics teachers often face tensions in their professional life and especially in teaching. These tensions include pedagogical conflicts emerging in teaching-learning processes

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(Jaworski & Potari, 2009); cognitive and emotional tensions revealed in the process of self-reflection (Chapman & Heater, 2010); tensions between different identities in relating to different professional obligations (Potari, Sakonidis, Chatzigoula, & Manaridis, 2010; Skott, 2001, 2009); conflicts related to teachers' expectations and perspectives regarding professional development (Yamagata-Lynch & Haudenschild, 2009). Tensions and conflicts often emerge in the context of classroom interaction (Barab, Barnett, Yamagata-Lynch, Squire, & Keating, 2002; Jaworski & Potari, 2009) or in teacher collaborative contexts (Goodchild & Jaworski, 2005; Sakonidis & Potari, 2014). They are mostly apparent when teachers are engaged in educational reform, for instance when working with a newly prescribed curriculum (Remillard & Bryans, 2004).

During the last few years, teacher collaboration has been considered a significant source of their professional learning, for instance when they plan and reflect upon interventions and innovations (e.g., Lewis, Perry, & Hurd, 2009). Sometimes such collaborative contexts function with no or little outside support and with no pre-specified goals for teacher learning, whereas in others they are part of more comprehensive teacher development programmes. In either case, professional learning and the development of teaching are often studied through participative perspectives that focus on social factors in order to capture the complexity of the situation (Potari, 2013). In some cases the emphasis is on the emerging tensions and conflicts themselves (e.g., Goodchild & Jaworski, 2005), while in others the focus is on ways of addressing such conflicts in intervention studies (e.g., Engeström, Engeström, & Suntio, 2002).

The study reported in the present paper concerns the tensions and conflicts that two groups of teachers experience as they interact with each other in a situation that is located somewhere between the extremes of either studying teacher learning without external support and being part of a larger intervention programme. The study is conducted at two schools in Greece at the time of the introduction of a newly prescribed curriculum. The schools were selected by the government as pilot schools for the new curriculum, and the first author, who is both a colleague of the research participants and member of the group who wrote the curricular materials, called the meetings and conducted the interviews. However, the study is not part of a more comprehensive teacher development scheme. The participating teachers expect – and do receive – some support for their discussions, but the questions raised and the ways of addressing them are generally a result of their discussions among themselves. The questions we ask in the present paper are what the character is of the contradictions the teachers experience in this particular context, in what ways teachers deal with them, and if there appear to be potentials for teacher learning in the discussions in this particular context? We draw on dialectical logic (Ilyenkov, 2009) and activity theory (AT) (Engeström, 2001; Leont'ev, 1978) to study the epistemological background to some of these contradictions and to understand them as potential starting points for shifts in teaching. Our substantive argument is that emerging contradictions that challenge teachers' instructional approaches may qualify "as sources of change and development" in Engeström's sense (Engeström, 2001, p. 137) and function as instigators of their professional learning. At a more formal level, we seek to demonstrate the potentials of our framework as one way of conceptualising teacher learning.

To make our points we first describe our framework by outlining aspects of AT and elaborating on our understanding of a key construct in the study, the one of dialectical opposition. We then provide information about the context of the study and present our methodological decisions, before demonstrating how we use the framework to study the epistemological background of the emerging contradictions and to understand them as potential starting points for teacher learning and shifts in teaching.

2 Theoretical framework

2.1 Teachers as curriculum developers: an enactment perspective

Teachers need to play a substantial role as a link between curricular intentions and the practices that evolve in their classrooms. In the context of current reform efforts, this situates the teacher at the centre of the curriculum enactment and creates new challenges and conflicts leading to a situation that Skott (2004) describes as one of "forced autonomy", a consequence of the demands put on the teacher. This implies that implementation is not a suitable metaphor for the introduction of educational reform, as it may carry connotations of a smooth and conflict-free execution of a new set of curricular intentions, irrespectively of contextual factors that emerge in schools and in classrooms. An alternative perspective on curriculum, the one of enactment, focuses learning opportunities that evolve in classrooms (Brown, 2009; Remillard, 2005). Teachers are not seen as transmitters of a curriculum formulated by experts outside the classroom. Rather they are considered active agents and designers, whose instructional actions may be influenced by curricular materials, but also shaped by their interactions with the students in the classroom and by other aspects of the specific and broader context. According to this participatory perspective "teachers and curriculum materials are engaged in a dynamic interrelationship that involves participation on the parts of both the teacher and the text" (Remillard, 2005, p. 221). As Choppin (2011) points out, a teacher's contribution to the interrelationship is based not merely on her reading of the rationale of the prescribed curriculum, but on classroom-based experiences with working with students and with the use of curricular materials.

