27 E-Textbooks in/for Teaching and Learning Mathematics

A Potentially Transformative Educational Technology

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INTRODUCTION

In *The Age of Discontinuity: Guidelines to Our Changing Society* (1992), Professor of Management Peter Drucker lays out ways in which technologies are transforming, and will continue to transform, industries throughout the world economy; for many workers, what characterizes work life now is the continual need to adapt to technological change. Such changes are not limited to the world of work: technology is transforming interactions with media, and this also relates to books. This chapter focuses on one way in which technology may transform educational processes and bring about new educational dynamics. Specifically we examine ways in which e-book technology might influence one genre of book, the (mathematics) textbook.

This chapter's focus on e-textbooks is a little different than the focus of many who do research on technology in mathematics education. For many years, technology in mathematics education has focused on support for student activity in classrooms. While two of the authors have written, in previous editions of this Handbook (Yerushalmy & Chazan, 2002, 2008), about curriculum development and technology, and the impact of various tools and approaches on teaching and learning, we explore in this chapter deep evolutions in the learning, teaching, and professional development processes that digital resources, in particular e-textbooks, may create as they potentially transform current educational practice. We suggest that these potential transformations in teaching and learning require new research perspectives (Gueudet, Pepin, & Trouche, 2012). Further, we suggest that there are interesting opportunities—and even an urgent need—to renew research perspectives on technology and teachers' participation in the design phase of technology tools, in particular on e-textbooks used and designed by teachers. In this chapter, we present a synthesis of research and development studies on e-textbooks, and analyze the crucial evolutions connected with their design and use.

At the same time that our focus on e-textbooks is slightly different than what is often done in the name of technology in mathematics education. This focus is consonant with greater interest in general in the role of textbooks in mathematics education. Textbook conferences in Europe and Asia (e.g. International Conference on School Mathematics Textbooks, East China Normal University, Shanghai, 2011; International Conference on Mathematics Textbook Research and Development, University of Southampton, July 2014) and the recent ICMI Study 22 Conference on Task Design in Mathematics Education" in Oxford (see proceedings: Margolinas, 2013) evidence the renewed interest in textbook research in mathematics education (Fan, Zhu, & Miao, 2013). Recent studies (e.g. *ZDM Special Issue 45*[5]) address new issues and point to new roles for the textbook (e.g. Yerushalmy, 2013), not only as a tool for students and teachers, but also as interface between policy and practice (Pepin, Gueudet, & Trouche, 2013a).

The textbook is commonly seen as the major curriculum resource in the classroom for teachers and students, and many authors claim that the textbook is an important artifact and a major source of provision of educational opportunities (Haggarty & Pepin, 2002, Schmidt, 2012). From the research literature, it is clear that textbooks are a vital ingredient for mathematics teachers' lesson preparations and their pedagogic practice (e.g. Gueudet, Pepin, & Trouche, 2013a). Perhaps as importantly, they may be seen as vehicles and tools for teacher learning and professional development (Brown 2009; Collopy, 2003). At the same time, mathematics textbooks are perceived to reflect the views expressed in national curricular documents, and hence what the country views as appropriate for their students to learn, and how (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997, Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). In short, much research has gone into analyzing traditional textbooks and their use in mathematics classrooms (Fan, 2013). The ICMI Study 22 discussion document states that "most teachers use textbooks and/or online packages of materials as their total or main source of tasks" (Margolinas, 2013, p. 11).

It is reasonable to argue that resources such as textbooks/e-textbooks and other materials (also digital materials, e.g. worksheets) are an important part of the context in which pupils and teachers work. In recognition of the central importance of such documents, the framework for the Third International Mathematics and Science Study (TIMSS) included large-scale cross-national analyses of mathematics curricula and textbooks as part of its examination of mathematics education and attainment in almost 50 nations. Concerns have been expressed about the *quality* of textbooks, for example, and about their persuasive influence. It appears that textbook content, and how it is used, are significant influences on students' opportunities to learn and their subsequent achievement (Robitaille & Travers, 1992). It is also commonly assumed that textbooks are one of the main sources for the content covered and the pedagogical styles used in classrooms (Valverde et al., 2002). Teachers often rely heavily on textbooks in their day-to-day teaching when they decide what to teach, how to teach it, and the kinds of tasks and exercises to assign to their students. Hence, it seems sensible to analyze e-/textbooks with respect to their "quality."

It is clear that both content and structure depend on the textbook's design, whether traditional or digital textbooks. In this chapter we investigate textbook design, focusing on its use and developments brought about by digital means, and we examine the ways in which the interfaces (e.g. between teachers, textbook authors and learners) may be different as compared to traditional textbooks. To provide specific examples, with e-textbooks, interfaces may change in terms of the following.

- Interactions amongst teachers: digital means offer opportunities for teachers to easily prepare lessons together, in particular if the necessary tools are provided by the e-textbook—hence more opportunities for collective work.

- Interactions between teachers and textbook authors: if teachers can change the content of the book (e.g. in terms of the sequencing of topic areas, and within topic areas the learning trajectories suggested, or indeed the tasks provided and the digital 'tools' used for particular tasks), these changes may be approved by the textbook author group, and subsequently included in the book—hence teachers become quasi-authors of the textbook, and the authority of the text changes.
- Interactions between teachers and learners: e-textbooks may provide interfaces for teachers and pupils to easier communicate, for example in terms of feedback on (written) homework (which may be in or out of class).

Textbooks are now often complemented by digital materials (e.g. files to be projected during the lesson by the teacher; exercises using particular software; etc.). ICMI Study 22 asked the following question:

> How can or should new digital formats influence textbook design: e.g. use of podcasts, twitter, and other social media; implications for design and coherence of materials (either original digital design or transfer from print) if teachers are able to select tasks in varied orders?
>
> (Margolinas, 2013, p. 19)

This question is complex and encompasses several aspects of the evolutions resulting from digital means, in particular the following: digital means provide new opportunities for the structuring of textbooks for their use by teachers, and they open up new possibilities for design and further evolutions. To address this issue we will start by working toward a definition of an e-textbook. To do so, we have to begin with a characterization of the role of textbooks in compulsory schooling. From there, we will move on to examine how e-textbooks have the potential to change both teachers and students' interactions with textbooks. Throughout these sections of the chapter, we will offer periodic "windows" into the use of e-textbooks in teaching and learning. Finally, we will illustrate how the presence of e-textbooks calls for reconceptualizations of constructs, such as "quality" and "coherence," which researchers have been using in research on textbooks; thus, one might say that e-textbooks are challenging existing constructs in mathematics education research.

E-TEXTBOOKS: WORKING TOWARDS A DEFINITION

Studying e-textbooks involves being able to identify their nature, to define what an e-textbook is. However, to date there is no clear notion or definition of an e-textbook. We cannot provide a "complete" definition, but we can work toward such a goal. For example, whilst we can claim that a textbook is a book, there is no corresponding claim for e-textbooks. We discuss this next, with both general considerations and examples provided by our research. Developing a deeper understanding of the nature of e-textbooks involves examining their (potential) features and their structure in particular, but also their design modes, and investigating the similarities and differences between paper textbooks and e-textbooks in terms of design.

