CLASSIFYING PROCESSES OF PROVING

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This paper outlines a preliminary classification of the kinds of justifications that students offer in mathematical contexts, i.e., their "proof schemes." The classification is based primarily on the work of students during teaching experiments and individual interviews, with secondary and post-secondary students. The dominant, natural proof schemes of most students--even university mathematics majors--are not ones accepted in the mathematical community as giving mathematical proofs. Transformational proof schemes are viewed as essential for advancing beyond these schemes; teaching experiments with university students suggest that many students can make pleasing progress toward expecting and giving acceptable mathematical proofs.

Many researchers have given attention to different aspects of the learning and teaching of proof (e.g., Bell, 1976; Chazan, 1993; Fischbein and Kedem, 1982; Hanna, 1990; Martin and Harel, 1989; Senk, 1985; Yerushalmy, 1993). These indicate that the ideas of proof are difficult for students to learn, at least as they are currently taught. A quote from Poincaré summarizes our position toward the teaching and learning of proof in mathematics:

> It is difficult for a teacher to teach something which does not satisfy him entirely, but the satisfaction of the teacher is not the unique goal of teaching; *one has at first to take care of what is the mind of the student* and what one wants it to become. [via Artigue, 1994; emphasis added]

Accordingly, we have been concerned with attempting to determine what *is* in the minds of students, when proof comes up in mathematics. Others have had the same concern. For example, Chazan (1993) noted that U.S. high school geometry students were skeptical that a deductive proof assured that there were no counterexamples to the assertion proved, and that a proof was only further evidence that a conjecture is true. Fischbein and Kedem (1982) found that among students in an Israeli program of studies involving the greatest concentration on mathematics only about one-third of the students who had endorsed a statement and its proof realized that further checks of specific instances would be superfluous.

Our approach has been to focus on justifications, and to view a mathematical proof as the type of justification that is usually accepted by the mathematical community. During interviews, mostly of university students in courses for mathematics majors, we have attempted to determine what sorts of justifications convince them, and what sorts of justifications they would offer in order to convince others. During teaching experiments with university students, the thrust has been to help students refine their own ideas about what constitutes justification in mathematics.