The Australian Curriculum

Learning areas	Mathematics
Subjects	Essential Mathematics,General Mathematics,Mathematical Methods,Specialist Mathematics
Year levels	Unit 1,Unit 2,Unit 3,Unit 4,Bridging Unit 1,Bridging Unit 2,Bridging Unit 3,Bridging Unit 4,Unit 1: Investigating the Ancient World,Unit 2: Ancient Societies,Unit 3: People, Power and Authority,Unit 4: Reconstructing the Ancient World,Unit 1: Natural and ecological hazards,Unit 2: Sustainable places,Unit 3: Land cover transformations,Unit 4: Global transformations,Unit 1: Understanding the Modern World,Unit 2: Movements for Change in the 20th century,Unit 3: Modern Nations in the 20th century,Unit 4: The Modern World since 1945

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The Australian Curriculum

Mathematics

Mathematics - How the Learning Area works

Overview of the senior secondary Australian Curriculum

ACARA has developed a senior secondary Australian Curriculum for English, Mathematics, Science and Humanities and Social Sciences.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

- **Essential Mathematics** focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.
- **General Mathematics** focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies

based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

- **Mathematical Methods** focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.
- **Specialist Mathematics** provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that has been designed to not be taken as a stand-alone subject.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander peoples' histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Each of the senior secondary mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Glossary

Essential Mathematics Glossary

General Mathematics Glossary

Mathematical Methods Glossary

Specialist Mathematics Glossary

Glossary

Abstract

Abstract scenario: a scenario for which there is no concrete referent provided.

Account

Account for: provide reasons for (something).
Give an account of: report or describe an event or experience.
Taking into account: considering other information or aspects.

Analyse

Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences.

Apply

Use, utilise or employ in a particular situation.

Assess

Determine the value, significance or extent of (something).

Coherent

Orderly, logical, and internally consistent relation of parts.

Communicates

Conveys knowledge and/or understandings to others.

Compare

Estimate, measure or note how things are similar or dissimilar.

Complex

Consisting of multiple interconnected parts or factors.

Considered

Formed after careful thought.

Critically analyse

Examine the component parts of an issue or information, for example the premise of an argument and its plausibility, illogical reasoning or faulty conclusions.

Critically evaluate

Evaluation of an issue or information that includes considering important factors and available evidence in making critical judgement that can be justified.

Demonstrate

Give a practical exhibition as an explanation.

Describe

Give an account of characteristics or features.

Design

Plan and evaluate the construction of a product or process.

Develop

In history: to construct, elaborate or expand.
In English: begin to build an opinion or idea.

Discuss

Talk or write about a topic, taking into account different issues and ideas.

Distinguish

Recognise point/s of difference.

Evaluate

Provide a detailed examination and substantiated judgement concerning the merit, significance or value of something.
In mathematics: calculate the value of a function at a particular value of its independent variables.

Explain

Provide additional information that demonstrates understanding of reasoning and/or application.

Familiar

Previously encountered in prior learning activities.

Identify

Establish or indicate who or what someone or something is.

Integrate

Combine elements.

Investigate

Plan, collect and interpret data/information and draw conclusions about.

Justify

Show how an argument or conclusion is right or reasonable.

Locate

Identify where something is found.

Manipulate

Adapt or change.

Non-routine

Non-routine problems: Problems solved using procedures not previously encountered in prior learning activities.

Reasonableness

Reasonableness of conclusions or judgements: the extent to which a conclusion or judgement is sound and makes sense.

Reasoned

Reasoned argument/conclusion: one that is sound, well-grounded, considered and thought out.

Recognise

Be aware of or acknowledge.

Relate

Tell or report about happenings, events or circumstances.

Represent

Use words, images, symbols or signs to convey meaning.

Reproduce

Copy or make close imitation.

Responding

In English: When students listen to, read or view texts they interact with those texts to make meaning. Responding involves students identifying, selecting, describing, comprehending, imagining, interpreting, analysing and evaluating.

Routine problems

Routine problems: Problems solved using procedures encountered in prior learning activities.

Select

Choose in preference to another or others.

Sequence

Arrange in order.

Solve

Work out a correct solution to a problem.

Structured

Arranged in a given organised sequence.
In Mathematics: When students provide a structured solution, the solution follows an organised sequence provided by a third party.

Succinct

Written briefly and clearly expressed.

Sustained

Consistency maintained throughout.

Synthesise

Combine elements (information/ideas/components) into a coherent whole.

Understand

Perceive what is meant, grasp an idea, and to be thoroughly familiar with.

Unfamiliar

Not previously encountered in prior learning activities.

Deduce

Arrive at a conclusion by reasoning.

Substantiate

Establish proof using evidence.

Essential Mathematics - How the Subject works

Rationale/Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language used to describe much of the physical world. Statistics is the study of ways of collecting and extracting information from data and of methods of using that information to describe and make predictions about the behaviour of aspects of the real world, in the face of uncertainty. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Essential Mathematics focuses on enabling students to use mathematics effectively, efficiently and critically to make informed decisions in their daily lives. Essential Mathematics provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts, in a range of workplace, personal, further learning and community settings. This subject offers students the opportunity to prepare for post-school options of employment and further training.

For all content areas of Essential Mathematics, the proficiency strands of understanding, fluency, problem solving and reasoning from the F–10 curriculum are still applicable and should be inherent in students' learning of the subject. Each of these proficiencies is essential, and all are mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. Students will encounter opportunities for problem solving, such as finding the volume of a solid so that the amount of liquid held in a container can be compared with what is written on the label, or finding the interest on a sum of money to enable comparison between different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

The content of the Essential Mathematics subject is designed to be taught within contexts that are relevant to the needs of the particular student cohort. The skills and understandings developed throughout the subject will be further enhanced and reinforced through presentation in an area of interest to the students.

Aims

Essential Mathematics aims to develop students':

- understanding of concepts and techniques drawn from mathematics and statistics
- ability to solve applied problems using concepts and techniques drawn from mathematics and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately.

Links to Foundation to Year 10

For all content areas of Essential Mathematics, the proficiency strands of Understanding, Fluency, Problem solving and Reasoning from the F–10 curriculum are still very much applicable and should be inherent in students' learning of the subject. Each strand is essential, and all are mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. They will encounter opportunities for problem solving, such as finding the volume of a solid to enable the amount of liquid that is held in the container to be compared with what is written on the label, or finding the interest on an amount in order to be able to compare different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

Representation of General capabilities

The seven general capabilities of *Literacy, Numeracy, Information and Communication Technology (ICT) capability, Critical and creative thinking, Personal and social capability, Ethical understanding*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions about gambling.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding in Mathematics

In the senior years students develop ethical behaviour in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Structure of Essential Mathematics

Essential Mathematics has four units each of which contains a number of topics. It is intended that the topics be taught in a context relevant to students’ needs and interests. In Essential Mathematics, students use their knowledge and skills to investigate realistic problems of interest which involve the application of mathematical relationships and concepts.

Unit 1	Unit 2	Unit 3	Unit 4
Calculations, percentages and rates Measurement Algebra Graphs	Representing and comparing data Percentages Rates and ratios Time and motion	Measurement Scales, plans and models Graphs Data collection	Probability and relative frequencies Earth geometry and time zones Loans and compound interest

Units

Unit 1 provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to their students. A variety of approaches could be used to achieve this. Two contexts which could be used in this unit are *Mathematics and foods* and *Earning and managing money*. However, these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 2 provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, and time and motion. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and cars* and *Mathematics and independent living*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 3 provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs, and data collection. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and design* and *Mathematics and medicine*. However these contexts may not be relevant for all students and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 4 provides students with the mathematical skills and understanding to solve problems related to probability, earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit are Mathematics of Finance and Mathematics of travelling. However these contexts may not be relevant for all students and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior

secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

1. Given a specified context (as described in the curriculum content)
2. With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
3. Students perform a specified action (described through a verb)
4. In relation to what is valued in the curriculum (specified as the object or subject)
5. With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Glossary

Essential Mathematics Glossary

Unit 1

Unit 1 Description

This unit provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of the four topics in this unit – 'Calculations, percentages and rates', 'Measurement', 'Algebra' and 'Graphs' – in contexts which are meaningful and of interest to their students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used are Mathematics and foods and Earning and managing money. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Unit 1 Learning Outcomes

Learning Outcomes

By the end of this unit students:

- understand the concepts and techniques in calculations, measurement, algebra and graphs
- apply reasoning skills and solve practical problems in calculations, measurement, algebra and graphs
- communicate their arguments and strategies when solving problems using appropriate mathematical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit 1 Content Descriptions

Topic 1: Calculations, percentages and rates

Examples in context

Calculations – for example:

- creating a budget for living at home and for living independently
- using timesheets, which include overtime, to calculate weekly wages
- converting between weekly, fortnightly and yearly incomes.

Percentages – for example:

- expressing ingredients of packaged food as percentages of the total quantity, or per serving size, or per 100 grams
- comparing the quantities, both numerically and in percentage terms, of additives within a product or between similar products, such as flavours
- calculating commissions, including retainers from sales information.

Rates – for example:

- using rates to compare and evaluate nutritional information, such as quantity per serve and quantity per 100g
- calculating heart rates as beats per minute, given the number of beats and different time periods
- applying rates to calculate the energy used in various activities over different time periods
- completing calculations with rates, including solving problems involving direct proportion in terms of rate; for example, if a person works for 3 weeks at a rate of $300 per week, how much do they earn?
- analysing and interpreting tables and graphs that compare body ratios such as hip height versus stride length, foot length versus height.

Calculations:

solve practical problems requiring basic number operations (ACMEM001)

apply arithmetic operations according to their correct order (ACMEM002)

ascertain the reasonableness of answers to arithmetic calculations (ACMEM003)

use leading-digit approximation to obtain estimates of calculations (ACMEM004)

use a calculator for multi-step calculations (ACMEM005)

check results of calculations for accuracy (ACMEM006)

recognise the significance of place value after the decimal point (ACMEM007)

evaluate decimal fractions to the required number of decimal places (ACMEM008)

round up or round down numbers to the required number of decimal places (ACMEM009)

apply approximation strategies for calculations. (ACMEM010)

Percentages:

calculate a percentage of a given amount (ACMEM011)

determine one amount expressed as a percentage of another (ACMEM012)

apply percentage increases and decreases in situations; for example, mark-ups, discounts and GST. (ACMEM013)

Rates:

identify common usage of rates; for example, km/h as a rate to describe speed, beats/minute as a rate to describe pulse (ACMEM014)

convert units of rates occurring in practical situations to solve problems (ACMEM015)

use rates to make comparisons; for example, using unit prices to compare best buys, comparing heart rates after exercise. (ACMEM016)

Topic 2: Measurement

Examples in context

Length – for example:

- determining the dimensions/measurements of food packaging
- determining the length of the lines on a sporting field to find the cost of marking it.