Textbooks as Artifacts of Compulsory Schooling

Books have been crucial tools for mathematical communication; texts, like Euclid's *Elements*, were at the same time both compendiums of known mathematical knowledge, as well as pedagogical texts that could form the basis for studies (Herbst, 2002) and communicate aspects of mathematical practice (Netz, 1999). More recently, the texts authored by Bourbaki were intended to play a similar role (Guedj, 1985). The French mathematician Cartier, from the Bourbaki School, wrote: "The mathematician belongs to the civilization of the book" (Cartier & Chemla, 2000, p. 166, our translation).

Textbooks, a new kind of mathematical text, came into existence with the advent of compulsory formal schooling and efforts to democratize knowledge (Schubring, 1987). These new kinds of mathematical texts were explicitly designed to communicate at the same time both to pupil and teacher the mathematics (e.g. to supplement texts like Euclid's *Elements*) and the values that the author/s regarded appropriate for common school knowledge. Textbooks now commonly include exposition; worked examples, exercises or questions, images, and more (Love & Pimm, 1996). While at some times and in some places these specialized texts—textbooks—are authored by people whose authority is taken for granted (Kidwell, Ackerberg-Hastings, & Roberts, 2008), in current times, these books are either created by central educational authorities or written by individuals or teams (mathematics teachers, educators, inspectors) to meet guidelines produced by such authorities.

Distinguishing mathematical texts for school from other kinds of mathematical texts is a complex task (Proust, 2012). Moreover, the role they play in schooling is of course crucial. Textbooks are meant to communicate to the teacher what it is that students are supposed to learn and in which order (Westbury, 1990), and something about how it is that students are to do so (Chazan & Yerushalmy, 2014). New electronic means of publishing texts have the potential to change the textbook industry, just as they have changed the publication of other books. When textbooks are published as paper books, they are written at one time and then produced. With this mode of production, the teacher interacts with a final product that is fixed and does not expand as it is used (except under the form of written notes in the margins of the pages). When e-books are published in bits and bytes, they now can potentially be continually edited and supplemented by a large number of people; as books are edited in this way, such changes in mode of publication can reshape the relationships between textbook author or curriculum developer, teacher, and student. In particular, the nature of the authority of the text in the classroom may shift. It is this potential that we seek to explore in our discussion of e-textbooks.

Design of E-Textbooks

Whether in the form of dedicated hardware, tablet PCs, or a software format, digital books have challenged the object we used to refer to as a "book." Clearly, the 21st-century reader will be increasingly reading materials in digital format, and may find them both useful and attractive. For a growing community of readers digital books have already changed the book culture. It did not take too long for the textbook publishing industry to follow these global changes and offer digital textbooks. At first textbook publishers addressed the higher education audience, and recently they have been targeting schools and schoolteachers, both as authors and as users.

The first generation of digital textbooks may be considered "old wine in new wineskin" (Gould, 2011); they were merely digitized versions of their paper counterparts, integrating a given fixed content, and supporting limited interactivity only by means of search and navigation of the digital document. The second generation of digital textbooks bears a noticeable change in the object itself, opening new occasions for interactions, personalization, and evolution of the content: an increasing number of school textbooks are now supplemented by continuously upgraded digital resources that can be found on the Web, such as the ClassroomAid blog (*http://classroom-aid.com/educational-apps*). Textbook publishers are addressing a wide range of expected changes in the affordances of the digital object, including material aspects of weight and cost; the quality and attractiveness of the material; the richness of the modes of presentation; and the opportunities for personalization. Publishers offer teachers the possibility to personalize digital textbooks for their courses, emphasizing flexibility and inexpensive dynamic changes that allow schoolteachers to personalize the textbook by selecting from existing chapters and content, and even individualizing the book for each student. Thus, the change in the object is also associated with an essential change concerning *design, teacher agency*, and *authorship*.

Different Types of E-Textbooks

Working towards a definition, we will introduce three types/models of e-textbooks and describe examples corresponding to each type. Each example describes a complex design but is used to illustrate specific characteristic/s that for us are central dimensions of e-textbooks and their use by teachers or students.

1. The **integrative e-textbook** refers to an "add-on" type model where the digital version of a (traditional) textbook is connected to other learning objects (see Figure 27.1): a digital book that is ideologically similar to a rigid paper textbook; i.e. it is a traditionally authored textbook and many users are likely to use it as a digital version of a paper textbook. In that sense, norms of authority, coherence, and quality are not changing. But the integrative e-textbook allows for users (teachers or developers) to add on or link to other learning objects that traditionally are not assumed to be part of a textbook. It is used by educational systems (schools, states, developers) as the core of a Virtual Learning Environment—learning management, course management, authoring tools to add or edit activities (by teachers), etc.
2. The **evolving or "living" e-textbook** refers to an accumulative/developing model where a core community (e.g. of teachers, IT specialists) has authored a digital textbook that is permanently under development due to the input of other practicing members/teachers (see Window 1 and Figure 27.2). The Sésamath system is an example of a sustained project of community-authored resources. The use of such a textbook by teachers who are not contributing is different from the use of the integrative model, because the evolving/"living" e-textbook emphasizes interactivity of "living" resources (and we will show that in the subsequent section).
3. The **interactive e-textbook** refers to a "tool kit" model where the e-textbook (authored to function only as an interactive textbook) is based upon a set of learning objects—tasks and interactives (diagrams and tools)—that can be linked and combined. The VisualMath example (see Window 2) is an example for this type of textbook. It is "traditionally" authored, thus representing the traditional view of external authority. However, different from the other models, (a) the tasks are based on interactives that are an integral part of the textbook (rather than being add-on tools); (b) it is designed to afford object-oriented navigation along mathematical objects and operations that provide mathematical opportunities that can be taught in various orders (like the museum visitors who visit art in different ways.)

The first example (Figure 27.1) refers to the "add-on" model where the digital version of a (traditional) textbook is linked to other learning objects. Korea, considered as one of the leading countries in mathematics and science achievements, became a leading innovator in the area of e-textbooks, especially in school mathematics and science. Korean publishers hold an integrative view in which textbooks remain the central learning resource, surrounded by other types of facilitating media.

Other educational systems are adopting a similar view of the new textbook (Taizan, Bhang, Kurokami, & Kwon, 2012). The Israeli education system requires that each textbook appears in at least one of three formats: a digitized textbook; a digitized textbook that is enriched with external links and multimodal materials; and/or a textbook that is specially designed to work in a digital environment and which includes online tools for authoring, learning, and management.

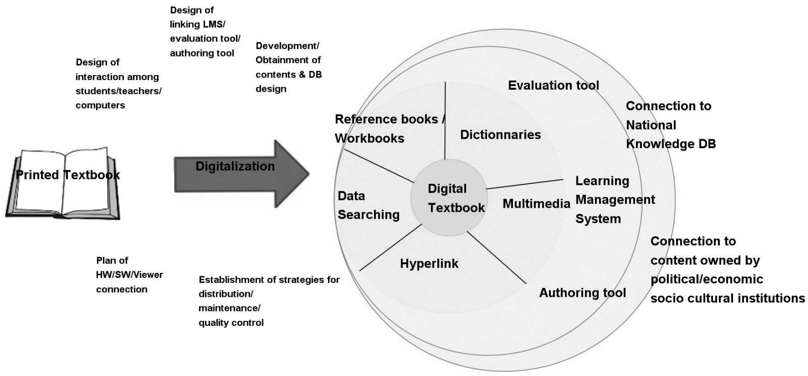


Figure 27.1 The integrated e-textbook. Source: Korea Education and Research Information Service (KERIS), 2007.