This perspective is inspired by cultural-historical activity theory with its emphasis on tool use and mediation. Curriculum materials function as tools, that is, as products of sociocultural and historical evolution that "both shape and are shaped by human action through their affordances and constraints" (Remillard, 2005, p. 221). As Remillard notes, the characteristic of this perspective is "its focus on the activity of using or participating with the curriculum resource and on the dynamic relationship between the teacher and curriculum" (p. 221). This perspective emphasizes the ways in which individual and social factors come together to influence the acts of teaching and their development. Cultural-historical activity theory (AT) provides a dialectical perspective on the ways in which teaching develops because of and through the contradictions that emerge as teachers engage with it.

2.2 Activity theory and teaching

As teachers engage in teaching, the goal of promoting their students' mathematical learning is intertwined with other professional obligations. Their actions are mediated by tools such as curricular documents, school textbooks and other teaching-learning materials. Also, they are framed by institutional constraints and commitments (e.g., examinations, time constraints, timetables) and school and classroom norms for the teacher's role and position in the classroom and at the institution in general, including issues pertaining to power and the division of labour. Consequently, teachers constantly balance the emphasis on student learning with practices and priorities that are linked to the discipline of mathematics or that evolve as they engage with their colleagues, the students, the parents, and many more.

AT offers a lens to study this context, as it tries to capture the complexity of teaching, by integrating dialectically the individual and the social/collective and focusing on how the individual engages in the societal activity of teaching. The activity is driven by a motive and directed towards an object (Leont'ev, 1978), in our case the motives of students' learning of mathematics and the fulfilment of teachers' other professional obligations. From this perspective, the unit of analysis is the activity system (AS) that incorporates social factors that frame the relations between the subject and the object with the mediation of tools (Fig. 1). These factors are related to the communities in which the subject acts, the rules of these communities, and the division of labour among participants in them (Engeström, 2001).

Engeström (2001) has pointed to five central ideas or principles of AT. These are that in AT (a) an AS is the unit of analysis; (b) an AS is characterised by "multi-voicedness", as it "is always a community of multiple points of view, traditions and interests" (p. 136); (c) the historicity of AS are acknowledged; (d) the contradictions of an AS are viewed as sources of development; (e) the possibility is recognised of expansive transformations, that is, of qualitative shifts in the functioning of the activity system as participants react to growing of contradictions within it, which in turn may lead to "a deliberate collective change effort" (p. 137). These principles have oriented prior studies in mathematics education (e.g., Goodchild & Jaworski, 2005; Jaworski & Potari, 2009; Potari, 2013), and also inform the present one. For our present purposes we need to elaborate on our understanding of the last two principles, the ones that deal explicitly with contradictions.

2.3 Contradictions: activity theory and dialectical oppositions

Every activity system is characterised by contradictions. They may emerge when an AS adopts new elements from the outside, such as a new tool or a new rule, causing a conflict with how it functions at present. Contradictions are not every day solvable problems but "historically accumulating structural tensions within and between activity systems" (Engeström, 2001, p. 137). Contradictions may create learning opportunities for the subject and may broaden the activity, for example leading to reconsideration of the actions and goals if an innovation is introduced (Engeström, 2001; Potari, 2013). However, contradictions, manifested as conflicts, are often related to negative emotions that may become obstacles for professional learning and push the individual towards resignation (Roth, 2007).

Roth and Radford (2011) write about "inner contradictions", referring to conflictual aspects of the same phenomenon that coexist dialectically and "cannot be removed". Like Roth & Radford, we draw on Ilyenkov (2009), who point to this "inner" aspect of contradictions as a key concept in dialectics. A contradiction, he says, is "the concrete unity of mutually exclusive opposites […] the real nucleus of dialectics, its central category" (p. 185). Furthermore, "any

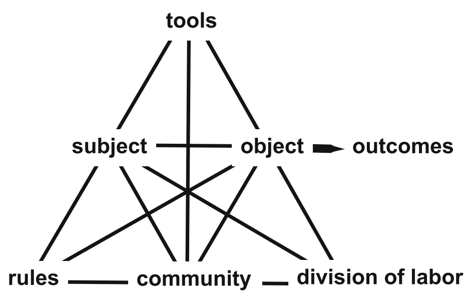


Fig. 1 The activity system (adapted from Engeström, 2001)

concrete, developing system includes contradictions as the principle of its self-movement and as the form in which the development is cast." (p. 191).