Mass – for example:

- comparing and discussing the components of different food types for the components of packaged food expressed as grams.

Area and volume – for example:

- determining the area of the walls of a room for the purpose of painting
- finding the volume of water collected from a roof under different conditions
- finding the volume of various cereal boxes.

Linear measure:

use metric units of length, their abbreviations, conversions between them, and appropriate levels of accuracy and choice of units (ACMEM017)

estimate lengths (ACMEM018)

convert between metric units of length and other length units (ACMEM019)

calculate perimeters of familiar shapes, including triangles, squares, rectangles, and composites of these. (ACMEM020)

Area measure:

use metric units of area, their abbreviations, conversions between them, and appropriate choices of units (ACMEM021)

estimate the areas of different shapes (ACMEM022)

convert between metric units of area and other area units (ACMEM023)

calculate areas of rectangles and triangles. (ACMEM024)

Mass:

use metric units of mass, their abbreviations, conversions between them, and appropriate choices of units (ACMEM025)

estimate the mass of different objects. (ACMEM026)

Volume and capacity:

use metric units of volume, their abbreviations, conversions between them, and appropriate choices of units (ACMEM027)

understand the relationship between volume and capacity (ACMEM028)

estimate volume and capacity of various objects (ACMEM029)

calculate the volume of objects, such as cubes and rectangular and triangular prisms. (ACMEM030)

Units of energy:

use units of energy to describe consumption of electricity, such as kilowatt hours (ACMEM031)

use units of energy used for foods, including calories (ACMEM032)

use units of energy to describe the amount of energy in activity, such as kilojoules (ACMEM033)

convert from one unit of energy to another. (ACMEM034)

Topic 3: Algebra

Examples in context

Formula substitution – for example:

- using formulas to calculate the volumes of various packaging
- using formulas to find the height of a male (H) given the bone radius (r)
- find weekly wage (W) given base wage (b) and overtime hours(h) at 1.5 times rate (r) W = b + 1.5 × h × r.

Single substitution:

substitute numerical values into algebraic expressions; for example, substitute different values of x to evaluate the expressions $\frac{3x}{5}$, $5(2x-4)$ (ACMEM035)

General substitution:

substitute given values for the other pronumerals in a mathematical formula to find the value of the subject of the formula. (ACMEM036)

Topic 4: Graphs

Examples in context

Reading and interpreting graphs – for example:

- analysing and interpreting a range of graphical information about global weather patterns that affect food growth
- interpreting a range of graphical information provided on gas and electricity bills.

Drawing graphs – for example:

- expressing ingredients of particular food types as percentages of the total quantity, or per serving size, or per 100 grams, and presenting the information in different formats; for example, column graphs, and pie graphs
- creating graphs to show the deductions from gross wages such as tax, the Medicare levy and superannuation.

Reading and interpreting graphs:

interpret information presented in graphs, such as conversion graphs, line graphs, step graphs, column graphs and picture graphs (ACMEM037)

interpret information presented in two-way tables (ACMEM038)

discuss and interpret graphs found in the media and in factual texts. (ACMEM039)

Drawing graphs:

determine which type of graph is best used to display a dataset (ACMEM040)

use spreadsheets to tabulate and graph data (ACMEM041)

draw a line graph to represent any data that demonstrate a continuous change, such as hourly temperature. (ACMEM042)

Unit 2

Unit 2 Description

This unit provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, the mathematics of finance, and time and motion. Teachers are encouraged to apply the content of the four topics in this unit – 'Representing and comparing data', 'Percentages', 'Rates and ratios' and 'Time and motion' – in a context which is meaningful and of interest to their students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used are Mathematics and cars and Mathematics and independent living. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Unit 2 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in representing and comparing data, percentages, rates and ratios, and time and motion
- apply reasoning skills and solve practical problems in representing and comparing data, percentages, rates and ratios, and time and motion
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Unit 2 Content Descriptions

Topic 1: Representing and comparing data

Examples in context

- analysing and interpreting a range of statistical information related to car theft, car accidents and driver behaviour
- using statistics and graphs to find the number of people in each blood type, given the population percentages of blood types in different countries
- using blood usage statistics to predict the amount of blood needed at different times of the year
- using blood donation statistics to predict how much blood will be needed and when.

Classifying data:

identify examples of categorical data (ACMEM043)

identify examples of numerical data. (ACMEM044)

Data presentation and interpretation:

display categorical data in tables and column graphs (ACMEM045)

display numerical data as frequency distributions, dot plots, stem and leaf plots, and histograms (ACMEM046)

recognise and identify outliers (ACMEM047)

compare the suitability of different methods of data presentation in real-world contexts. (ACMEM048)

Summarising and interpreting data:

identify the mode (ACMEM049)

calculate measures of central tendency, the arithmetic mean and the median (ACMEM050)

investigate the suitability of measures of central tendency in various real-world contexts (ACMEM051)

investigate the effect of outliers on the mean and the median (ACMEM052)

calculate and interpret quartiles, deciles and percentiles (ACMEM053)

use informal ways of describing spread, such as spread out/dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers (ACMEM054)

calculate and interpret statistical measures of spread, such as the range, interquartile range and standard deviation (ACMEM055)

investigate real-world examples from the media illustrating inappropriate uses, or misuses, of measures of central tendency and spread. (ACMEM056)

Comparing data sets:

compare back-to-back stem plots for different data-sets (ACMEM057)

complete a five number summary for different datasets (ACMEM058)

construct box plots using a five number summary (ACMEM059)

compare the characteristics of the shape of histograms using symmetry, skewness and bimodality. (ACMEM060)

Topic 2: Percentages

Examples in context

- calculating stamp duty costs involved in buying a car, using percentages and tables
- calculating depreciation of a vehicle over time
- using statistics and graphs to find the number of people in each blood type, given the population percentages of blood types in different countries.

Percentage calculations:

review calculating a percentage of a given amount (ACMEM061)

review one amount expressed as a percentage of another. (ACMEM062)

Applications of percentages:

determine the overall change in a quantity following repeated percentage changes; for example, an increase of 10% followed by a decrease of 10% (ACMEM063)

calculate simple interest for different rates and periods. (ACMEM064)

Topic 3: Rates and ratios

Examples in context

Rates – for example:

- using rates to find fuel consumption for different vehicles under different driving conditions
- calculating food, clothing, transport costs per day, week or month using tables, spreadsheets, and estimation
- calculating clothing costs per week or month using tables, spreadsheets, and estimation.

Ratios – for example:

- discussing various ratios used in bicycle gears
- comparing ratios such as people per household.

Ratios:

demonstrate an understanding of the elementary ideas and notation of ratio (ACMEM065)

understand the relationship between fractions and ratio (ACMEM066)

express a ratio in simplest form (ACMEM067)

find the ratio of two quantities (ACMEM068)

divide a quantity in a given ratio (ACMEM069)

use ratio to describe simple scales. (ACMEM070)

Rates:

review identifying common usage of rates such as km/h (ACMEM071)

convert between units for rates; for example, km/h to m/s, mL/min to L/h (ACMEM072)

complete calculations with rates, including solving problems involving direct proportion in terms of rate (ACMEM073)

use rates to make comparisons (ACMEM074)

use rates to determine costs; for example, calculating the cost of a tradesman using rates per hour, call-out fees. (ACMEM075)

Topic 4: Time and motion

Examples in context

Time – for example:

- calculating reaction times through experiments.

Distance – for example:

- calculating distances travelled to school and the time taken, considering different average speeds.

Speed – for example:

- calculating stopping distances for different speeds by using formulas for different conditions such as road type, tyre conditions and vehicle type.

Time:

use units of time, conversions between units, fractional, digital and decimal representations (ACMEM076)

represent time using 12-hour and 24-hour clocks (ACMEM077)

calculate time intervals, such as time between, time ahead, time behind (ACMEM078)

interpret timetables, such as bus, train and ferry timetables (ACMEM079)

use several timetables and electronic technologies to plan the most time-efficient routes (ACMEM080)

interpret complex timetables, such as tide charts, sunrise charts and moon phases (ACMEM081)

compare the time taken to travel a specific distance with various modes of transport (ACMEM082)

Distance:

use scales to find distances, such as on maps; for example, road maps, street maps, bushwalking maps, online maps and cadastral maps (ACMEM083)

optimise distances through trial-and-error and systematic methods; for example, shortest path, routes to visit all towns, and routes to use all roads. (ACMEM084)

Speed:

identify the appropriate units for different activities, such as walking, running, swimming and flying (ACMEM085)

calculate speed, distance or time using the formula speed = distance/time (ACMEM086)

calculate the time or costs for a journey from distances estimated from maps (ACMEM087)

interpret distance-versus-time graphs (ACMEM088)

calculate and interpret average speed; for example, a 4-hour trip covering 250 km. (ACMEM089)

Unit 3

Unit 3 Description

This unit provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs, and data collection. Teachers are encouraged to apply the content of the four topics in this unit – 'Measurement', 'Scales, plans and models', 'Graphs' and 'Data collection' – in a context which is meaningful and of interest to the students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used in this unit are Mathematics and design and Mathematics and medicine. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Unit 3 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in measurement, scales, plans and models, graphs, and data collection
- apply reasoning skills and solve practical problems in measurement, scales, plans and models, graphs, and data collection
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Unit 3 Content Descriptions

Topic 1: Measurement

Examples in context

- calculating and interpreting dosages for children and adults from dosage panels on medicines, given age or weight
- calculating and interpreting dosages for children from adults' medication using various formulas (Fried, Young, Clark) in milligrams or millilitres
- calculating surface areas of various buildings to compare costs of external painting.

Linear measure:

review metric units of length, their abbreviations, conversions between them, estimation of lengths, and appropriate choices of units

(ACMEM090)

calculate perimeters of familiar shapes, including triangles, squares, rectangles, polygons, circles, arc lengths, and composites of these.

