they're-equal proof, even though the proof by construction was based on a 2x2 case with numerical entries. The students, most of whom realized the drawbacks of arguments based on specific numerical cases, regarded the argument with the specific case as a generic argument and preferred it because "you can see how it works."

The Structural Proof Schemes. The general characterization of these schemes is that they are special transformational proof schemes in which conjectures and facts are representations of situations from different realities that share a common structure. The structure is characterized by a collection of accepted facts. There are three subcategories, which will be described only briefly here. It is important to keep in mind that these must be transformational in nature; otherwise there is the danger of resorting to rote memory in settings where they could be used. The postulational proof scheme is a structural proof scheme in which the structure is characterized by a collection of permanently accepted facts. This scheme is essential in studying the theory of vector spaces, for example. The spatial-postulational proof scheme is a postulational proof scheme whose realities are based in intuitions of space. The postulates in Hilbert's *Grundlagen der Geometrie*, for example, could provide the characterization with which to justify statements in geometry. Finally, the axiomatization proof scheme is a structural proof scheme in which the structure is characterized by a collection of tentatively accepted facts. This scheme is essential in studying questions of consistency, independence, completeness, etc.

Implications

The symbolic and ritual proof schemes, grounded as they are in meaningless symbol manipulation or surface features, have nothing to recommend them; perhaps with a greater emphasis on the giving of justifications instruction can help students to avoid them. Students must be educated to value and to want to know justifications; the source of the results, not just the results, must be emphasized. The authoritarian proof scheme, on the other, hand is a two-edged sword. In the culture of schools or of knowledge acquisition, it can be valuable. The concern is to move away from a complete reliance on it and its suffocating effect on the giving of justifications. For example, in the teaching experiments, a conjecture was no longer labelled "theorem," simply because the label "theorem" seemed to reduce the students' effort, willingness, and even the ability for some students to justify the conjecture. The label "theorem" apparently rendered the relationship into something to obey rather than to reason about. The use of small groups, in which there is no obvious authority figure, seems to foster more openness to evaluating justifications; there the student is a more genuine partner in justifying statements than in a teacher-led justification. The empirical proof schemes, with their roots in everyday thinking, are important and valuable. The inductive proof scheme is so strong, however, that instruction must deliberately combat it to show its defects.

To become "sufficiently mature in the mathematical culture" or to progress toward Poincaré's what-one-wants-the student's-mind-to-become, it is clear that a student must move beyond the external and empirical proof schemes. Of greatest