In dialectics, contradictions get a different meaning than in logic:

> Strictly speaking it [the logical contradiction] relates to the use of terms and not to the process of the movement of a concept. […] [In dialectics] another law is dominant, the law of the unity or coincidence of opposites, a coincidence, moreover, that goes as far as their identity. It is that which constitutes the real core of dialectics as the logic of thought that follows the development of reality. (Ilyenkov, 2009, p. 198)

For example, in group meetings the way the students could become aware about the different meanings associated with the minus sign was discussed. This symbol in -5 shows a negative number, but in $-x$ indicates the opposite of x, even if this number ($-x$) is positive. We conceive this as an inner contradiction related to the use of this sign.

For our present purposes we build on Ilyenkov's understanding of dialectics and classify observed contradictions in our data according to the possible presence and character of dialectical oppositions (DOs), that is, of "the concrete unity of mutually exclusive opposites" (Ilyenkov, p. 185). In our case DOs refer to the unity of different aspects of mathematical concepts or how concepts are used and transformed in teaching. DO is a theoretical construct that may interpret some or most of the emerging contradictions. We use DOs to characterise significant aspects of the discussion among the teachers as seen from our perspective, even if the dialectical character may not be acknowledged by the teachers themselves. For instance, we use the DO object-process for aspects involving the concepts, relations and properties on the one hand and the execution of operations or algorithms on the other. Although the emerging contradiction concerns the teacher's attempt to focus on the notion of an equation and the student's preference of the procedural approach, DO offers an epistemological way of interpreting this contradiction as object–process.

2.4 Contradictions in research in mathematics education and in this study

Contradictions and their educational significance have been dealt with in mathematics education research for decades, sometimes in ways that appear compatible with the notion of DOs. Steiner (1985) suggests that it is important for both epistemological and educational reasons to understand the interrelatedness and mutual dependence of apparent opposites such as "skill vs. understanding, structure building vs. problem solving, [and] axiomatics vs. constructivism", referring to the concept of complementarity as an adequate tool (p.15). Also working with complementarity, Sfard (1991) argues that mathematics may be conceived in two epistemologically different ways, namely operationally and structurally. While these understandings are fundamentally different, they may be conceived of as complementary rather than as incompatible. Other attempts in mathematics education to adopt a non-dichotomous view of seeming incompatibilities include discussions of the relation between intuition and logic (Fischbein, 1987) and between symbols and meaning (Presmeg, 1992).

More recently, the number of references to contradictions in the research literature has increased, and some studies use AT to identify, describe and interpret contradictions in teaching and in teachers' professional development (Barab et al., 2002; Engeström et al., 2002; Goodchild & Jaworski, 2005; Jaworski & Potari, 2009; Potari, 2013). In these recent studies contradictions refer mainly to pedagogical or professional issues, with less attention paid to mathematical and epistemological ones.

In the context of AT, professional learning is considered embedded in what is referred to as an expansive cycle (Cole & Engeström, 1993), a process that is triggered by the emergence of contradictions. An expansive cycle of an activity system is a developmental process that contains both internalization and externalization. It begins with almost exclusive internalization, in which individuals become competent members of an existing activity as it is routinely carried out. As contradictions, in the evolution of the AS, become more and more disruptive and demanding, critical self-reflection and the search for solutions increase. Creative externalization, that is, the search for novel solutions, first appears in the form of discrete individual violations and innovations. Externalization reaches its peak when a new model for the activity is adopted and implemented by other subjects. As the new model stabilizes itself, internalization becomes again dominant. Engeström and his colleagues (Engeström et al., 2002) as well as other researchers (Goodchild & Jaworski, 2005) supported teachers in identifying and overcoming contradictions in attempts to open expansive cycles. We adopt Engeström's position that professional learning often is "not stable, not even defined or understood ahead of time" and "there is no competent teacher" who knows what must be learned (Engeström, 2001, p. 137–138). We do so, however in a situation in which teachers' professional learning is triggered by the introduction of a new significant tool, a new set of curricular materials, as well as by the presence of some outside support (the presence of the first author), who to some extent may be conceived as an external authority by the research participants.

We build on the notion of expansive cycles in our attempt to understand the role of a range of different contradictions for teachers' professional learning. In particular, we focus on the part of the expansive cycle that refers to the first forms of creative externalization as "discrete individual violations and innovations" (Cole & Engeström, 1993, p. 40). The new materials conflict with established practices, fuelling the emergence of a multitude of specific contradictions in teaching and teacher development settings. Most of them may be interpreted dialectically as DOs: the opposing aspects may be seen to form an inseparable unity. Thus, we use the notion of a DO as an interpretive tool to develop categories of contradictions that emerge in our data. For example, the participating teachers in one of the schools express their disapproval of students' tendency to focus on keywords rather than conceptual understanding. For instance, students interpret "at least" in tasks as a prompt to use Least common multiple in problems when working with divisibility. This contradiction may be interpreted as a manifestation of the DO between the object (concepts and relations in divisibility) and the process (a procedural approach in problem solving).