Thus, while part of the digital textbook can also function as printed text, the e-textbook integrates functions and uses (by teachers) that traditionally were not part of textbook use, such as grading students' work and organizing their course.

The second example is the Sésamath e-textbook, developed in France (see Window 1). This example refers to a system approach model, where a core group/community (of teachers, IT specialists) has developed a hybrid textbook that is continuously evolving through the input of other practicing members. An e-textbook can be entirely designed by teachers, like in the case of Sésamath. In this case teachers intervened as individual authors of the e-textbook. However, even in the case of commercial publishers and "expert" authors (e.g. "expert teachers", inspectors, teacher educators), teachers are not expected to be passive users of e-textbooks; they are expected to personalize the book (see example in Window 2).

Window 1—Sésamath E-Textbook, a Living Resource System Collaboratively Designed

Sésamath, a French online association of mathematics teachers (most of them teaching in lower secondary school, grades 6–9), started in 2001. Its spirit is summarized on its Web site (www.sesamath.net/) as "Mathematics for all" (Figure 27.2). The association started with a gathering of approximately 20 mathematics teachers who shared their personal Web sites and subsequently designed a "drill-and-practice" software program called *Mathenpoche* (which stands for *mathematics in the pocket)*. Mathenpoche has very quickly become very successful, and it has been used by many teachers and students. The possibilities and opportunities for organizing collaborative work, for offering a set of flexible resources and tools, and for questioning (e.g. through discussions with researchers) its structure and development mode, led Sésamath to become a major reference in the French educational landscape (Trouche, Drijvers, Gueudet, & Sacristan, 2013).

Since, 2005 Sésamath has designed textbooks for grades 6–10 and has now become one of the most popular textbooks in France: 300,000 textbooks have been sold, representing 20% of a very competitive French textbook market (Vieillard-Baron, 2009). Four elements can explain this:

1. The *mode of design* of these textbooks (Sabra & Trouche, 2011) involves a large number of actors. Many teachers (approximately one hundred, for each textbook) have contributed to the design in a *collaborative* and *iterative* way, as "authors of content," or "designers of didactical scenarios," or "testers," or "experimentators" in classes (a single teacher could have several roles, or change roles at different moments).
   The textbook resulting from this process is expected to fit the wishes and needs of a large number of teachers.

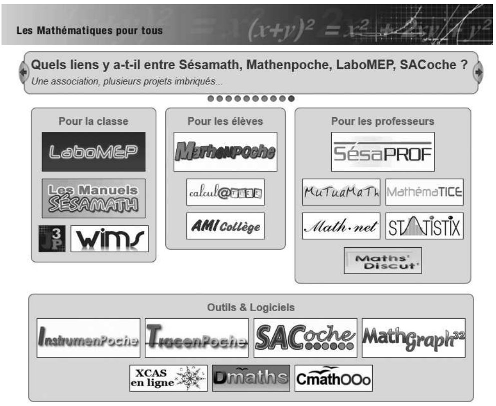


Figure 27.2 The front page of the Sésamath Web site.

2. Far from being a simple textbook, the Sésamath textbooks constitute a *hybrid* system of resources for teaching (i.e., including a classical structure in chapters, online supplements, and animated corrections). Following their development helps to understand this systemic aspect:
   - the first model of Sésamath textbooks was a *single static book*, available both online (as a. pdf file, but also in. odt format, which allowed teachers to make modifications) and in hard copy, accompanied by separated animations online, a set of Mathenpoche exercises, etc. (i.e. a real *resource system*, see Figure 27.1);
   - the second model was a *flexible and dynamic digital textbook* that a teacher could organize according to his/her needs, with animation and extra exercises integrated in each chapter;
   - The third model was both a flexible and dynamic digital textbook *and* a laboratory for collaboratively adjusting the textbook to the needs and projects of the community (school, team of teachers). This laboratory, named *LaboMEP* (which stands for *Laboratory for Mathenpoche*) allows teachers to develop and share their own lessons, but also to differentiate their teaching according to the results of their students (using an application—Pepite—that has been developed through the collaboration of Sésamath with researchers).
3. Sésamath has found an economic model that is well-suited to the world of free software (Vieillard-Baron, 2009): all Sésamath resources, including the textbooks, can be freely downloaded. A printed-paper version can be bought for half the price of other mathematics textbooks in France. The sale of the books allows Sésamath to employ seven teachers, who constitute the technical infrastructure of the association (all authors work without remuneration).
4. Sésamath resources are truly *alive*, as the association itself is *alive* (Gueudet & Trouche, 2009b). Once the textbooks have been produced, they are constantly discussed in the community of users (in discussions lists, forums, etc.), and they are regularly adjusted and "redesigned." For example, the grade 6 Sésamath textbook was comprehensively revised in 2013, and it includes now open-ended problems and more complex tasks than before.

We provide the third example to illustrate the interactive model showing the design of an interactive e-textbook that is based upon a set of independent but conceptually connected learning objects, which address specific content and can be linked and combined to create learning trajectories that reflect different pedagogical ideas (see Window 2).

Window 2—Visual Math Design Process and Structure

VisualMath has been designed to challenge traditional notions of what school mathematics is and how it can be taught and learned (more about the rationale and background can be found in the chapters within the first and second editions of this *Handbook* (Yerushalmy & Chazan, 2002, 2008). The VisualMath Function's based algebra e-textbook is part of a geometry, algebra and calculus curriculum development project that spans more than 25 years. It supports the "what" and the means for the "how" to teach and learn a full school-algebra course based on the concept of function. The tasks and the interactives are all organized around a relatively small number of mathematical objects (functions) and operations (with the functions) represented and interactively manipulated by graphs, sketches, numbers, and symbols. This object-oriented mapping of the curriculum proposes a coherent organization of the algebra curriculum.

The term *VisualMath* indicates the emphasis of the program on mathematical representations and especially on visualization. In this respect, the program is a source for important ideas, as well as methods for students to act on those ideas supported by visual feedback (Chazan & Yerushalmy, 2003). As teachers are expected to guide mathematical inquiry and take a more active role in the ways different resources are used to communicate ideas in their class, a bound book that represents the curriculum plan and assumes a linear progression for all students may not best serve teaching. Thus, the learning units of the e-textbook are designed to support a variety of progressions and sequences. Throughout the development of VisualMath, various formats of printed textbooks and of digital resources were developed and studied.

Paper textbooks were part of the first round of development (1992–2002): digital interactive multiple-representation tools were designed to be part of the work with the textbook, and these were often used (by teachers) for enrichment, rather than as an integral necessary resource. The tasks require making sense of problems, spending longer on analyzing givens, constraints, relationships, and goals. Although problem solving can always be helped by use of appropriate tools, it should be carried out strategically, constructing viable arguments and critiquing the reasoning of others. Expositions were replaced by proposals for hands-on explorations followed by whole group teacher-led discussions.

2. The VisualMath e-textbook was designed anew, based on the printed VisualMath algebra books, as a Web.1 learning environment. (The development by Yerushalmy, Katriel, and Shternberg (2004) is further detailed in Yerushalmy, 2013.) Its design was a joint effort of the teams who developed the tools for explorations and teacher-users groups who participated as developers of the VisualMath tasks. The VisualMath Algebra e-textbook accommodates two objects (functions), the linear and the quadratic, and six operations on/with the objects (represent, modify, transform, analyze, combine, and compare).