(ACMEM091)

find the area of irregular figures by decomposition into regular shapes (ACMEM094)

Area measure:

review metric units of area, their abbreviations, and conversions between them (ACMEM092)

use formulas to calculate areas of regular shapes, including triangles, squares, rectangles, parallelograms, trapeziums, circles and sectors (ACMEM093)

find the surface area of familiar solids, including cubes, rectangular and triangular prisms, spheres and cylinders (ACMEM095)

find the surface area of pyramids, such as rectangular- and triangular-based pyramids (ACMEM096)

use addition of the area of the faces of solids to find the surface area of irregular solids. (ACMEM097)

Mass:

review metric units of mass (and weight), their abbreviations, conversions between them, and appropriate choices of units (ACMEM098)

recognise the need for milligrams (ACMEM099)

convert between grams and milligrams. (ACMEM100)

Volume and capacity:

review metric units of volume, their abbreviations, conversions between them, and appropriate choices of units (ACMEM101)

recognise relations between volume and capacity, recognising that $1\text{cm}^3 = 1\text{mL}$ and $1\text{m}^3 = 1\text{kL}$ (ACMEM102)

use formulas to find the volume and capacity of regular objects such as cubes, rectangular and triangular prisms and cylinders (ACMEM103)

use formulas to find the volume of pyramids and spheres. (ACMEM104)

Topic 2: Scales, plans and models

Examples in context

- drawing scale diagrams of everyday two-dimensional shapes
- interpreting common symbols and abbreviations used on house plans
- using the scale on a plan to calculate actual external or internal dimensions, the lengths of the house and the dimensions of? particular rooms
- using technology to translate two-dimensional house plans into three-dimensional buildings
- creating landscape designs using technology.

Geometry:

recognise the properties of common two-dimensional geometric shapes and three-dimensional solids (ACMEM105)

interpret different forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams (ACMEM106)

use symbols and conventions for the representation of geometric information; for example, point, line, ray, angle, diagonal, edge, curve, face and vertex. (ACMEM107)

Interpret scale drawings:

interpret commonly used symbols and abbreviations in scale drawings (ACMEM108)

find actual measurements from scale drawings, such as lengths, perimeters and areas (ACMEM109)

estimate and compare quantities, materials and costs using actual measurements from scale drawings; for example, using measurements for packaging, clothes, painting, bricklaying and landscaping. (ACMEM110)

Creating scale drawings:

understand and apply drawing conventions of scale drawings, such as scales in ratio, clear indications of dimensions, and clear labelling (ACMEM111)

construct scale drawings by hand and by using software packages. (ACMEM112)

Three dimensional objects:

interpret plans and elevation views of models (ACMEM113)

sketch elevation views of different models (ACMEM114)

interpret diagrams of three-dimensional objects. (ACMEM115)

Right-angled triangles:

apply Pythagoras' theorem to solve problems (ACMEM116)

apply the tangent ratio to find unknown angles and sides in right-angled triangles (ACMEM117)

work with the concepts of angle of elevation and angle of depression (ACMEM118)

apply the cosine and sine ratios to find unknown angles and sides in right-angled triangles (ACMEM119)

solve problems involving bearings. (ACMEM120)

Topic 3: Graphs

Examples in context

- interpreting graphs showing growth ranges for children (height or weight or head circumference versus age)
- interpreting hourly hospital charts showing temperature and pulse

- interpreting graphs showing life expectancy with different variables.

Cartesian plane:

demonstrate familiarity with Cartesian coordinates in two dimensions by plotting points on the Cartesian plane (ACMEM121)

generate tables of values for linear functions, including for negative values of x (ACMEM122)

graph linear functions for all values of x with pencil and paper and with graphing software. (ACMEM123)

Using graphs:

interpret and use graphs in practical situations, including travel graphs and conversion graphs (ACMEM124)

draw graphs from given data to represent practical situations (ACMEM125)

interpret the point of intersection and other important features of given graphs of two linear functions drawn from practical contexts; for example, the 'break-even' point. (ACMEM126)

Topic 4: Data collection

Examples in context

- analysing data obtained from medical sources, including bivariate data.

Census:

investigate the procedure for conducting a census (ACMEM127)

investigate the advantages and disadvantages of conducting a census. (ACMEM128)

Surveys:

understand the purpose of sampling to provide an estimate of population values when a census is not used (ACMEM129)

investigate the different kinds of samples; for example, systematic samples, self-selected samples, simple random samples (ACMEM130)

investigate the advantages and disadvantages of these kinds of samples; for example, comparing simple random samples with self-selected samples. (ACMEM131)

Simple survey procedure:

identify the target population to be surveyed (ACMEM132)

investigate questionnaire design principles; for example, simple language, unambiguous questions, consideration of number of choices, issues of privacy and ethics, and freedom from bias. (ACMEM133)

Sources of bias:

describe the faults in the collection of data process (ACMEM134)

describe sources of error in surveys; for example, sampling error and measurement error (ACMEM135)

investigate the possible misrepresentation of the results of a survey due to misunderstanding the procedure, or misunderstanding the reliability of generalising the survey findings to the entire population (ACMEM136)

investigate errors and misrepresentation in surveys, including examples of media misrepresentations of surveys. (ACMEM137)

Bivariate scatterplots:

describe the patterns and features of bivariate data (ACMEM138)

describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak). (ACMEM139)

Line of best fit:

identify the dependent and independent variable (ACMEM140)

find the line of best fit by eye (ACMEM141)

use technology to find the line of best fit (ACMEM142)

interpret relationships in terms of the variables (ACMEM143)

use technology to find the correlation coefficient (an indicator of the strength of linear association) (ACMEM144)

use the line of best fit to make predictions, both by interpolation and extrapolation (ACMEM145)

recognise the dangers of extrapolation (ACMEM146)

distinguish between causality and correlation through examples. (ACMEM147)

Unit 4

Unit 4 Description

This unit provides students with the mathematical skills and understanding to solve problems related to probability, Earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of the three topics in this unit – 'Probability and relative frequencies', 'Earth geometry and time zones' and 'Loans and compound interest' – in a context which is meaningful and of interest to the students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used in this unit are Mathematics of finance and Mathematics of travelling. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Unit 4 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- apply reasoning skills and solve practical problems in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- communicate their arguments and strategies when solving mathematical problems using appropriate mathematical or statistical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit 4 Content Descriptions

Topic 1: Probability and relative frequencies

Examples in context

- using data to calculate the relative frequencies of the different countries of origin of visitors to a particular tourist venue or country
- using data to calculate the relative frequencies of the amounts of household expenditure is this sentence incomplete?

Probability expressions:

interpret commonly used probability statements, including 'possible', 'probable', 'likely', 'certain' (ACMEM148)

describe ways of expressing probabilities formally using fractions, decimals, ratios, and percentages. (ACMEM149)

Simulations:

perform simulations of experiments using technology (ACMEM150)

recognise that the repetition of chance events is likely to produce different results (ACMEM151)

identify relative frequency as probability (ACMEM152)

identify factors that could complicate the simulation of real-world events. (ACMEM153)

Simple probabilities:

construct a sample space for an experiment (ACMEM154)

use a sample space to determine the probability of outcomes for an experiment (ACMEM155)

use arrays or tree diagrams to determine the outcomes and the probabilities for experiments. (ACMEM156)

Probability applications:

determine the probabilities associated with simple games (ACMEM157)

determine the probabilities of occurrence of simple traffic-light problems. (ACMEM158)

Topic 2: Earth geometry and time zones

Location:

locate positions on Earth's surface given latitude and longitude using GPS, a globe, an atlas, and digital technologies (ACMEM159)

find distances between two places on Earth on the same longitude (ACMEM160)

find distances between two places on Earth using appropriate technology. (ACMEM161)

Time:

understand the link between longitude and time (ACMEM162)

solve problems involving time zones in Australia and in neighbouring nations, making any necessary allowances for daylight saving (ACMEM163)

solve problems involving Greenwich Mean Time and the International Date Line (ACMEM164)

find time differences between two places on Earth (ACMEM165)

solve problems associated with time zones; for example, internet and phone usage (ACMEM166)

solve problems relating to travelling east and west, incorporating time zone changes. (ACMEM167)

Topic 3: Loans and compound interest

Examples in context

- using formula, graphs and spreadsheets to calculate the outcomes of investment accounts with compound interest
- using percentages, rates and spreadsheets to investigate personal loan calculations
- calculating and analysing the costs, hidden traps, advantages and disadvantages of payment plans with interest free periods, using rates and percentages.

Compound interest:

review the principles of simple interest (ACMEM168)

understand the concept of compound interest as a recurrence relation (ACMEM169)

consider similar problems involving compounding; for example, population growth (ACMEM170)

use technology to calculate the future value of a compound interest loan or investment and the total interest paid or earned (ACMEM171)

use technology to compare, numerically and graphically, the growth of simple interest and compound interest loans and investments (ACMEM172)

use technology to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan or investment. (ACMEM173)

Reducing balance loans (compound interest loans with periodic repayments):

use technology and a recurrence relation to model a reducing balance loan (ACMEM174)

investigate the effect of the interest rate and repayment amount on the time taken to repay a loan. (ACMEM175)

Units 1 and 2 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in measurement, financial mathematics and statistics to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to display and organise mathematical and statistical information to solve routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine and non-routine problems • selects and applies techniques in measurement, financial mathematics and statistics to solve routine and non-routine problems • uses digital technologies appropriately to display and organise mathematical and statistical information to solve routine and non-routine problems	• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine problems • selects and applies techniques in measurement, financial mathematics and statistics to solve routine problems • uses digital technologies to display and organise mathematical and statistical information to solve routine problems	• demonstrates familiarity with concepts of measurement, financial mathematics and statistics • uses simple techniques in measurement, financial mathematics and statistics • uses digital technologies to display and organise simple mathematical and statistical information	

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates clear and reasoned observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of results and solutions to routine and non-routine problems in a variety of contexts	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine problems • describes the reasonableness of results and solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical and symbolic form • describes observations using mathematical and statistical language • describes solutions to routine problems • describes the appropriateness of the results of calculations	

Units 3 and 4 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in measurement, scales, graphs and statistics to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to display and organise mathematical and statistical information to solve routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine and non-routine problems • selects and applies techniques in measurement, scales, graphs and statistics to solve routine and non-routine problems • uses digital technologies appropriately to display and organise mathematical and statistical information to solve routine and non-routine problems	• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine problems • selects and applies techniques in measurement, scales, graphs and statistics to solve routine problems • uses digital technologies to display and organise mathematical and statistical information to solve routine problems	• demonstrates familiarity with concepts of measurement, scales, graphs and statistics • uses simple techniques in measurement, scales, graphs and statistics • uses digital technologies to display and organise simple mathematical and statistical information	

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates clear and reasoned observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of results and solutions to routine and non-routine problems in a variety of contexts	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical and symbolic form • describes observations using mathematical and statistical language • describes the solutions to routine problems • describes the appropriateness of the results of calculations	

General Mathematics - How the Subject works

Rationale/Aims

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe many aspects of the world in the twenty-first century. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve practical problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

General Mathematics is designed for those students who want to extend their mathematical skills beyond Year 10 level but whose future studies or employment pathways do not require knowledge of calculus. The subject is designed for students who have a wide range of educational and employment aspirations, including continuing their studies at university or TAFE.