Summing up, we argue that the concept of DO may be used as an interpretive device for understanding the character of the contradictions that emerge in teachers' planning and in teaching and that it allows us to reconsider the epistemological dimensions of mathematics (e.g., content vs. processes) and mathematics teaching and learning. It also helps us identify and understand the development of the individual teacher's teaching and its potential contribution to the transformation of the collective activity.

3 Methodology

3.1 The context of the study

A new set of reform-oriented curricular materials was introduced and piloted in a small number of schools in Greece in 2011–12 and 2012–13. The new materials, teachers' guides, etc.

emphasize students' mathematical reasoning and argumentation, connections within and outside mathematics, communication through the use of tools, and students' metacognitive awareness. It also attributes a central role to the teacher in designing instruction.

In 2012–13 we collaborated with teachers in three of the junior secondary schools that piloted the new materials. The collaboration took place in group meetings at the respective schools, as the first author, who was also a member of the team that developed the curriculum materials, supported the teachers by providing explanations about the rationale of the reform and of the proposed teaching materials. The main tasks undertaken in the groups (from now on referred to as reflection groups) were planning lessons and reflecting on their experiences with teaching different modules of the designed curriculum. The teachers decided on the themes of the discussions, and the role of the first author was to participate in the discussions by challenging teachers to describe and justify their teaching decisions. In this paper we refer to the collaboration with school A (8 meetings with 5 teachers) and school B (10 meetings with 2 teachers), and we draw on the transcriptions of the meetings and of semi-structured interviews conducted with each teacher at the beginning and at the end of the year.

3.2 The schools and the participants

School A is an experimental school with an innovative spirit in a Greek major city, and the teachers participating in the study have long teaching experience and are familiar with educational innovations. In this paper we refer especially to two teachers, Marina and Linda. They both have more than 25 years of teaching experience and additional qualifications beyond their teacher certification: Marina has a masters' degree in mathematics and Linda has one in mathematics education. Also, they both have experiences with innovative teaching approaches, and they have participated in teacher collaborative groups developing classroom materials. Further, Marina has written papers for conferences and for journals for mathematics teachers, and is more informed than Linda about the research activities of the mathematics education community in Greece. Both teachers have a reflective approach to innovations in general, adopting some of them and rejecting others, and they have strong views about their instructional choices. Concerning the new mandated curriculum, Marina says that she considers it a "legitimizing umbrella over my practice", explaining that it is in line with practices that she has tried to promote in her classroom for years. This is a comment with which Linda explicitly agrees (Marina, 1st interview).

School B is a school in a suburb of a major city in Greece, and the students are to a greater extent than those from school A from working class backgrounds. This school also has an innovative orientation, especially in using digital technology in teaching. At the beginning of the study, none of the participating teachers from school B has additional formal qualifications beyond their pre-service teacher education. One of them, Peter, has experience with using computers in school mathematics from his involvement as educator in teacher education. Manolis, the other teacher, has no such experiences. Peter and Manolis have 15 to 20 years of teaching experience each.

3.3 The process of data analysis

The data presented in this paper are generated at the beginning and towards the end of the study. The data material consists of transcriptions of audiotaped conversations and interviews as well as of written documents (worksheets, lesson plans). The written documents were used to develop a better understanding of the content of the conversations. The transcriptions were analysed with methods inspired by grounded theory, but without the objectivist connotations sometimes associated with them (Charmaz, 2006). We fully acknowledge that our pre-understandings, most notably the framework that we use, inform our interpretations of the data. In line with Dey (1999), we do not assume to approach the data material with an empty head, but with an open mind. This is reflected in our coding. The initial open coding resulted in the identification of discussion themes for each meeting, forming thematic units which were subsequently analysed further. For example, in the 4th meeting in school A, one thematic unit was labelled "the use of models in operations with integers". Brief narratives were formed for each unit, where teachers' concerns, choices and reflections were described with minimal interpretation. The average length of the narratives was about half a page for two pages of transcribed discussions. In each unit teachers' choices, their rationale, and the emerging contradictions were identified. We found three types of indicators of contradictions useful, namely (1) disagreements among the participants; (2) disagreements between participants and an external source (e.g., the students or the curricular materials); (3) apparent incompatibilities between different utterances made by a single teacher. As an example of the last of these, a teacher in school B, Manolis, says that he appreciates communication between students, but requires students to be quiet in the classroom. Each identified contradiction, was formulated as a dipole (e.g., the choice of tasks aimed at conceptual understanding or at procedural fluency). For every identified contradiction we used descriptive codes related to its content (e.g., students' difficulties, selection of tasks etc.), the agents (e.g., a contradiction between participants and the curriculum), and teachers' awareness about the contradiction. While up to that point, our analysis was mainly grounded, in the following steps AT began to orient our interpretations which led to the construct of DOs. If possible, the contradictions were subsequently, classified in categories of DOs. Thus, we used theoretical analysis to interpret the outcomes of the grounded analysis.