The design borrowed images from a "museum setting" and is consistent with the distinction Kress and van Leeuwen (1996) made to describe linear and nonlinear texts. It accommodates two exhibit halls (units): the linear and the quadratic. Entering the hall, the tour leads to various galleries (units) according to the central mathematical operations of functions (on Reals): Modify, Transform, Operate and Compare two functions. Each gallery consists of a central piece that reflects the essence of the mathematical concept at hand. The "art" is located in a few spaces, engaging the visitor in learning a concept in different ways: there are interactive tasks (for problem solving), special-purpose tools (hands-on explorations within limited scope of the gallery), interactive exercises' generators (to improve skills), and an integrative project. As the quantity of the "art" is too vast to be covered in a single course, decisions of choice and sequencing should be made by the teacher. Major design efforts were invested in offering interactive diagrams. An interactive diagram is a relatively small and simple software application (applet) built around a preconstructed example (in contrast to tools). An interactive diagram combines the characteristic features of a static diagram and of a tool.

3. The Algebra VisualMath e-textbook is currently being redesigned attempting to support new teachers' engagements with materials. It is developed to function as Web.2 environment and includes a new set of editing options that allow users to create courses, design their personal version of the e-textbook, redesign the sequences, and rewrite tasks from the authored materials.

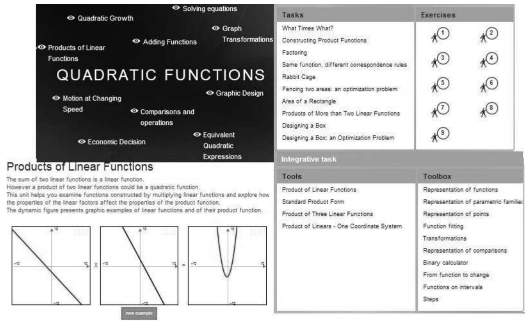


Figure 27.3 The Graph Transformations unit of the VisualMath e- textbook: It offers a tool box; a central piece of exposition; and three working spaces (on the left): Activity Tools, Tasks, and Exercises.

We suggest here that a common feature of e-textbooks is that they constitute networks of digital material that encompass the mathematical content of a traditional textbook, but they also include diverse supplementary tools, in particular those that support the learning of the mathematical content. They can have very different structures, according to the authors, publishers, or national policies. Nevertheless, many e-textbooks seem to offer possibilities for adaptations by teachers, for teacher collaboration, and for building personal paths for students. Hence, we contend that we can *define an e-textbook as an evolving structured set of digital resources, dedicated to teaching, initially designed by different types of authors, but open for redesign by teachers, both individually and collectively*.

The three models and examples presented illustrate these common features and possible differences. Sésamath and VisualMath share a perspective on how students learn mathematics: interactively, meeting various tasks and choosing representations. Both e-textbooks propose different kinds of digital resources, their structure is not linear, and they can be adapted and shaped by teachers according to their particular contexts and teaching objectives. Interestingly, the Sésamath e-textbook was designed solely by teachers; it did not, at least in the early stages, incorporate mathematics education research results (Gueudet et al., 2013a). However, it has been continuously modified according to teacher-users' remarks. VisualMath was designed by experts in the context of a long-term research and development project. In the currently designed version teachers will be able to personalize the text by reordering or rewriting tasks and adding new tasks and live resources. It will be important to study patterns of use, such as choosing a path within the offered original materials, creating a "personal book" from the given materials, or adding and dropping tasks and live resources. It will also be important to observe whether the inserted changes and personalization will serve only the teachers in their classrooms, or whether teachers will attempt to share their work in online communities, as is happening with Sésamath.

We develop these examples further in subsequent sections, to investigate the evolutions developing from the digital format, and moreover to examine not only the features/designs of e-textbooks, but also their use by students or teachers.

USE OF E-TEXTBOOKS BY STUDENTS AND TEACHERS

Most authors agree that mathematics textbooks address both the teacher and the learner (Love & Pimm, 1996). Rezat (2006), who challenges this dichotomy, suggests four triangular connections related to the use of any textbook: student-textbook-mathematical knowledge; student-teacher-textbook; teacher-textbook-mathematical knowledge (didactical); and student-teacher-mathematical knowledge (represented by the textbook and mediated by the teacher). A recent typology for analyzing digital programs in mathematics by Choppin and colleagues (2014) offers three themes corresponding to the compound relations of resources, learning, and teaching: students' interactions with the program, curriculum uses and adaptation, and assessment. In this section we approach general issues illustrated by two examples: the first focuses on student-text-knowledge, and the second on the student-teacher-textbook triad specifically related to digital interactive textbooks (supported by recent limited research results).

E-Textbooks and Changes in the Relationship Between Students and Textbooks

Whilst indeed a textbook is most commonly conceptualized as an artifact that is mediated and used primarily by the teacher, some recent studies analyze student-textbook-mathematical knowledge relations. Engagement with mathematics is a critical component when designing learning experiences with e-textbooks. As it is a too large an issue to be addressed in one section, we choose one aspect of engagement of students' problem solving with e-textbooks: learning with interactive visuals. Offering qualitative observations, Rezat (2013) describes how students choose tasks for practicing on their own by identifying three utilization schemes, amongst them visual appearance similarity. Diagrams are core visual elements of mathematics textbooks and increasingly more dominant in reform textbooks (Love & Pimm, 1996). Yerushalmy's (2005) review of studies involving students' engagement with diagrams in mathematical text stresses that diagrams (intending to present information) are likely to implicitly engage the viewer in meaningful interpretations. Research of problem solving by students suggests that diagrams can support structuring students' ideas by making them meaningfully visible and concrete, and they may focus students on core aspects of the problem and engagement in their own sense-making process (Murata, 2008). Engagement with diagrams has also been studied by Bremigan (2005) who found that the modification of given diagrams or the construction of new diagrams in students' solutions are related to success in problem-solving. Naftaliev and Yerushalmy (2013) studied structures of design and affordances of interactive diagrams and analyzed students' problem solving experiences with e-textbooks. Window 3 (below) offers an example:

Window 3—Students' Engagements With Interactive Tasks of the VisualMath E-Textbook

Yaniv, Lior, and Daniel, 13–14-year-old algebra students, were observed while solving an unfamiliar task in the Linear Function modeling unit. The task aims at understanding properties of linear graphs that are mathematical models of animated motion. The diagram consists of two representations of motion by seven cars: a simulation, and a hot-linked position-time graph. The graph and the simulation are only partially linked: motion occurs simultaneously on the simulation and on the graph but there is no color match, so the identification process requires extracting data from the simulation and the graph in order to link them.

Students began their work by activating the animation and observing the motion of the cars (which they called "dots") as well as the related animated dots on the graphs. They ran through the entire race several times, then identified a conflict between the two animated representations: one "car" out of the six dots does not move, but all seven dots on the graphs are in motion! They stopped the animation and attempted to retell the story of the motion by

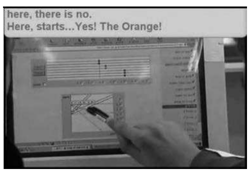


Figure 27.4 A student working on the Linear Function modeling unit.

viewing the given static components and identified the constant function graph that describes the static state.