The proficiency strands of the F-10 curriculum – Understanding, Fluency, Problem solving and Reasoning – are still relevant and are inherent in all aspects of this subject. Each of these proficiencies is essential, and all are mutually reinforcing. Fluency, for example, might include learning to perform routine calculations efficiently and accurately, or being able to recognise quickly from a problem description the appropriate mathematical process or model to apply. Understanding, furthermore, that a single mathematical process can be used in seemingly different situations, helps students to see the connections between different areas of study and encourages the transfer of learning. This is an important part of learning the art of mathematical problem solving. In performing such analyses, reasoning is required at each decision-making step and in drawing appropriate conclusions. Presenting the analysis in a logical and clear manner to explain the reasoning used is also an integral part of the learning process.

Throughout the subject there is also an emphasis on the use and application of digital technologies.

Aims

General Mathematics aims to develop students':

- understanding of concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- ability to solve applied problems using concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate the results of a mathematical or statistical problem-solving activity in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Links to Foundation to Year 10

The General Mathematics subject provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F-10 curriculum.

Representation of General capabilities

The seven general capabilities of *Literacy, Numeracy, Information and Communication Technology (ICT) capability, Critical and creative thinking, Personal and social capability, Ethical understanding*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in mathematics

In the senior years, literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in mathematics

The students who undertake this subject will develop their numeracy skills at a more sophisticated level than in Foundation to Year 10. This subject contains financial applications of mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age.

ICT in mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, using data, addressing problems, and operating systems in authentic situations.

Critical and creative thinking in mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions do not match, it is due to a flaw in the theory or in the method of applying the theory to make predictions, or both. They revise, or reapply, their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and in making accurate predictions.

Personal and social capability in mathematics

In the senior years students develop personal and social competence in mathematics by setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making.

The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for decision making, life-long learning, citizenship and self-management. As part of their mathematical explorations and investigations, students work collaboratively in teams, as well as independently.

Ethical understanding in mathematics

In the senior years students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation, the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and the examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in mathematics

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origins in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number, and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Structure of General Mathematics

General Mathematics is organised into four units. The topics in each unit broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic, geometric and statistical thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction.

Unit 1	Unit 2	Unit 3	Unit 4
Consumer arithmetic Algebra and matrices Shape and measurement	Univariate data analysis and the statistical investigation process Applications of trigonometry Linear equations and their graphs	Bivariate data analysis Growth and decay in sequences Graphs and networks	Time series analysis Loans, investments and annuities Networks and decision mathematics

Units

Unit 1 has three topics: 'Consumer arithmetic', 'Algebra and matrices', and 'Shape and measurement'. 'Consumer arithmetic' reviews the concepts of rate and percentage change in the context of earning and managing money, and provides fertile ground for the use of spreadsheets. 'Algebra and matrices' continues the F-10 study of algebra and introduces the new topic of matrices. 'Shape and measurement' extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Unit 2 has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs', and 'Applications of trigonometry'. 'Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation. . 'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression, and bearings in

navigation ‘Linear equations and their graphs’ uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations

Unit 3 has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’, and ‘Graphs and networks’. ‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. ‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. ‘Graphs and networks’ introduces students to the language of graphs and the way in which graphs, represented as a collection of points and interconnecting lines, can be used to analyse everyday situations such as a rail or social network.

Unit 4 has three topics: ‘Time series analysis’, ‘Loans, investments and annuities’, and ‘Networks and decision mathematics’. ‘Time series analysis’ continues students’ study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process. ‘Loans and investments’ aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. ‘Networks and decision mathematics’ uses networks to model and aid decision making in practical situations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained a priori without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

1. Given a specified context (as described in the curriculum content)
2. With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
3. Students perform a specified action (described through a verb)
4. In relation to what is valued in the curriculum (specified as the object or subject)
5. With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills in these subjects.

Glossary

General Mathematics Glossary

Unit 1

Unit 1 Description

This unit has three topics: ‘Consumer arithmetic’, ‘Algebra and matrices’, and ‘Shape and measurement’.

‘Consumer arithmetic’ reviews the concepts of rate and percentage change in the context of earning and managing money, and provides a fertile ground for the use of spreadsheets.

‘Algebra and matrices’ continues the F-10 study of algebra and introduces the new topic of matrices.

‘Shape and measurement’ builds on and extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Classroom access to the technology necessary to support the computational aspects of the topics in this unit is assumed.

Unit 1 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement
- apply reasoning skills and solve practical problems arising in consumer arithmetic, algebra and matrices, and shape and measurement
- communicate their arguments and strategies, when solving problems, using appropriate mathematical language
- interpret mathematical information, and ascertain the reasonableness of their solutions to problems
- choose and use technology appropriately and efficiently.

Unit 1 Content Descriptions

Topic 1: Consumer arithmetic

Applications of rates and percentages:

review rates and percentages (ACMGM001)

calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework (ACMGM002)

calculate payments based on government allowances and pensions (ACMGM003)

prepare a personal budget for a given income taking into account fixed and discretionary spending (ACMGM004)

compare prices and values using the unit cost method (ACMGM005)

apply percentage increase or decrease in various contexts; for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest (ACMGM006)

use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars (ACMGM007)

calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio. (ACMGM008)

Use of spreadsheets:

use a spreadsheet to display examples of the above computations when multiple or repeated computations are required; for example, preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year. (ACMGM009)

Topic 2: Algebra and matrices

Linear and non-linear expressions:

substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate (ACMGM010)

find the value of the subject of the formula, given the values of the other pronumerals in the formula (ACMGM011)

use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights. (ACMGM012)

Matrices and matrix arithmetic:

use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks (ACMGM013)

recognise different types of matrices (row, column, square, zero, identity) and determine their size (ACMGM014)

perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate (ACMGM015)

use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person. (ACMGM016)

Topic 3: Shape and measurement

Pythagoras Theorem:

review Pythagoras' Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions. (ACMGM017)

Mensuration:

solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites (ACMGM018)

calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the volume of water contained in a swimming pool (ACMGM019)

calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the surface area of a cylindrical food container. (ACMGM020)

Similar figures and scale factors:

review the conditions for similarity of two-dimensional figures including similar triangles (ACMGM021)

use the scale factor for two similar figures to solve linear scaling problems (ACMGM022)

obtain measurements from scale drawings, such as maps or building plans, to solve problems (ACMGM023)

obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures (ACMGM024)

obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids. (ACMGM025)

Unit 2

Unit 2 Description

This unit has three topics: ‘Univariate data analysis and the statistical investigation process’, ‘Linear equations and their graphs’; and ‘Applications of trigonometry’.

‘ Univariate data analysis and the statistical investigation process’ develops students’ ability to organise and summarise univariate data in the context of conducting a statistical investigation.

‘Linear equations and their graphs’ uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations.

‘Applications of trigonometry’ extends students’ knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Unit 2 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in univariate data analysis and the statistical investigation process, linear equations and their graphs, and applications of trigonometry
- apply reasoning skills and solve practical problems in univariate data analysis and the statistical investigation process, linear equations and their graphs, and the applications of trigonometry
- implement the statistical investigation process in contexts requiring the analysis of univariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Unit 2 Content Descriptions

Topic 1: Univariate data analysis and the statistical investigation process

The statistical investigation process:

review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM026)

Making sense of data relating to a single statistical variable:

classify a categorical variable as ordinal, such as income level (high, medium, low), or nominal, such as

place of birth (Australia, overseas), and use tables and bar charts to organise and display the data (ACMGM027)

classify a numerical variable as discrete, such as the number of rooms in a house, or continuous, such as the temperature in degrees Celsius (ACMGM028)

with the aid of an appropriate graphical display (chosen from dot plot, stem plot, bar chart or histogram), describe the distribution of a numerical dataset in terms of modality (uni or multimodal), shape (symmetric versus positively or negatively skewed), location and spread and outliers, and interpret this information in the context of the data (ACMGM029)

determine the mean and standard deviation of a dataset and use these statistics as measures of location and spread of a data distribution, being aware of their limitations. (ACMGM030)

Comparing data for a numerical variable across two or more groups:

construct and use parallel box plots (including the use of the 'Q1 – 1.5 x IQR' and 'Q3 + 1.5 x IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and to interpret and communicate the differences observed in the context of the data (ACMGM031)

compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate; interpret the differences observed in the context of the data; and report the findings in a systematic and concise manner (ACMGM032)

implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups; for example, are Year 11 students the fittest in the school? (ACMGM033)

Topic 2: Applications of trigonometry

Applications of trigonometry:

review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle (ACMGM034)

determine the area of a triangle given two sides and an included angle by using the rule $Area = \frac{1}{2}ab\sin C$, or given three sides by using Heron's rule, and solve related practical problems (ACMGM035)

solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule (ACMGM036)

solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation. (ACMGM037)

Topic 3: Linear equations and their graphs

Linear equations:

identify and solve linear equations (ACMGM038)

develop a linear formula from a word description (ACMGM039)

Straight-line graphs and their applications:

construct straight-line graphs both with and without the aid of technology (ACMGM040)

determine the slope and intercepts of a straight-line graph from both its equation and its plot (ACMGM041)

interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation (ACMGM042)

construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required. (ACMGM043)

Simultaneous linear equations and their applications:

solve a pair of simultaneous linear equations, using technology when appropriate (ACMGM044)

solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations. (ACMGM045)

Piece-wise linear graphs and step graphs:

sketch piece-wise linear graphs and step graphs, using technology when appropriate (ACMGM046)

interpret piece-wise linear and step graphs used to model practical situations; for example, the tax paid as income increases, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time, the charging scheme for sending parcels of different weights through the post. (ACMGM047)

Unit 3

Unit 3 Description

This unit has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’ and ‘Graphs and networks’.

‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

‘Graphs and networks’ introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network.

Classroom access to technology to support the graphical and computational aspects of these topics is assumed.

Unit 3 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in bivariate data analysis, growth and decay in sequences, and graphs and networks
- apply reasoning skills and solve practical problems in bivariate data analysis, growth and decay in sequences, and graphs and networks
- implement the statistical investigation process in contexts requiring the analysis of bivariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Unit 3 Content Descriptions

Topic 1: Bivariate data analysis

The statistical investigation process:

review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results.