We used the thematic units to identify shifts in the teachers' discussion about their participation in teaching activity. Such shifts may concern changes in the teacher's decisions about instruction or refer to changes in her instructional approach in the classroom, as these changes are reflected in the group discussions. Interpreting these shifts, we linked them to the contradictions emerging in the same thematic units. We selected thematic units of the same content from the different meetings and the interviews to trace the shifts.

In the process of the analysis, similar contradictions were collected in clusters. Then, we used DOs known from philosophy and from mathematics education research to label the categories and subcategories of contradictions. Schematically, while the identification of contradictions was an outcome of our (theoretically informed) analytical approach inspired by grounded theory, the construct of DOs was an outcome of our attempt to classify them on the basis of our theoretical framework. In the next steps of characterizing the DOs and linking them with shifts in teaching, the lenses of dialectics and AT played a more prominent role in the analysis.

4 Results

We identified 171 contradictions in the data and interpreted 141 of them as having a dialectic character. The research participants were not always explicitly aware of what to us appeared as contradictions, and especially the ones between different utterances by the same person were often not identified by the person in question and consequently not acted upon. Other contradictions were recognised by the participants and triggered discussions among them, but were not resolved in the process. In these situations the discussion ended in conflictual disagreement between or among research participants. However, in many other cases teachers sought to deal with a contradiction, and we identified three ways of doing so: (a) the synthesis of the opposing poles; (b) the adoption of one of the poles and the participants' individual or communal attempts to distance themselves from the other; (c) the explicit reference to both poles as relevant to mathematics teaching and learning. Sometimes these ways led to a shift in the teacher's teaching towards more innovative approaches.

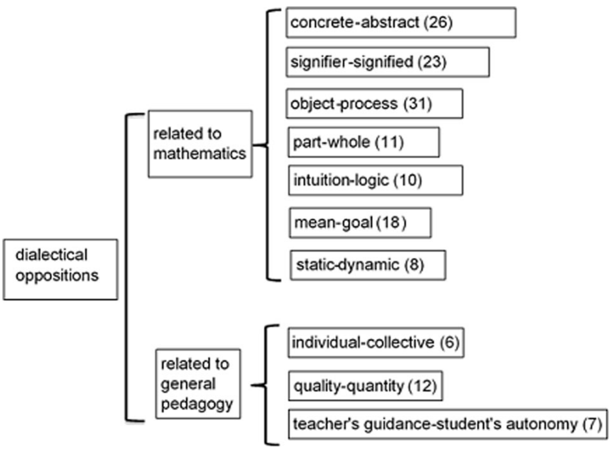
Below, in Section 4.1, we elaborate on the dialectical nature of the 141 contradictions and present a classification of them, based on the construct of DO. In Section 4.2, we illustrate the potential of our framework, as we use it to study shifts in teaching as they are revealed in the group discussions by focusing on the way two teachers, Marina and Linda, integrate geometrical transformations into their teaching.

4.1 Classification of contradictions

In Fig. 2 the categories of contradictions classified as DOs are presented in the form of a systemic network (Bliss, Monk, & Ogborn, 1983) with some of the contradictions falling in more than one DO (the numbers add up to more than 141). The DOs are related to mathematics or to general pedagogy and in this paper we discuss those of the first category. The DOs *concrete-abstract*, *signifier-signified* and *object-process* are the most frequent in our study as more than half of the identified contradictions fall under these categories. These DOs represent central features of mathematics and mathematics learning such as the importance of abstraction, the use of symbols, and the dual nature of mathematical objects as concepts and processes.