Yaniv: Time doesn't pass for it [for the dot].
Lior: Time is constant—doesn't move.
Yaniv: Time passed for the dot but it [the dot] doesn't use it because it doesn't have speed.
Lior: No, the time does move. The dot doesn't move during that time. . . For all of them. . . time is passing but it [points at a dot along the path] simply doesn't move during that time.
Daniel: Maybe. It doesn't run. Time always changes but there is a common starting point and then time starts. And it is impossible to say that because it doesn't move its time stops.
Lior: Ah! I know, because in. . . wait a minute. . . in this dimension, the dimension of the time it is impossible to control.
Daniel: You can't control because it is the x-axis and here time always moves and can't stand still. Only on the y-axis can you change [the position].

In the discussion the students started to treat the graph as a description of a process taking place over time. They discussed the graph in the static state and began interpreting it. They reached a conclusion about time as an independent variable: time always changes and it cannot be stopped, and the passage of time cannot be controlled. The work they performed was mental: formulating hypotheses and reaching conclusions.

Next, they activated the animation to verify the hypotheses and to choose the appropriate graph for the dot that does not move. Although they reached the right conclusion and seemed to understand the situation, they were still surprised when they identified a constant function line as the graph that matched the static dot. They continued to explain the correct albeit peculiar result they arrived at regarding the static point, and continued to explore relations between motion and distance-time graphs. By imagining rather than executing motion they learned the two central pieces of knowledge concerning motion: rate of change and directional motion.

While it was expected that students would engage more due to the animation, and would experiment and try out different answers, a noteworthy finding in the demonstrated problem-solving process is the unusually extensive mental work, encompassing an entire cycle of logical argumentation (raising assumptions, deriving conclusions through conjectures and refutations), which eventually led to the solution. Our evidence suggests that interactive tasks offer engaging reading by providing readers with explicit options for manipulating given examples and especially diagrams (Naftaliev & Yerushalmy, 2013). Although characteristics of problem-solving processes of complex challenges cannot be attributed to a single factor, we were able to elicit the central role of the boundaries, designed into interactive tasks, in the processes of interactive learning. Whilst live-learning resources are often empowered by attractive video clips and animation-based demonstrations, we found that important fine tuning should be considered in the design of e-textbook tasks to include 1) a well-planned interactive example that presents the necessary repertoire of examples that would provide cognitive support (such as the conflicting animations in the example above) rather than cognitive load; 2) partial control and partial links between representations that help to capture interesting "views," but also a balance between the experimentations and the analytic phases. More generally, students' interactive learning of algebra occurred whilst reading and solving tasks of interactive text that we studied in a long-term research project. The interactive tasks offered experimentation with multiple-linked-representations that reflect back the student's actions. We have evidence that these tasks led to processes of personalization of the text and to the development of main mathematical ideas within the boundaries designed in the interactive diagrams. The design of the diagrams organized and directed the process of development of the students' knowledge, but the students controlled the task, produced the given presentation of the task, and formulated new questions.

Beyond the challenging problem solving, other interactive scenarios that e-textbooks are often designed to support are *dynamic expositions* (as the one described in Window 4) and *exercises* for practice. Dynamic exposition is designed to present an example and to afford limited user control: for example, dragging a given figure or running a video or simulation are frequent elements used in either teachers' illustrations or students' own first exploration of a new term. Dynamic exposition contributes to the learner's awareness of which features (of an object) make it a specific example, and which features can be varied to form a class of similar or related examples. The widespread use of examples in mathematics textbooks serves as a means of communication and mediation between learners and ideas. If examples are well selected, the variations between examples are the means by which students can distinguish between essential and redundant features (Goldenberg & Mason, 2008, Watson & Shipman, 2008). One of the unique features of the computerized environment lies in the possibility for systematically creating multiple related examples. In contrast, the design assumption in regard to exercises is that they are performed after students have constructed correct (though not necessarily complete) concept images and concept definitions (Tall & Vinner, 1981). The strength of technology in this phase of learning includes the large quantity of exercises generated semi-randomly (random within constraints) for practice, and the different types of feedback that can be designed to support skills practice with understanding, even in domains that traditionally offer rote exercises. The self-generation and analysis of examples and practice with understanding are two central topics currently within the scope of the design of assessment systems that could be embedded within interactive e-textbooks (but are beyond the scope of this chapter).

E-Textbooks and Changes in the Relationship Between Teachers and Textbooks

In the previous section, we discussed ways in which e-textbooks may change students' work. Now, we will consider potential changes to the role of the teacher, and student-teacher interactions, in class and out of class.

We adhere to the theoretical perspective of the documentational approach (Gueudet & Trouche, 2009a). This means that we focus on the interactions between teachers and the resources that they use in/for their teaching. Learning from Pepin & Gueudet (2013) and amending their categories for our use, we categorize teacher e-resources in the following way:

- e-text resources: e.g. e-textbooks, online teacher curricular guidelines, Web sites, and online student worksheets;
- e-based "material" resources: e.g. interactive whiteboards, computer software, applets;
- e-based works and exchanges: e.g. mail exchanges with colleagues, pupils, and parents; pupil work submitted electronically (such as online homework).

According to Adler (2000), a resource is anything likely to *re-source* the work of the teacher. Teachers search for resources, retain some resources and discard others, modify them, associate them, set them up in class—and as they perform this documentational work, they develop a structured *resource system* (Gueudet & Trouche, 2009a).

Understanding the modifications resulting from the e-textbook means understanding how a teacher's integration and appropriation of the e-textbook in the teacher's resource system modifies this system, and the implications for the teacher's work and professional development.

Window 4—Vera's Use of Sésamath E-Textbook

Vera teaches mathematics at a middle school. She uses the Sésamath e-textbook (Window 1), amongst other textbooks, for all her classes. With the video projector, she shows extracts of the e-textbook in class, including lesson texts, animations with Dynamic Geometry software, and interactive exercises.

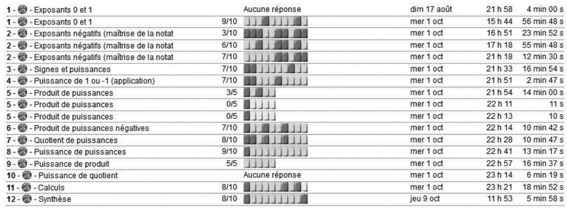
Figure 27.5 Solving an interactive exercise in class.

Vera also programs exercise sessions on LaboMEP (Window 1 and Window 5), which the students can do in class, but mostly complete as homework. She has access to the individual and collective summary of their activity, and uses it to plan remedial activities for selected students.

Her grade 6 students have the Sésamath paper textbook as their common text. However, for this class Vera does not follow the curriculum plan of the Sésamath textbook (which is directly transferred from the official national curriculum). She has her own textbook, *Helice* (English translation: helix), and follows its spiral progression, hence deepening pupils' understanding by revisiting particular topics (Gueudet et al., 2013b).

In a teacher's resource system, e-textbooks are associated with other resources: resources linked with the projection in class (the computer and video projector in the classroom, the screen, sometimes an interactive whiteboard, Dynamic Geometry software); resources linked with the preparation of worksheets to be handed out to students (files on the teacher's computer that are printed out and handed to students). Other resources are linked with out-of-class work: specific sessions that can be programmed for the students, and a Virtual Learning Environment. Considering these changes in the teacher's resource system evidences changes in the teacher's activity and proposes potential further changes (which would require further investigation).