(ACMGM048)

Identifying and describing associations between two categorical variables:

construct two-way frequency tables and determine the associated row and column sums and percentages (ACMGM049)

use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association (ACMGM050)

describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data. (ACMGM051)

Identifying and describing associations between two numerical variables:

construct a scatterplot to identify patterns in the data suggesting the presence of an association (ACMGM052)

describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak) (ACMGM053)

calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association. (ACMGM054)

Fitting a linear model to numerical data:

identify the response variable and the explanatory variable (ACMGM055)

use a scatterplot to identify the nature of the relationship between variables (ACMGM056)

model a linear relationship by fitting a least-squares line to the data (ACMGM057)

use a residual plot to assess the appropriateness of fitting a linear model to the data (ACMGM058)

interpret the intercept and slope of the fitted line (ACMGM059)

use the coefficient of determination to assess the strength of a linear association in terms of the explained variation (ACMGM060)

use the equation of a fitted line to make predictions (ACMGM061)

distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation (ACMGM062)

write up the results of the above analysis in a systematic and concise manner. (ACMGM063)

Association and causation:

recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them (ACMGM064)

identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner. (ACMGM065)

The data investigation process:

implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female)? is there an association between height and foot length? (ACMGM066)

Topic 2: Growth and decay in sequences

The arithmetic sequence:

use recursion to generate an arithmetic sequence (ACMGM067)

display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations (ACMGM068)

deduce a rule for the nth term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions (ACMGM069)

use arithmetic sequences to model and analyse practical situations involving linear growth or decay; for example, analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation. (ACMGM070)

The geometric sequence:

use recursion to generate a geometric sequence (ACMGM071)

display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations (ACMGM072)

deduce a rule for the nth term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions (ACMGM073)

use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay; for example, analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour, the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate. (ACMGM074)

Sequences generated by first-order linear recurrence relations:

use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form (ACMGM075)

recognise that a sequence generated by a first-order linear recurrence relation can have a long term increasing, decreasing or steady-state solution (ACMGM076)

use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems; for example, investigating the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted, or the amount owing on a reducing balance loan

after each payment is made. (ACMGM077)

Topic 3: Graphs and networks

The definition of a graph and associated terminology:

explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network (ACMGM078)

identify practical situations that can be represented by a network, and construct such networks; for example, trails connecting camp sites in a National Park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition (ACMGM079)

construct an adjacency matrix from a given graph or digraph. (ACMGM080)

Planar graphs:

explain the meaning of the terms: planar graph, and face (ACMGM081)

apply Euler's formula, $v + f - e = 2$, to solve problems relating to planar graphs. (ACMGM082)

Paths and cycles:

explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge (ACMGM083)

investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only) (ACMGM084)

explain the meaning of the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem, planning a garbage bin collection route (ACMGM085)

explain the meaning of the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem (by trial-and-error methods only). (ACMGM086)

Unit 4

Unit 4 Description

This unit has three topics: ‘Time series analysis’; ‘ Loans, investments and annuities’ and ‘Networks and decision mathematics’.

‘Time series analysis’ continues students’ study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process.

‘Loans and investments and annuities’ aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments.

‘Networks and decision mathematics’ uses networks to model and aid decision making in practical situations.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Unit 4 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in time series analysis; loans, investments and annuities; and networks and decision mathematics
- apply reasoning skills and solve practical problems in time series analysis; loans, investments and annuities; and networks and decision mathematics
- implement the statistical investigation process in contexts requiring the analysis of time series data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Unit 4 Content Descriptions

Topic 1: Time series analysis

Describing and interpreting patterns in time series data:

construct time series plots (ACMGM087)

describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers; for example, one-off unanticipated events. (ACMGM088)

Analysing time series data:

smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process (ACMGM089)

calculate seasonal indices by using the average percentage method (ACMGM090)

deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process (ACMGM091)

fit a least-squares line to model long-term trends in time series data. (ACMGM092)

The data investigation process:

implement the statistical investigation process to answer questions that involve the analysis of time series data. (ACMGM093)

Topic 2: Loans, investments and annuities

Compound interest loans and investments:

use a recurrence relation to model a compound interest loan or investment, and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment (ACMGM094)

calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)

with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value. (ACMGM096)

Reducing balance loans (compound interest loans with periodic repayments):

use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan (ACMGM097)

with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans; for example, determining the monthly repayments required to pay off a housing loan. (ACMGM098)

Annuities and perpetuities (compound interest investments with periodic payments made from the investment):

use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity (ACMGM099)

with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case); for example, determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount. (ACMGM100)

Topic 3: Networks and decision mathematics

Trees and minimum connector problems:

explain the meaning of the terms tree and spanning tree identify practical examples (ACMGM101)

identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm (ACMGM102)

use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns. (ACMGM103)

Project planning and scheduling using critical path analysis (CPA):

construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal (ACMGM104)

use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project (ACMGM105)

use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)

use the critical path to determine the minimum time for a project to be completed (ACMGM107)

calculate float times for non-critical activities. (ACMGM108)

Flow networks:

solve small-scale network flow problems including the use of the 'maximum-flow minimum- cut' theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink). (ACMGM109)

Assignment problems:

use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning (ACMGM110)

determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems. (ACMGM111)

Units 1 and 2 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	•

organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts	routine problems			

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	

solutions to routine and non-routine problems			

Units 3 and 4 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics. • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	•

and statistical information to solve a range of routine and non-routine problems in a variety of contexts				

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	

solutions to routine and non-routine problems			

Mathematical Methods - How the Subject works

Rationale/Aims

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of Mathematical Methods are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this subject provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, the subject Mathematical Methods is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of Mathematical Methods, the proficiency strands of the F-10 curriculum are still applicable and should be inherent in students' learning of this subject. These strands are Understanding, Fluency, Problem solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of mathematics in this subject. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Mathematical Methods is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the F-10 curriculum from the strands, Number and Algebra, Measurement and Geometry and Statistics and Probability. In Mathematical Methods there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3 where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Aims

Mathematical Methods aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems

- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Links to Foundation to Year 10

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In order to study Mathematical Methods, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended for the study of Mathematical Methods:

- ACMNA264: Define rational and irrational numbers, and perform operations with surds and fractional indices
- ACMNA269: Factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts
- ACMSP278: Calculate and interpret the mean and standard deviation of data, and use these to compare datasets.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical undertanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Structure of Mathematical Methods

Mathematical Methods is organised into four units. The topics broaden students’ mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content,

applications, level of sophistication and abstraction. The probability and statistics topics lead to an introduction to statistical inference.

Unit 1	Unit 2	Unit 3	Unit 4
Functions and graphs Trigonometric functions Counting and probability	Exponential functions Arithmetic and geometric sequences and series Introduction to differential calculus	Further differentiation and applications Integrals Discrete random variables	The logarithmic function Continuous random variables and the normal distribution Interval estimates for proportions

Units

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of probability and statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an ‘instantaneous rate of change’. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

In Unit 3, the study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

In Unit 4, the logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit students are introduced to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding

and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

1. Given a specified context (as described in the curriculum content)
2. With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
3. Students perform a specified action (described through a verb)
4. In relation to what is valued in the curriculum (specified as the object or subject)
5. With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Glossary

Mathematical Methods Glossary

Unit 1

Unit 1 Description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence. Access to technology to support the computational aspects of these topics is assumed.

Unit 1 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
- solve problems using algebra, functions, graphs, trigonometric functions and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Unit 1 Content Descriptions

Topic 1: Functions and graphs

Lines and linear relationships:

determine the coordinates of the midpoint of two points (ACMMM001)

examine examples of direct proportion and linearly related variables (ACMMM002)

recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003)

find the equation of a straight line given sufficient information; parallel and perpendicular lines (ACMMM004)

solve linear equations. (ACMMM005)

Review of quadratic relationships:

examine examples of quadratically related variables (ACMMM006)

recognise features of the graphs of $y = x^2$, $y = a(x - b)^2 + c$, and $y = a\,(x - b)\,(x - c)$ including their parabolic nature, turning points, axes of symmetry and intercepts (ACMMM007)

solve quadratic equations using the quadratic formula and by completing the square (ACMMM008)

find the equation of a quadratic given sufficient information (ACMMM009)

find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010)

recognise features of the graph of the general quadratic $y = ax^2 + bx + c$ (ACMMM011)

Inverse proportion:

examine examples of inverse proportion (ACMMM012)

recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes. (ACMMM013)

Powers and polynomials:

recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}$, $n = -1$ and $n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014)

identify the coefficients and the degree of a polynomial (ACMMM015)

expand quadratic and cubic polynomials from factors (ACMMM016)

recognise features of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM017)

factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018)

solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained. (ACMMM019)

Graphs of relations:

recognise features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020)

recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry. (ACMMM021)

Functions:

understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022)

use function notation, domain and range, independent and dependent variables (ACMMM023)

understand the concept of the graph of a function (ACMMM024)

examine translations and the graphs of $y = f(x) + a$ and $y = f(x + b)$ (ACMMM025)

examine dilations and the graphs of $y = cf(x)$ and $y = f(kx)$ (ACMMM026)

recognise the distinction between functions and relations, and the vertical line test. (ACMMM027)

Topic 2: Trigonometric functions

Cosine and sine rules:

review sine, cosine and tangent as ratios of side lengths in right-angled triangles (ACMMM028)

understand the unit circle definition of $\cos\theta,\ \sin\theta$ and $\tan\theta$ and periodicity using degrees (ACMMM029)

examine the relationship between the angle of inclination of a line and the gradient of that line (ACMMM030)

establish and use the sine and cosine rules and the formula $Area = \frac{1}{2}bc\sin A$ for the area of a triangle. (ACMMM031)

Circular measure and radian measure:

define and use radian measure and understand its relationship with degree measure (ACMMM032)

calculate lengths of arcs and areas of sectors in circles. (ACMMM033)

Trigonometric functions:

understand the unit circle definition of $\cos\theta,\ \sin\theta$ and $\tan\theta$ and periodicity using radians (ACMMM034)

recognise the exact values of $\cos\theta,\ \sin\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035)

recognise the graphs of $y = \sin x,\ y = \cos x$, and $y = \tan x$ on extended domains (ACMMM036)

examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$ (ACMMM037)

examine period changes and the graphs of $y = \sin bx$, y=\cos bx\), and $y = \tan bx$ (ACMMM038)

examine phase changes and the graphs of $y = \sin(x + c)$, $y = \cos(x + c)$ and $y = \tan(x + c)$ and the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$ (ACMMM039)

prove and apply the angle sum and difference identities (ACMMM041)

identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042)

solve equations involving trigonometric functions using technology, and algebraically in simple cases. (ACMMM043)

Topic 3: Counting and probability

Combinations:

understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects (ACMMM044)

use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects

taken from a set of n distinct objects (ACMMM045)

expand $(x+y)^n$ for small positive integers n (ACMMM046)

recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$) (ACMMM047)

use Pascal's triangle and its properties. (ACMMM048)

Language of events and sets:

review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049)

use set language and notation for events, including $\overline{A}$ (or A') for the complement of an event A, $A?B$ for the intersection of events A and B, and $A?B$ for the union, and recognise mutually exclusive events (ACMMM050)

use everyday occurrences to illustrate set descriptions and representations of events, and set operations. (ACMMM051)

Review of the fundamentals of probability:

review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052)

review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certaint (ACMMM053)

review the rules: $P\left(\overline{A}\right) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054)

use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)

Conditional probability and independence:

understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)

use the notation $P(A|B)$ and the formula $P(A|B) = P(A \cap B)/P(B)$ (ACMMM057)

understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)

establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059)

use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events. (ACMMM060)

Unit 2

Unit 2 Description

The algebra section of this unit focuses on exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an ‘instantaneous rate of change’. These concepts are reinforced numerically, by calculating difference quotients both geometrically, as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimisation problems.