Below, we elaborate on the DO *object-process* as an exemplar of our use of the construct of DO, providing some examples of contradictions from the data. This opposition is closely related to epistemological dimensions of mathematics and to the learning of mathematics. It expresses, as Sfard (2008) suggests, that narratives about mathematical processes may reify into discursive mathematical objects, which in turn "may be used both as prescriptions for

Fig. 2 Dialectical oppositions as categories of contradictions emerged in the data



processes and as the products of these processes" (p. 182). In our data it appears in the forms below:

a) Concepts, relations and properties vs. execution of operations or algorithms. For example, Peter says that many students have difficulties with the concept of equation, although they are comfortable with the procedures for solving equations: "[the student] memorizes a procedure like a rote, she is able to do miracles … but she cannot answer … what is the meaning of solving an equation … she doesn't know that in every equation we try to find the number that we must substitute to the unknown to have equality" (school B, 4th meeting, [B4]).
b) Structure vs. process. For example, Marina says that students often perceive the expressions 3 (a+b+c) and 3a+3b+3c as a call to execute a series of operations without recognizing their structure as sum or product: "children see addition and multiplication, while we see product and sum … so I want to emphasize the structure …" (school A, 2nd meeting [A2])
c) Conceptual understanding vs. procedural fluency. For example, in his lesson on simplification of fractions with algebraic expressions in numerator and denominator, Peter asks children to break the fraction into other fractions in such a way that the properties of powers to become obvious (e.g., $\frac{3x^2y}{xy^3} = 3 \cdot \frac{x^2}{x} \cdot \frac{y}{y^3}$). He explains this choice: "… because I have noticed that most of them delete the exponents mechanically. … I want [students] to connect simplification with division and power properties" (B5).

4.2 Contradictions and shifts in the context of teaching geometrical transformations

Geometrical transformations play a more important role in the new materials than before. Some elements of reflectional and rotational symmetry existed in the previous syllabus and in the textbook, but geometrical transformations (translation, reflection and rotation) are introduced as a distinct topic in the new curricular materials, primarily in grade 8. The rationale is to support students' development of spatial sense and to use transformations when tackling issues of congruence and similarity. The topic emphasizes the transformation of a figure as a whole and seeks to support intuitive and dynamic approaches to geometric shapes and their properties. The focus is on the relationship between the two figures (original and image), highlighting the relation of congruence or similarity and attributing to the transformations the character of a proving tool.

Geometrical transformations are discussed repeatedly in the reflection groups, as the topic has not been taught much before. Fifteen contradictions emerge, most of them were categorised as DOs: part-whole, means-goals, static-dynamic, intuition-logic and concrete-abstract. For example, the DO part-whole was expressed as an opposition between the transformation of points of a figure and of the figure as a whole. In his introductory lesson, Peter (school B) selects tasks requiring students to use paper and pencil to reflect a triangle over an axis by reflecting its vertices. After two lessons, he asks them to use Geogebra's ready-made tools to transform a figure.

DOs identified in connection with geometrical transformations are generally of an epistemological character. Often the teachers do not seem to be aware of these contradictions and so few attempts are made to deal with them in their planning and teaching. In what follows, we are interested in what happens, when teachers do become aware of these contradictions and decide to deal with them. Below we exemplify our use of DO and AT to conceptualize the role of contradictions in the transformation of teaching activity.

In Greece, school geometry is traditionally rooted in "Euclid's elements" and is taught in a rather rigorous deductive manner. The use of transformations as a proving tool is an alternative to the Euclidean perspective on school geometry, since the intuitive use of the moving figure is often seen as incompatible with Euclidean geometry. This issue was highlighted in the discussions in school A. In this section we study the contrasting ways that Marina and Linda cope with this contradiction. Marina and Linda have been teaching geometrical transformations in grade 8 in the first year of enacting the new curriculum. The discussion below concerns whether geometrical transformations are to have a role in teaching congruence of triangles in grade 9.

In the fourth meeting (A4), Marina refers to her introductory lesson on triangle congruence in grade 9. She asks "How can we ascertain that these two triangles are congruent?" and is pleased that some of her students respond that they are congruent "if the triangles match after translation or reflection or rotation". She refers to Freudenthal and says that he claimed that Euclidean geometry is overused in school and she considers using tasks with geometrical transformations when teaching the congruence of triangles. However, she is wondering how she can do this, as "there is a need of investigation and inquiry before doing so". She describes her goal saying "I want them [the students] to understand that when we compare angles or segments or generally elements of polygons, we have two tools. One is transformations and the other the criteria of triangle congruence". In these, Marina appears to reconsider the way she is carrying out the teaching activity in respect to the congruence of triangles. Linda listens to Marina and finds her thoughts interesting, but she claims that "every topic has its purpose" adding that "there is a purpose to learn how to write [a justification], to observe the shape, to distinguish the given data from the required claims, to make conclusions, and to prove … [Congruence] has its meaning". Linda implies that these goals can be achieved through teaching congruence without involving transformations.