Designing worksheets for students with a summary of the main properties, and "holes" to be filled in (in class) is certainly common practice. This practice can be supported if the teacher has access to the textbook's. doc or. odt files. Similarly, the use of projection devices is frequent, which can also be fostered by the use of an e-textbook, which constitutes, both for the teacher and the students, a shared reference easy to project. The collective solving of exercises by the whole class, and managed by the teacher, does not require an e-textbook. However, students seem more motivated when an interactive exercise is displayed (see Figure 27.5). In terms of out-of-class work, e-textbooks can offer possibilities, for example, for differentiation (attending to individual pupils' needs) and checking work. With particular programs linked to e-textbooks, homework can be checked by the teacher out of class (see Figure 27.6, the summary provided for each pupil by LaboMEP), and the results and formative assessments communicated to individual pupils. Supplementary work can be provided in differentiated form and different homework can be provided to individual students. Moreover, students seem more motivated to work on interactive exercises, and may be more engaged in related homework—at least in the present context of relative novelty of this kind of digital material. Taking into account the needs of individual students, or particular student groups can be



1 - - Exposants 0 et 1		Aucune réponse	dim 17 août	21 h 58	4 min 00 s
1 - - Exposants 0 et 1	9/10		mer 1 oct	15 h 44	56 min 48 s
2 - - Exposants négatifs (maîtrise de la notat	3/10		mer 1 oct	16 h 51	23 min 52 s
2 - - Exposants négatifs (maîtrise de la notat	6/10		mer 1 oct	17 h 18	55 min 48 s
2 - - Exposants négatifs (maîtrise de la notat	7/10		mer 1 oct	21 h 18	12 min 30 s
3 - - Signes et puissances	7/10		mer 1 oct	21 h 33	16 min 54 s
4 - - Puissance de 1 ou -1 (application)	7/10		mer 1 oct	21 h 51	2 min 47 s
5 - - Produit de puissances	3/5		mer 1 oct	21 h 54	14 min 00 s
5 - - Produit de puissances	0/5		mer 1 oct	22 h 11	11 s
5 - - Produit de puissances	0/5		mer 1 oct	22 h 13	10 s
6 - - Produit de puissances négatives	7/10		mer 1 oct	22 h 14	10 min 42 s
7 - - Quotient de puissances	8/10		mer 1 oct	22 h 28	10 min 47 s
8 - - Puissance de puissances	9/10		mer 1 oct	22 h 41	13 min 17 s
9 - - Puissance de produit	5/5		mer 1 oct	22 h 57	16 min 37 s
10 - - Puissance de quotient		Aucune réponse	mer 1 oct	23 h 14	6 min 19 s
11 - - Calculs	8/10		mer 1 oct	23 h 21	18 min 52 s
12 - - Synthèse	8/10		jeu 9 oct	11 h 53	5 min 58 s

Figure 27.6 Following the work of a student in LaboMEP.

considered an important dimension for the quality of mathematics teaching and management of pupil/teacher work. Whilst we do not claim here that these possibilities are actually used by many teachers—this would require further investigation—the e-textbook certainly offers interesting possibilities in this respect. Nevertheless, in the cases we studied (Gueudet & Trouche, 2009a, Gueudet et al., 2013a), we noticed that the use of the e-textbook was linked to the provision of differentiated tasks. In the perspective of documentational genesis we propose that this can be interpreted as knowledge development: using the e-textbook fosters the development of professional knowledge about the management of tasks in a heterogeneous class.

We also observed, in the case of an e-textbook being designed by a group of teachers (e.g. Sésamath), that the users/teachers were able to introduce/suggest modifications to the content, and in this way participate in the design process of the e-textbook. Evidently, not all suggestions by users yield modifications to the content (shared by the Sésamath authors)—in particular because not all the suggestions are relevant, but also because continuous modifications are time-consuming for the authors (Trgalovà & Jahn, 2013). Considering various potential contributions and contributors to the e-textbook, how can the quality and the coherence of an e-textbook be examined and defined? We discuss this in the next section.

E-TEXTBOOKS AND CHANGING RESEARCH CATEGORIES: TOWARDS A RECONCEPTUALIZATION OF TEXTBOOK QUALITY AND COHERENCE

In this chapter we have described the design of e-textbooks, developed a working definition of e-textbooks, and identified three different types/models of e-textbooks. We have also described their use by teachers and students. We have provided evidence that an e-textbook can be collaboratively designed, and that its content can be coauthored, in terms of modifications, by its users. These developments imply major evolutions in how textbooks operate in schools and raise questions about how (a) the "quality"; and (b) the "coherence" of textbooks change as textbooks become e-textbooks. What does mathematical quality/coherence mean, if the teacher can change the e-textbook elements? It is clear that these evolutions require serious consideration, and possibly a reconceptualization of the notions of "quality" and "coherence" with respect to e-textbooks.

A Reconceptualization of Quality

Quality of Textbooks

The quality of textbooks has been the subject of many studies, not least because they play a vital role in educational practice and curricula provide a "crucial link between standards and accountability measures" (Confrey & Stohl, 2004). Evaluation studies can provide information on how effective a particular curriculum is, and for whom and under what conditions. In the United States a National Research Council committee (2004) examined a large array of evaluation studies. The committee did not evaluate the curriculum materials directly (or indeed rank particular programs), neither did it provide criteria for evaluating textbooks, but it identified four types of evaluation studies: (1) content analyses (examining the content of curriculum materials: e.g. accuracy, depth of coverage, logical sequencing of topics, etc.); (2) comparative studies (comparing two or more curricula and their effects on student learning); (3) case studies (documenting how programs and components of curricula of a particular curriculum play out in classrooms); and (4) synthesis studies (summarizing several evaluation studies across a particular curriculum). This work highlights the importance of the quality of curriculum materials.

Concerning more specific studies for the examination of the notion of "quality," selected authors consider a macro level and analyze quality with a more "global" appreciation; e.g. Charalambous, Delaney, Hsu, and Mesa (2010) propose a "horizontal" analysis of the textbook where the textbook "is examined as a whole, as part of technology in the educational system" (p. 119). Studies on this level include those that focus on general textbook characteristics (e.g. Schmidt et al., 1997, Stevenson & Bartsch, 1992).

Other studies consider a micro level; they analyze textbooks for quality, according to their particular aims (e.g. Oesterholm & Bergqvist, 2013). Often, in these studies a single mathematical concept is examined (e.g. Li, 2000, Mesa, 2010), viewing the textbook as an "environment for construction of knowledge" (Herbst, 1995, p. 3). Charalambous and colleagues (2010) would call this a "vertical analysis." They developed a three-layered framework to analyze textbooks (from three countries) to investigate the learning opportunities offered by textbooks. In our view, learning opportunities are closely linked to "knowledge construction" and to "learning mathematics with understanding." During the past decades there has been much concern and discussion about learning opportunities linked to students' conceptual understanding and with respect to students' developing expertise in terms of mathematical thinking, reasoning, and problem-solving (e.g. National Research Council, 1989, Hiebert & Carpenter, 1992, Hiebert et al., 1997).