Access to technology to support the computational aspects of these topics is assumed.

Unit 2 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Unit 2 Content Descriptions

Topic 1: Exponential functions

Indices and the index laws:

review indices (including fractional indices) and the index laws (ACMMM061)

use radicals and convert to and from fractional indices (ACMMM062)

understand and use scientific notation and significant figures. (ACMMM063)

Exponential functions:

establish and use the algebraic properties of exponential functions (ACMMM064)

recognise the qualitative features of the graph of $y = a^x (a > 0)$ including asymptotes, and of its translations $y = a^x + b$ and $y = a^{x+c}$ (ACMMM065)

identify contexts suitable for modelling by exponential functions and use them to solve practical problems (ACMMM066)

solve equations involving exponential functions using technology, and algebraically in simple cases. (ACMMM067)

Topic 2: Arithmetic and geometric sequences and series

Arithmetic sequences:

recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$ (ACMMM068)

use the formula $t_n = t_1 + (n-1)\,d$ for the general term of an arithmetic sequence and recognise its linear nature (ACMMM069)

use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest (ACMMM070)

establish and use the formula for the sum of the first n terms of an arithmetic sequence. (ACMMM071)

Geometric sequences:

recognise and use the recursive definition of a geometric sequence:$t_{n+1} = rt_n$ (ACMMM072)

use the formula $t_n = r^{n-1}t_1$ for the general term of a geometric sequence and recognise its exponential nature (ACMMM073)

understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r (ACMMM074)

establish and use the formula $S_n = t_1\frac{r^n-1}{r-1}$ for the sum of the first n terms of a geometric sequence (ACMMM075)

use geometric sequences in contexts involving geometric growth or decay, such as compound interest. (ACMMM076)

Topic 3: Introduction to differential calculus

Rates of change:

interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f (ACMMM077)

use the Leibniz notation δx and δy for changes or increments in the variables x and y (ACMMM078)

use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$ (ACMMM079)

interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$ (ACMMM080)

The concept of the derivative:

examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081)

define the derivative $f'(x)$ as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (ACMMM082)

use the Leibniz notation for the derivative: $\frac{dy}{dx}=\lim_{\delta x\to 0}\frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx}=f'(x)$ where $y=f(x)$ (ACMMM083)

interpret the derivative as the instantaneous rate of change (ACMMM084)

interpret the derivative as the slope or gradient of a tangent line of the graph of $y=f(x)$ (ACMMM085)

Computation of derivatives:

estimate numerically the value of a derivative, for simple power functions (ACMMM086)

examine examples of variable rates of change of non-linear functions (ACMMM087)

establish the formula $\frac{d}{dx}(x^n)=nx^{n-1}$ for positive integers n by expanding $(x+h)^n$ or by factorising $(x+h)^n-x^n$ (ACMMM088)

Properties of derivatives:

understand the concept of the derivative as a function (ACMMM089)

recognise and use linearity properties of the derivative (ACMMM090)

calculate derivatives of polynomials and other linear combinations of power functions. (ACMMM091)

Applications of derivatives:

find instantaneous rates of change (ACMMM092)

find the slope of a tangent and the equation of the tangent (ACMMM093)

construct and interpret position-time graphs, with velocity as the slope of the tangent (ACMMM094)

sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x\to\infty$ and $x\to-\infty$ (ACMMM095)

solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains. (ACMMM096)

Anti-derivatives:

calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line. (ACMMM097)

Unit 3

Unit 3 Description

In this unit the study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Unit 3 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit 3 Content Descriptions

Topic 1: Further differentiation and applications

Exponential functions:

estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$ (ACMMM098)

recognise that e is the unique number a for which the above limit is 1 (ACMMM099)

establish and use the formula $\frac{d}{dx}(e^x) = e^x$ (ACMMM100)

use exponential functions and their derivatives to solve practical problems. (ACMMM101)

Trigonometric functions:

establish the formulas $\frac{d}{dx}(\sin x) = \cos x, \text{ and } \frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102)

use trigonometric functions and their derivatives to solve practical problems. (ACMMM103)

Differentiation rules:

understand and use the product and quotient rules (ACMMM104)

understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions (ACMMM105)

apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$,, $\frac{1}{x^n}$, $x\sin x$, $e^{-x}\sin x$ and $f(ax+b)$ (ACMMM106)

The second derivative and applications of differentiation:

use the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x (ACMMM107)

understand the concept of the second derivative as the rate of change of the first derivative function (ACMMM108)

recognise acceleration as the second derivative of position with respect to time (ACMMM109)

understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)

understand and use the second derivative test for finding local maxima and minima (ACMMM111)

sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection (ACMMM112)

solve optimisation problems from a wide variety of fields using first and second derivatives. (ACMMM113)

Topic 2: Integrals

Anti-differentiation:

recognise anti-differentiation as the reverse of differentiation (ACMMM114)

use the notation $\int f(x)\,dx$ for anti-derivatives or indefinite integrals (ACMMM115)

establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$ (ACMMM116)

establish and use the formula $\int e^x dx = e^x + c$ (ACMMM117)

establish and use the formulas, $\int \sin x dx = -\cos x + c$ and $\int \cos x dx = \sin x + c$ (ACMMM118)

recognise and use linearity of anti-differentiation (ACMMM119)

determine indefinite integrals of the form $\int f(ax+b)\,dx$ (ACMMM120)

identify families of curves with the same derivative function (ACMMM121)

determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$ (ACMMM122)

determine displacement given velocity in linear motion problems. (ACMMM123)

Definite integrals:

examine the area problem, and use sums of the form $\sum_i f(x_i)\ \delta x_i$ as area under the curve $y = f(x)$ (ACMMM124)

interpret the definite integral $\int_a^b f(x)\,dx$ as area under the curve $y = f(x)$ if $f(x) > 0$ (ACMMM125)

recognise the definite integral $\int_a^b f(x)\,dx$ as a limit of sums of the form $\sum_i f(x_i)\ \delta x_i$ (ACMMM126)

interpret $\int_a^b f(x)\,dx$ as a sum of signed areas (ACMMM127)

recognise and use the additivity and linearity of definite integrals. (ACMMM128)

Fundamental theorem:

understand the concept of the signed area function $F(x) = \int_a^x f(t)\,dt$ (ACMMM129)

understand and use the theorem $F(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, and illustrate its proof geometrically (ACMMM130)

understand the formula $\int_a^b f(x)\,dx = F(b) - F(a)$ and use it to calculate definite integrals. (ACMMM131)

Applications of integration:

calculate the area under a curve (ACMMM132)

calculate total change by integrating instantaneous or marginal rate of change

(ACMMM133)

calculate the area between curves in simple cases

(ACMMM134)

determine positions given acceleration and initial values of position and velocity.

(ACMMM135)

Topic 3: Discrete random variables

General discrete random variables:

understand the concepts of a discrete random variable and its associated probability function, and their use in modelling data (ACMMM136)

use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)

recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)

examine simple examples of non-uniform discrete random variables (ACMMM139)

recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases (ACMMM140)

recognise the variance and standard deviation of a discrete random variable as a measures of spread, and evaluate them in simple cases (ACMMM141)

use discrete random variables and associated probabilities to solve practical problems. (ACMMM142)

Bernoulli distributions:

use a Bernoulli random variable as a model for two-outcome situations (ACMMM143)

identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)

recognise the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p (ACMMM145)

use Bernoulli random variables and associated probabilities to model data and solve practical problems. (ACMMM146)

Binomial distributions:

understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial (ACMMM147)

identify contexts suitable for modelling by binomial random variables (ACMMM148)

determine and use the probabilities $\mathrm{P}(\mathrm{X}=\mathrm{r})=\binom{\mathrm{n}}{\mathrm{r}}\mathrm{p}^{\mathrm{r}}(1-\mathrm{p})^{\mathrm{n}-\mathrm{r}}$ associated with the binomial distribution with parameters n and p ; note the mean np and variance $np(1-p)$ of a binomial distribution (ACMMM149)

use binomial distributions and associated probabilities to solve practical problems. (ACMMM150)

Unit 4

Unit 4 Description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In Mathematical Methods statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Unit 4 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probabilty and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Unit 4 Content Descriptions

Topic 1: The logarithmic function

Logarithmic functions:

define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$ (ACMMM151)

establish and use the algebraic properties of logarithms (ACMMM152)

recognise the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$ (ACMMM153)

interpret and use logarithmic scales such as decibels in acoustics, the Richter Scale for earthquake magnitude, octaves in music, pH in chemistry (ACMMM154)

solve equations involving indices using logarithms (ACMMM155)

recognise the qualitative features of the graph of $y = \log_a x \ (a > 1)$ including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x + c)$ (ACMMM156)

solve simple equations involving logarithmic functions algebraically and graphically (ACMMM157)

identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems.