In the next meeting (A5) Marina describes how her students in grade 9 work with the congruence of triangles in combination with geometrical transformations to prove the congruence of segments or angles. She argues that there are tasks that can show the students when one approach is more appropriate than the other, like the task: "If A and A', B and B', C and C' are three pairs of diametrically opposing points, then the triangles ABC and A'B'C' are congruent" (Fig. 3a). This can be easily tackled by an 180° rotation, while the use of the criteria of triangle congruence is very complex. On the basis of such tasks, epistemological issues concerning the rigor and the intuition inherent in different approaches are also discussed. For example, in the discussion with Linda and the researcher, Marina notes that the previous task can be solved with "pure transformation" without further justification. Nevertheless, in the case of an isosceles triangle one should justify that the median is axis of symmetry before using reflection: "There is a missing justification here" (Fig. 3b). These epistemological discussions are indications of Marina's growing awareness about the contradicting aspects but also about the potential for dialectical integrations. Later in the discussion, Marina points out that her students used transformations as an alternative to triangle congruence after having engaged them in such approaches. She also notices that this happened regularly in the class she taught last year, but not very often in the one is teaching now. Linda follows the discussion, appreciating Marina's approach as a "nice idea" and saying that she likes children working in both ways (triangle congruence and geometrical transformations).

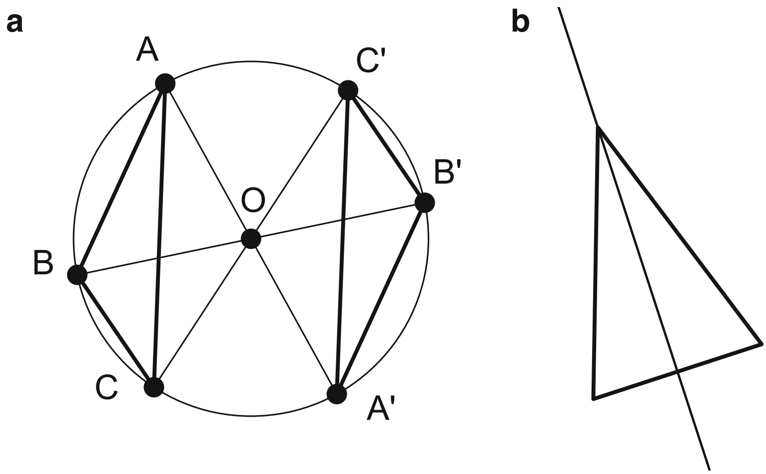


Fig. 3 **a**, **b**. Tasks on geometrical transformations

In the sixth meeting (A6) Marina has completed the topic of congruence and she notices that the students used both geometrical transformations and Euclidean arguments in their justifications. Reflecting on her use of transformations in the classroom, Marina explains:

> Marina: The introduction of transformations in grade 8 gives you the opportunity to change the framework [of proving] in grade 9 … [for the students] to see that you can cope with the proof of geometrical properties with two strategies … using transformations and triangle congruence … And it was done easily … it came from the students. … And I think it is very nice that for the first time there is the possibility to get away from Euclidean geometry…

In the 8th meeting (A8) Marina reflects on her preference for transformations and mentions a seminar on transformations she attended 3 years ago and her experimental teaching in another school. These references indicate previous activities that influence Marina's choices. Marina mentions that some students use transformations in other topics, such as trigonometry (to visualize the equality sinx=sin(180-x) as a reflection on the y-axis), indicating that they use them as an operational tool to visualize and prove congruence. In this discussion (unlike in previous meetings) Linda expresses a hesitation to such intertwining of different topics. She says: "I like transformations per se. I don't like overusing them later in congruence … I don't find the reason to [do so]". She also expresses her difficulty with the new approach by saying "there is a refusal … not refusal … difficulty for a teacher to change … a teacher with a certain approach for 40 years…".

5 Discussion

Examining Marina's approach, as it appears in the discussions, a shift in her teaching of congruence can be traced. At the beginning she realizes the possibility of combining congruence and geometrical transformations and the "need of investigation before doing so" (A4). Later, she selects tasks to highlight the potential of transformations in congruence (A5). Her initial goals are that “[the students] understand that when we compare angles or segments … we have two tools” (A4). Later in the meetings, she formulates more clear epistemological perspectives: “to change the framework [of proving], … you can cope with two strategies, … the possibility to get away from Euclidean geometry” (A6). These shifts seem to have been facilitated through Marina’s work in her classroom and her reflections during discussions with the collegial others including the researcher.

Linda acknowledges that geometrical transformations can be used as proving tools for congruence, since some students have developed skills related to intuition and visual reasoning. However, she prefers not to combine congruence with geometrical transformations pursuing the affordances of Euclidean geometry. Finally, she recognizes that it is difficult to change the approach you have established “through the last 40 years” (A8).