Quality of Online Resources

Assessing the quality of online resources is a complex issue. Several research works have considered the issue of quality of online resources in mathematics and in science, in particular with a focus on their potential for investigation. The Intergeo project (Kortenkamp et al., 2009) concerns resources using Dynamic Geometry, and the research introduces nine dimensions for studying the quality of such a resource for inquiry: metadata (names of the authors, theme, date etc.), technical aspect, mathematical content, instrumental content, added-value of dynamic geometry, didactical implementation (learning objectives, adaptation to different kinds of pupils etc.), pedagogical implementation (organization in the class, management of different parts etc.), integration in a teaching sequence, and ergonomic aspects (ease of identification of the different texts, links etc.). Moreover, considering the importance of a resource's adequacy with respect to the users' needs, Intergeo has designed an assessment of quality by the users themselves (Trgalovà, Jahn, & Soury-Lavergne, 2009). This assessment can potentially lead to a further design of the resource, by and involving the users. Drawing on this work and on other studies in science education, Bueno-Ravel et al. (2009) propose seven dimensions for assessing the quality of an online resource for inquiry-based science learning and teaching: scientific aspects, scaffolding inquiry aspects, possible customization, ergonomics, choice of media, possible involvement of the users in the resource design, and possibilities offered for collective work.

In line with the move from traditional curriculum resources to online resources, the notion of "resource evaluation" has been replaced by the notion of "resource quality" (see Pepin et al., 2013b). Recent research about the quality of online resources for the teaching and learning of mathematics (Trouche et al., 2013) stresses in particular the importance of the

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ease of *appropriation* of the resource by the user. The involvement of users in the resource design, more generally the possibility of communication between the designers and the users of an online resource, can thus be retained as an important dimension for quality. This dimension can be related with the issue of connectivity (Hoyles et al., 2010), which encompasses in particular the potential for creating virtual communities. We draw on this idea to propose a reconceptualization of quality, in the case of e-textbooks.

Quality of E-Textbooks

It can be assumed that e-textbooks combine the features of textbooks and of online resources. We thus suggest studying the quality of e-textbooks by combining the two perspectives evoked earlier.

Connectivity is certainly an important dimension for the quality of an e-textbook. The potential to create virtual communities, connecting users and users (both teachers and students), as well as users and designers, can be considered as *connectivity at a macro level*. Moreover, at this macro level, the connectivity of the e-textbook can also mean that it actually interacts with other resources via Web links, or on a platform for example (see Window 5). More generally, this **macro level** for connectivity could include the following criteria, which we group under two headings, in order to assess the quality of an e-textbook:

EXISTING CONNECTIONS

- Connections between resources provided by the e-textbook;
- Connections to other resources outside the e-textbook (e.g. Geogebra);
- Connections across grades;
- Connections to the national curriculum.

POTENTIAL CONNECTIONS AFFORDED BY THE E-TEXTBOOK

- Connections between e-textbook and teacher resource system (for synergistic effects);
- Connections in terms of teacher collective work, e.g. connecting teacher resource system in a school, or professional community (see LaboMEP in Window 5);
- Connections between teacher and students;
- Connections to "larger" teacher needs/relevance (e.g. e-textbook as a means to prepare lessons) or pupils (e.g. revision for tests)
- Connections between teachers using the textbook and the authors of the textbook.

Moreover, connectivity can also be considered at a **micro level** concerning specific mathematics content. At this level, it means that the e-textbook offers different kinds of articulated materials, and offers connections and links between different features. We consider these at two levels:

EXISTING CONNECTIONS

- Connections between different topic areas;
- Connections between different semiotic representations (text, figures, static and dynamic), in order to develop a flexible awareness of the different facets of the mathematical content);
- Connections between different software for carrying out the same task, in order to develop a flexible use of technology.

POTENTIAL CONNECTIONS AFFORDED BY THE E-TEXTBOOK

- Connecting different concepts of a same conceptual field (Vergnaud, 1996);
- Connecting different strategies for problem solving;
- Connecting different moments of appropriating a given concept (e.g. spiral progression).

In Window 5 we show selected connections in the case of the Sésamath textbook.

Window 5—Sésamath Textbook Connectivity

SÉSAMATH TEXTBOOKS, CONNECTIVITY AT THE MICRO LEVEL

Each exercise in the Sésamath e-textbooks is connected with "complements." These complements can include, for example, Dynamic Geometry constructions to be done by the students, or with interactive helps (dynamic presentations of a solving method). Thus, we can claim that the Sésamath textbooks offer a lot of possible connections between representations.

At the same time, in previous works (Gueudet et al., 2013b) analyzing the Sésamath grade 6 textbook we have observed that the primary-secondary transition was not really taken into account, since the authors were not familiar with primary school curriculum—and this is an important missing connection, a criterion to retain in assessing quality. We have also noticed that the organization of the book, according to the official curriculum entries, does not favor connections between mathematical topics.

SÉSAMATH TEXTBOOK WITHIN LABOMEP, CONNECTIVITY AT THE MACRO LEVEL

The Sésamath e-textbooks are part of a wider Virtual Learning Environment, LaboMEP (see Window 1). Once connected in LaboMEP, the teacher has access to all Sésamath resources (left column, Figure 27.7), including the e-textbooks. The teacher can choose extracts of the e-textbook such as lessons or exercises and insert them into a personal session she/he is building (central column). All his/her sessions are recorded and saved in his/her personal space (right column). Each session can include extracts from the textbook and also online exercises, geometric constructions with Dynamic Geometry software, or even external Web links. Thus the Sésamath textbooks are connected, via LaboMEP, to a very large set of potential resources. The teacher can also share his/her sessions with other colleagues, groups of colleagues can design *a shared resources repertoire* that appears as shared resources in the left column; s/he

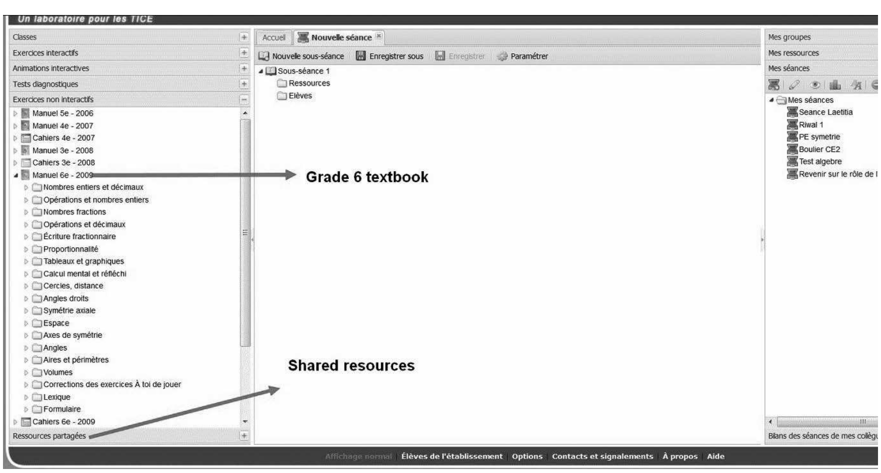

Figure 27.7 LaboMEP, teacher environment for programming sessions.

can also communicate with Sésamath members. But LaboMEP is above all a communication tool between the teacher and his/her students, via the sessions programmed by the teacher for them.