(ACMMM158)

Calculus of logarithmic functions:

define the natural logarithm $\ln x = \log_e x$ (ACMMM159)

recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$ (ACMMM160)

establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (ACMMM161)

establish and use the formula $\int \frac{1}{x} dx = \ln\ x\ + c$ for $x > 0$ (ACMMM162)

use logarithmic functions and their derivatives to solve practical problems. (ACMMM163)

Topic 2: Continuous random variables and the normal distribution

General discrete random variables:

use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)

understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts (ACMMM165)

recognise the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases (ACMMM166)

General continuous random variables:

understand the effects of linear changes of scale and origin on the mean and the standard deviation. (ACMMM167)

Normal distributions:

identify contexts such as naturally occurring variation that are suitable for modelling by normal random variables (ACMMM168)

recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution (ACMMM169)

calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems. (ACMMM170)

Topic 3: Interval estimates for proportions

Random sampling:

understand the concept of a random sample (ACMMM171)

discuss sources of bias in samples, and procedures to ensure randomness (ACMMM172)

use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli. (ACMMM173)

Sample proportions:

understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the sample proportion $\hat{p}$ (ACMMM174)

examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)

simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}\ -p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both n and p (ACMMM176)

Confidence intervals for proportions:

the concept of an interval estimate for a parameter associated with a random variable (ACMMM177)

use the approximate confidence interval $\left(\hat{p}-z\sqrt{(\hat{p}(1-\hat{p})/n},\ \ \hat{p}+z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution (ACMMM178)

define the approximate margin of error $E=z\sqrt{(\hat{p}(1-\hat{p})/n}$ and understand the trade-off between margin of error and level of confidence (ACMMM179)

use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p (ACMMM180)

Units 1 and 2 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information and to solve a range of routine and	• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, calculus and statistics that apply to routine problems • selects and applies techniques in functions, calculus and statistics to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, calculus and statistics • uses simple techniques in functions, calculus and statistics in routine problems • demonstrates familiarity mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	•

	non-routine problems in a variety of contexts				

Reasoning and Communication

A	B	C	D	E
• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies the limitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies the limitations of simple models used	

developing solutions to routine and non-routine problems			

Units 3 and 4 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information	• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, integration and distributions that apply to routine problems • selects and applies techniques in functions, integration and distributions to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, integration and distributions • uses simple techniques in functions, integration and distributions in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	•

and to solve a range of routine and non-routine problems in a variety of contexts				

Reasoning and Communication

A	B	C	D	E
• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies thelimitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies limitations of simple models used	

developing solutions to routine and non-routine problems			

Specialist Mathematics - How the Subject works

Rationale/Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout Specialist Mathematics. There is also a sound logical basis to this subject, and in mastering the subject students will develop logical reasoning skills to a high level.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of Specialist Mathematics will be able to appreciate the true nature of mathematics, its beauty and its functionality.

Specialist Mathematics has been designed to be taken in conjunction with Mathematical Methods. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematical Methods and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Specialist Mathematics is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still applicable and should be inherent in students' learning of the subject. These strands are Understanding, Fluency, Problem solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem solving. In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problems, for example integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject.

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1 vectors for two-dimensional space are introduced and then in Unit 3 vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in Mathematical Methods, is applied in Vectors in Unit 3 and applications of calculus

and statistics in Unit 4.

Aims

Specialist Mathematics aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately.
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Links to Foundation to Year 10

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still very much applicable and should be inherent in students' learning of the subject. The strands of Understanding, Fluency, Problem solving and Reasoning are essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving.In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes an important role, and the ability to present the solution of any problem in a logical and clear manner is of paramount significance. The ability to transfer skills learned to solve one class of problems, such as integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject. In order to study Specialist Mathematics, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended as preparation for Specialist Mathematics:

- ACMMG273: Establish the sine, cosine and area rules for any triangle, and solve related problems
- ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies
- ACMNAP266: Investigate the concept of a polynomial, and apply the factor and remainder theorems to solve problems.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and

communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains topics that will equip students for the ever-increasing demands of the information age.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skillfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Structure of Specialist Mathematics

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit provides a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Unit 1 and then in Unit 3 they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn prepares students for solving simultaneous equations in three variables.

Unit 1	Unit 2	Unit 3	Unit 4
Combinatorics Vectors in the plane Geometry	Trigonometry Matrices Real and complex numbers	Complex numbers Functions and sketching graphs Vectors in three dimensions	Integration and applications of integration Rates of change and differential equations Statistical inference

Units

Unit 1 contains three topics that complement the content of Mathematical Methods. The proficiency strand, 'Reasoning', of the F–10 curriculum is continued explicitly in the topic 'Geometry' through a discussion of developing mathematical arguments. This topic also provides the opportunity to summarise and extend students' studies in Euclidean Geometry, knowledge which is of great benefit in the later study of topics such as vectors and complex numbers. The topic 'Combinatorics' provides techniques that are very useful in many areas of mathematics, including probability and algebra. The topic 'Vectors in the plane' provides new perspectives on working with two-dimensional space, and serves as an introduction to techniques which can be extended to three-dimensional space in Unit 3. These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Unit 2 contains three topics, 'Trigonometry', 'Matrices' and 'Real and complex numbers'. 'Matrices' provides new perspectives for working with two-dimensional space, 'Real and complex numbers' provides a continuation of the study of numbers. The topic 'Trigonometry' contains techniques that are used in other topics in both this unit and Units 3 and 4. All of these topics develop students' ability to construct mathematical arguments. The technique of proof by the principle of mathematical induction is introduced in this unit.

Unit 3 contains three topics, 'Complex numbers', 'Vectors in three dimensions', and 'Functions and sketching graphs'. The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3 the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in Mathematical Methods is extended and utilised in the sketching of graphs and the

solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students' knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes,enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

Unit 4 contains three topics: 'Integration and applications of integration', 'Rates of change and differential equations' and 'Statistical inference'. In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students' previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: 'Concepts and Techniques' and 'Reasoning and Communication'. These two dimensions reflect students' understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

1. Given a specified context (as described in the curriculum content)
2. With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
3. Students perform a specified action (described through a verb)
4. In relation to what is valued in the curriculum (specified as the object or subject)
5. With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as 'analyse' and 'describe' have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Glossary

Specialist Mathematics Glossary

Unit 1

Unit 1 Description

Unit 1 of Specialist Mathematics contains three topics – ‘Combinatorics’, ‘Vectors in the plane’ and ‘Geometry’ – that complement the content of Mathematical Methods. The proficiency strand, Reasoning, of the F–10 curriculum is continued explicitly in ‘Geometry’ through a discussion of developing mathematical arguments. While these ideas are illustrated through deductive Euclidean geometry in this topic, they recur throughout all of the topics in Specialist Mathematics. ‘Geometry’ also provides the opportunity to summarise and extend students’ studies in Euclidean Geometry. An understanding of this topic is of great benefit in the study of later topics in the course, including vectors and complex numbers.

‘Vectors in the plane’ provides new perspectives for working with two-dimensional space, and serves as an introduction to techniques that will be extended to three-dimensional space in Unit 3.

‘Combinatorics’ provides techniques that are useful in many areas of mathematics including probability and algebra. All these topics develop students’ ability to construct mathematical arguments.

These three topics considerably broaden students’ mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Access to technology to support the computational aspects of these topics is assumed.

Unit 1 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in combinatorics, geometry and vectors
- apply reasoning skills and solve problems in combinatorics, geometry and vectors
- communicate their arguments and strategies when solving problems
- construct proofs in a variety of contexts including algebraic and geometric
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit 1 Content Descriptions

Topic 1: Combinatorics

Permutations (ordered arrangements):

solve problems involving permutations (ACMSM001)

use the multiplication principle (ACMSM002)

use factorial notation (ACMSM003)

solve problems involving permutations and restrictions with or without repeated objects (ACMSM004)

The inclusion-exclusion principle for the union of two sets and three sets:

determine and use the formulas for finding the number of elements in the union of two and the union of three sets. (ACMSM005)

The pigeon-hole principle:

solve problems and prove results using the pigeon-hole principle. (ACMSM006)

Combinations (unordered selections):

solve problems involving combinations (ACMSM007)

use the notation $\binom{n}{r}$ or ${}^{n}C_{r}$ (ACMSM008)

derive and use simple identities associated with Pascal's triangle. (ACMSM009)

Topic 2: Vectors in the plane

Representing vectors in the plane by directed line segments:

examine examples of vectors including displacement and velocity (ACMSM010)

define and use the magnitude and direction of a vector (ACMSM011)

represent a scalar multiple of a vector (ACMSM012)

use the triangle rule to find the sum and difference of two vectors. (ACMSM013)

Algebra of vectors in the plane:

use ordered pair notation and column vector notation to represent a vector (ACMSM014)

define and use unit vectors and the perpendicular unit vectors ***i*** and ***j*** (ACMSM015)

express a vector in component form using the unit vectors ***i*** and ***j*** (ACMSM016)

examine and use addition and subtraction of vectors in component form (ACMSM017)

define and use multiplication by a scalar of a vector in component form (ACMSM018)

define and use scalar (dot) product (ACMSM019)

apply the scalar product to vectors expressed in component form (ACMSM020)

examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)

define and use projections of vectors (ACMSM022)

solve problems involving displacement, force and velocity involving the above concepts. (ACMSM023)

Topic 3: Geometry

The nature of proof:

use implication, converse, equivalence, negation, contrapositive (ACMSM024)

use proof by contradiction (ACMSM025)

use the symbols for implication ($\Rightarrow$), equivalence ($\Longleftrightarrow$), and equality ($=$) (ACMSM026)

use the quantifiers ‘for all’ and ‘there exists’ (ACMSM027)

use examples and counter-examples. (ACMSM028)

Circle properties and their proofs including the following theorems:

An angle in a semicircle is a right angle (ACMSM029)

The angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc (ACMSM030)

Angles at the circumference of a circle subtended by the same arc are equal (ACMSM031)

The opposite angles of a cyclic quadrilateral are supplementary (ACMSM032)

Chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length (ACMSM033)

The alternate segment theorem (ACMSM034)

When two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord (ACMSM035)

When a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant. $(AM \times BM = TM^2)$. (ACMSM036)

Suitable converses of some of the above results (ACMSM037)

Solve problems finding unknown angles and lengths and prove further results using the results listed above. (ACMSM038)

Geometric proofs using vectors in the plane including:

The diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)

Midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)

The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides. (ACMSM041)

Unit 2

Unit 2 Description

Unit 2 of Specialist Mathematics contains three topics – ‘Trigonometry’, ‘Real and complex numbers’ and ‘Matrices’…

‘Trigonometry’ contains techniques that are used in other topics in both this unit and Unit 3. ‘Real and complex numbers’ provides a continuation of students’ study of numbers, and the study of complex numbers is continued in Unit 3. This topic also contains a section on proof by mathematical induction. The study of matrices is undertaken, including applications to linear transformations of the plane.

Access to technology to support the computational aspects of these topics is assumed.