In our interpretation, the discussions of the use of transformations and Euclidean criteria for triangle congruence reveal a contradiction between these two approaches as two different tools in proving geometrical properties. In the background of this contradiction lies the epistemological difference between rigorous, deductive foundation of knowledge in Euclidean geometry and more intuitive, visual, dynamic aspects of geometrical transformations in the sense that are introduced in the new curriculum materials. We see this epistemological difference as an indication of the DO between *intuition* and *logic*. Marina’s awareness of the contradiction between the two tools and their epistemological character allows her to attempt a shift in her teaching approach. Before she taught the two topics with no explicit connection between them; now she seeks to use the two tools in combination, synthesizing them in the students’ mathematical activity. For Linda, in contrast, the relationship between the two approaches remains a contradiction that does not dissolve into a unity of the opposing elements of a DO, and she continues to consider the two approaches incompatible.

Phrased in the terminology of activity theory, Linda and Marina share similar experiences and perspectives with a new set of rules and tools in the form of the new curricular materials. For both of them significant communities include the school they both work at, which adopted the new curriculum, and the same reflection group that discusses approaches to teaching according to the new curriculum materials. Yet, there are significant differences between how they describe their educational priorities and practices as they relate to geometrical transformations. More specifically, Linda appears less comfortable with the ambition of extending the scope of this particular field so as to include questions of congruence. This is so although she willingly, and in some cases to greater extent than Marina, adopts other innovations suggested in the new materials.

The apparent differences can be considered as differences between subjective understandings of the object of the activity and of the respective goals. They may possibly and in part be explained by the different communities they have participated in and the tools offered by these communities. Marina’s more comprehensive experiences with mathematics and her engagement in a learning community specifically committed to discussing geometrical transformations may be important. Besides, aspects of the functioning of other communities frame her instructional approach in significant ways. For instance, the school’s decision that she was to teach the same class as it progressed from grade 8 to grade 9 played a significant role for her experiences with teaching geometrical transformations and allowed her to play a significant role for the social and socio-mathematical norms that emerged. Her experiences with these communities appear to have reciprocally influenced her approach to the rules and tools offered by the new curricular materials and the discussions in the reflection group in ways that made the apparent contradiction between Euclidean and transformational approaches to geometry dissolve, at least in part.

While the explanations above for how Marina developed her new approach are somewhat speculative, we suggest that there are strong indications that the development of her new approach to the teaching of congruence is based on her shifting considerations of Euclidean and transformational approaches to geometry. The two approaches gradually come to function as different perspectives on and provide different tools for addressing geometrical problems. It seems, though, an exaggeration to suggest that they merge to create a "unity or coincidence of opposites, a coincidence […] that goes as far as their identity" (Ilyenkov, 2009, p. 198; cf. the quotation in Section 2.3). In this sense they do not, for Marina, qualify as a full-fledged DO, and neither does the more general duality of intuition-logic. The moral of the story is, however, that the recognition of the contradiction becomes a learning tool for Marina and that it is partly dissolved. In the process it takes on some of the characteristics of a DO, as she seeks to relate what were previously distinct approaches to geometry by reconciling them when working in a particular field, the one of congruence.

Barab et al. (2002, p. 104) argue that "when systemic tensions are brought into a healthy balance they can facilitate a meaningful interplay that enriches and adds dynamism to the learning process". It appears from our analysis that acknowledging the contradiction and deciding to incorporate both opposite aspects dialectically, may broaden the horizon of the activity (Engeström, 2001). Our claim is in accordance with Chapman's and Heater's position (2010) that key issues on teacher change are: the experience of authentic tensions based on actual, personal classroom experiences, the willingness to take ownership of the change, and the acceptance of a degree of uncertainty. Hence, teachers' decisions about contradictions become the object of the research (Stouraitis, 2016).

We do not know if the identified shifts, as they appear in Marina's tales of her teaching, will be sustained, let alone if they can be expanded in the collective activity of mathematics teaching at her school and beyond. Such an investigation requires more time and different research methods than the present study. What we can claim from this study is that the construct of dialectical opposition has potentials for understanding and contributing to teacher learning. For the teachers in a collaborative context, contradictions may and may not dissolve into dialectical oppositions and do so to different degrees. In either case, discussions about character and possible resolution of the contradiction may provide opportunities for teachers to engage differently in mathematics teaching and learning and carry potentials for shifts in their contributions to the practices that evolve in their classrooms. This turns the philosophical claim about the contradictions as driving force for any dynamic system to an empirical question supported by our study.

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