Unit 2 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in trigonometry, real and complex numbers, and matrices
- apply reasoning skills and solve problems in trigonometry, real and complex numbers, and matrices
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit 2 Content Descriptions

Topic 1: Trigonometry

The basic trigonometric functions:

find all solutions of $\mathrm{f}\left(\mathrm{a}\left(\mathrm{x}-\mathrm{b}\right)\right)=\mathrm{c}$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM042)

graph functions with rules of the form $\mathrm{y}=\mathrm{f}(\mathrm{a}\left(\mathrm{x}-\mathrm{b}\right))$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM043)

Compound angles:

prove and apply the angle sum, difference and double angle identities. (ACMSM044)

The reciprocal trigonometric functions, secant, cosecant and cotangent:

define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them. (ACMSM045)

Trigonometric identities:

prove and apply the Pythagorean identities (ACMSM046)

prove and apply the identities for products of sines and cosines expressed as sums and differences (ACMSM047)

convert sums $a\cos x + b\ \sin x$ to $R\ \cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ and apply these to sketch graphs, solve equations of the form $a\cos x + b\sin x = c$ and solve problems (ACMSM048)

prove and apply other trigonometric identities such as $\cos 3x = 4\ \cos^3 x - 3\cos x$ (ACMSM049)

Applications of trigonometric functions to model periodic phenomena:

model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model. (ACMSM050)

Topic 2: Matrices

Matrix arithmetic:

understand the matrix definition and notation (ACMSM051)

define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse (ACMSM052)

calculate the determinant and inverse of **2x2** matrices and solve matrix equations of the form **AX=B** , where **A** is a **2x2** matrix and **X** and **B** are column vectors. (ACMSM053)

Transformations in the plane:

translations and their representation as column vectors (ACMSM054)

define and use basic linear transformations: dilations of the form $(x, y) \longrightarrow (\lambda_1 x, \lambda_2 y)$, rotations about the origin and reflection in a line which passes through the origin, and the representations of these transformations by **2x2** matrices (ACMSM055)

apply these transformations to points in the plane and geometric objects (ACMSM056)

define and use composition of linear transformations and the corresponding matrix products (ACMSM057)

define and use inverses of linear transformations and the relationship with the matrix inverse (ACMSM058)

examine the relationship between the determinant and the effect of a linear transformation on area (ACMSM059)

establish geometric results by matrix multiplications; for example, show that the combined effect of two reflections in lines through the origin is a rotation. (ACMSM060)

Topic 3: Real and complex numbers

Proofs involving numbers:

prove simple results involving numbers. (ACMSM061)

Rational and irrational numbers:

express rational numbers as terminating or eventually recurring decimals and vice versa (ACMSM062)

prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$ (ACMSM063)

An introduction to proof by mathematical induction

understand the nature of inductive proof including the 'initial statement' and inductive step (ACMSM064)

prove results for sums, such as $1 + 4 + 9 \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer *n* (ACMSM065)

prove divisibility results, such as $3^{2n+4} - 2^{2n}$ is divisible by 5 for any positive integer *n*.

(ACMSM066)

Complex numbers:

define the imaginary number **i** as a root of the equation $x^2 = -1$ (ACMSM067)

use complex numbers in the form **a+bi** where **a** and **b** are the real and imaginary parts (ACMSM068)

determine and use complex conjugates (ACMSM069)

perform complex-number arithmetic: addition, subtraction, multiplication and division. (ACMSM070)

The complex plane:

consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates (ACMSM071)

examine addition of complex numbers as vector addition in the complex plane (ACMSM072)

understand and use location of complex conjugates in the complex plane. (ACMSM073)

Roots of equations:

use the general solution of real quadratic equations (ACMSM074)

determine complex conjugate solutions of real quadratic equations (ACMSM075)

determine linear factors of real quadratic polynomials. (ACMSM076)

Unit 3

Unit 3 Description

Unit 3 of Specialist Mathematics contains three topics: ‘Vectors in three dimensions’, ‘Complex numbers’ and ‘Functions and sketching graphs’. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students’ knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in Mathematical Methods, is extended and applied in sketching graphs and solving problems involving integration.

Access to technology to support the computational aspects of these topics is assumed.

Unit 3 Learning Outcomes

By the end of this unit, students will:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching
- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit 3 Content Descriptions

Topic 1: Complex numbers

Cartesian forms:

review real and imaginary parts $Re\,(z)$ and $Im(z)$ of a complex number z (ACMSM077)

review Cartesian form (ACMSM078)

review complex arithmetic using Cartesian forms. (ACMSM079)

Complex arithmetic using polar form:

use the modulus $|z|$ of a complex number z and the argument $Arg\,(z)$ of a non-zero complex number z and prove basic identities involving modulus and argument (ACMSM080)

convert between Cartesian and polar form (ACMSM081)

define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these (ACMSM082)

prove and use De Moivre's theorem for integral powers. (ACMSM083)

The complex plane (the Argand plane):

examine and use addition of complex numbers as vector addition in the complex plane (ACMSM084)

examine and use multiplication as a linear transformation in the complex plane (ACMSM085)

identify subsets of the complex plane determined by relations such as $|z-3i| \leq 4$ $\frac{\pi}{4} \leq Arg(z) \leq \frac{3\pi}{4}$, $Re\,(z) > Im(z)$ and $|z-1| = 2|z-i|$ (ACMSM086)

Roots of complex numbers:

determine and examine the n^{th} roots of unity and their location on the unit circle (ACMSM087)

determine and examine the n^{th} roots of complex numbers and their location in the complex plane. (ACMSM088)

Factorisation of polynomials:

prove and apply the factor theorem and the remainder theorem for polynomials (ACMSM089)

consider conjugate roots for polynomials with real coefficients (ACMSM090)

solve simple polynomial equations. (ACMSM091)

Topic 2: Functions and sketching graphs

Functions:

determine when the composition of two functions is defined (ACMSM092)

find the composition of two functions (ACMSM093)

determine if a function is one-to-one (ACMSM094)

consider inverses of one-to-one function (ACMSM095)

examine the reflection property of the graph of a function and the graph of its inverse. (ACMSM096)

Sketching graphs:

use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y = |x|$ (ACMSM098)

examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = |f\,(x)\,|$ and $y = f(|x|)$ (ACMSM099)

sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree. (ACMSM100)

Topic 3: Vectors in three dimensions

The algebra of vectors in three dimensions:

review the concepts of vectors from Unit 1 and extend to three dimensions including introducing the unit vectors ***i, j*** and ***k***. (ACMSM101)

prove geometric results in the plane and construct simple proofs in three-dimensions. (ACMSM102)

Vector and Cartesian equations:

introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres (ACMSM103)

use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case (ACMSM104)

determine a vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions (ACMSM105)

examine the position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet (ACMSM106)

use the cross product to determine a vector normal to a given plane (ACMSM107)

determine vector and Cartesian equations of a plane and of regions in a plane. (ACMSM108)

Systems of linear equations:

recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations (ACMSM109)

examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables. (ACMSM110)

Vector calculus:

consider position of vectors as a function of time (ACMSM111)

derive the Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas (ACMSM112)

differentiate and integrate a vector function with respect to time (ACMSM113)

determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration (ACMSM114)

apply vector calculus to motion in a plane including projectile and circular motion. (ACMSM115)

Unit 4

Unit 4 Description

Unit 4 of Specialist Mathematics contains three topics: ‘Integration and applications of integration’, ‘Rates of change and differential equations’ and ‘Statistical inference’.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout Specialist Mathematics.

In this unit all of the students’ previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

Unit 4 Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference
- apply reasoning skills and solve problems in applications of calculus and statistical inference
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Unit 4 Content Descriptions

Topic 1: Integration and applications of integration

Integration techniques:

integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ and $1 + \tan^2 x = \sec^2 x$ (ACMSM116)

use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))\,g'(x)$ (ACMSM117)

establish and use the formula $\int \frac{1}{x} dx = \ln|x| + c$ for x ≠ 0 (ACMSM118)

find and use the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM119)

find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM120)

integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ (ACMSM121)

use partial fractions where necessary for integration in simple cases (ACMSM122)

integrate by parts. (ACMSM123)

Applications of integral calculus:

calculate areas between curves determined by functions (ACMSM124)

determine volumes of solids of revolution about either axis (ACMSM125)

use numerical integration using technology $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$ of the exponential random variable with parameter $\lambda > 0$,, and use the exponential random variables and associated probabilities and quantiles to model data and solve practical problems. (ACMSM127)

use numerical integration using technology (ACMSM126)

Topic 2: Rates of change and differential equations

use implicit differentiation to determine the gradient of curves whose equations are given in implicit form (ACMSM128)

Related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (ACMSM129)

solve simple first-order differential equations of the form $\frac{dy}{dx} = f(x)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)\, g(y)$ using separation of variables (ACMSM130)

examine slope (direction or gradient) fields of a first order differential equation (ACMSM131)

formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved. (ACMSM132)

Modelling motion:

examine momentum, force, resultant force, action and reaction (ACMSM133)

consider constant and non-constant force (ACMSM134)

understand motion of a body under concurrent forces (ACMSM135)

consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$ for acceleration. (ACMSM136)

Topic 3: Statistical inference

Sample means:

examine the concept of the sample mean **X** as a random variable whose value varies between samples where **X** is a random variable with mean **μ** and the standard deviation **σ** (ACMSM137)

simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to

illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large (ACMSM138)

simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where s is the sample standard deviation. (ACMSM139)

Confidence intervals for means:

understand the concept of an interval estimate for a parameter associated with a random variable (ACMSM140)

examine the approximate confidence interval $\left(\overline{\mathrm{X}}-\frac{\mathrm{zs}}{\sqrt{n}},\ \overline{\mathrm{X}}+\frac{\mathrm{zs}}{\sqrt{n}}\right)$,, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution (ACMSM141)

use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ (ACMSM142)

use $\overline{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ (ACMSM143)

collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality. (ACMSM144)

Units 1 and 2 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems • selects and applies techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs simple mathematical proofs, and adapts previously seen mathematical proofs • uses digital technologies	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • reproduces previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen simple mathematical proofs • uses digital technologies to display some mathematical information in routine problems	demonstrate limited familiarity wi simple concepts of vectors, combinatoric geometry, matrices, trigonometry and complex numbers uses simple techniques i a structured context demonstrate limited familiarity wi mathematica models demonstrate limited familiarity wi mathematica proofs uses digital technologies for arithmeti calculations and to displa limited mathematica information

• constructs mathematical proofs in a variety of contexts, and adapts previously seen mathematical proofs • uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	appropriately to graph, display and organise mathematical information to solve a range of routine and non-routine problems			

Reasoning and Communication

A	B	C	D	E
• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing	• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical judgments and arguments, including simple proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical and symbolic form in routine problems • communicates mathematical arguments, including previously seen proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical or symbolic form in routine problems • communicates mathematical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	

solutions to routine and non-routine problems				

Units 3 and 4 Achievement Standard

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of concepts of functions, calculus, vectors and statistics in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts • constructs mathematical proofs in a variety of contexts using a range of techniques	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics in routine and non-routine problems • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs mathematical proofs in a variety of contexts and adapts previously seen mathematical proofs • uses digital technologies appropriately to graph, display and organise	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • constructs simple mathematical proofs and adapts previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen mathematical proofs • uses digital technologies to display some mathematical information in routine problems	•

• uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	mathematical information to solve a range of routine and non-routine problems			

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments, including simple proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates mathematical and statistical arguments, including previously seen proofs, using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	

limitations of models used when developing solutions to routine and non-routine problems			