Connectivity is considered here as a feature of digital content. Nevertheless, in particular at the micro level, it suggests the possibility of defining a new, more general notion of connectivity. Hiebert et al. (1997) argue that "learning mathematics with understanding" is linked to "making connections." "Connectedness" is not a new idea in the field of education, or indeed in mathematics education. Hiebert and Carpenter (1992), for example, believe that it is essential to make connections in mathematics if one intends to develop mathematical understanding. They emphasize the importance of learning with and for understanding. According to them, understanding can be defined as:

> [T]he way information is represented and structured. A mathematical idea or fact is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections.
>
> (Hiebert & Carpenter, 1992, p. 67)

The digital means can be used, within an e-textbook, to provide a network of representations. Nevertheless, such possibilities also exist in traditional textbooks. These textbooks can also foster the development of connections by the students, by offering rich mathematical tasks. It is indeed important that students have "frequent opportunities to engage in dynamic mathematical activity that is grounded in rich, worthwhile mathematical tasks" (Henningsen & Stein, 1997, p. 525), and it is argued that this is an essential component for understanding in order for connections to be made by the learner (Hiebert et al., 1997). Thus, we suggest that the idea of connectivity at the micro level could be extended beyond the case of digital textbooks, to define a kind of "mathematical connectivity," which could be an essential feature for quality of any textbook. This requires nevertheless further studies, complementing this chapter.

Coherence of the Design and Coherence for Use

Similar to our considerations earlier, we propose to reconsider/conceptualize the notion of "coherence." The coherence of a textbook can encompass many different aspects, as pointed out in Gueudet et al. (2013a): correctness of the mathematical content; consistency with the national curriculum; correctness of the sequencing of notions, of the articulation between the course and the exercises; etc. Yerushalmy and Chazan (2008), drawing on Tall (2002), propose that coherence involves how the character of the objects of study is handled.

We want to emphasize here three specific dimensions of e-textbooks that should be taken into account to evaluate their coherence.

1. *Initial design coherence*: In contrast to traditional textbooks that are often designed by a small number/team of experts (e.g. a coordinate designer chooses a number of "acquainted" colleagues; see Gueudet et al., 2013b), e-textbooks are commonly designed by a larger collective (e.g. by teachers working together, or teachers and inspectors/teacher educators working together). In the case of a small expert teams, these typically meet on a regular basis to insure the coherence of the process of design, whereas a large team of designers (as in the Sésamath case), not chosen but volunteered, is not likely to ever meet as a whole, but divides the design into compartmentalized tasks for each subteam (e.g. division according to topic areas). Hence the strategies for "insuring coherence" become crucial for e-textbooks with large designer teams. In this case, coherence can also mean that "the individual mathematical intentions of the individual authors, their epistemological stances, are well coordinated" (Gueudet et al., 2013a, p. 328). Assessing the coherence of such a textbook requires identifying these intentions

and stances to evaluate their coordination, the production, during the design process, of a shared intention shaping the choices of content.

2. *Product coherence* (strongly linked to quality): Yerushalmy and Chazan (2008) show how technological tools (e.g. graphing tools, symbol manipulators, spreadsheets) carry with them particular orientations toward the representations of function used in algebra classrooms. Using the notion of "discontinuities" (Tall, 2002), Yerushalmy and Chazan suggest that algebra curricula must negotiate the discontinuities between these symbol systems. Thus, the rollout of students' use of technological tools can be in conflict with or support textbook expositions. This is a different sense of "coherence" (sequencing of topics and uses of technology) compared to ones used with traditional print curricula: for example, graphing tools can offer different affordances in terms of different views of equations that may or may not be in tension with textbook exposition. Linking to this, it can be argued that e-textbooks (which are supported by and designed for particular technological tools) can provide a particular/different coherence, combining different views and practices (for a particular topic area), different sequencing of ideas within that topic area, and making accessible (through technology) to learners different ideas, perhaps by using different representations and hence different views or notions. Perhaps the integrated design of software and exposition in e-textbook environments has the potential to lead to greater coherence.
3. *Coherence-in-use*: In terms of implications for learning, the most important issue linked to coherence is likely to be the "coherence" of what teachers propose to their students, drawing on the textbook/curriculum materials (Pepin, 2012). As several authors point out (e.g. Shield & Dole, 2013, Lloyd, 2009) analyzing textbooks and other curriculum materials (e.g. in professional development activities) provide only limited support for the development of teachers' pedagogic practice, and coherence-in-use is dependent on how teachers actually use the resources. In many cases "teacher guides" provide guidelines of how authors/designer teams consider the use of the textbooks, and these guidelines link to the designers' pedagogical intentions.

We explain and exemplify the three dimensions in Window 6. The *initial design coherence* might be studied drawing on texts actually inserted in the book, or associated with it, which present the authors' stance and their didactical objectives.

Window 6—Sésamath Textbook Coherence, the Authors Didactical Choices

The Sésamath e-textbook for grade 6 starts with a classical table of contents complemented by the line of buttons above the text, labeled by the chapter numbers, and displaying the title of the chapter when the mouse is on the button. Then two pages present choices of structure (Figure 27.8), in terms of nature of activities proposed. Two other pages present the numerical complement present on each page. Three pages propose advice to the pupils, for the use of the textbook. Thus, we contend that an autonomous use of the textbook by pupils seems to be one objective of the authors.

The chapters correspond to the titles of the official curriculum (e.g. fractions, symmetry), except for two chapters dedicated to the articulation with primary school (N0 about integers and G0 about geometry), and one "synthesis chapter" that proposes more elaborate problems (for example "complex tasks," as required by the official curriculum). There is also a dictionary (*lexique*) at the "end" of the book: pupils can refer to it, if they meet an expression or word they do not know. This is particularly useful for mathematical language/expressions. The organization of each chapter is classical (except for the synthesis chapter): introductory activities, course, direct application exercises, exercises, multiple choice (self-testing) and mathematical games. A very important choice of the authors (also aligned with the institutional

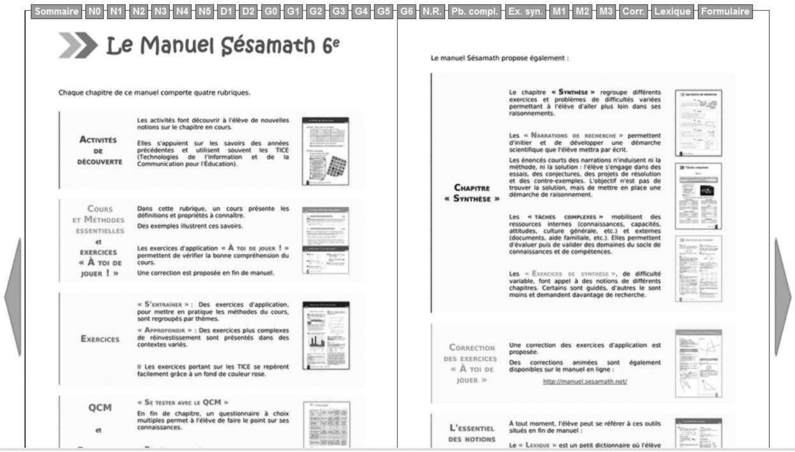

Figure 27.8 Sésamath grade 6 e-textbook, explanations about the structure of the textbook.

expectations) is an important use of information and communications technology (ICT), in the introductory activities and in exercises (identified by a special color).

E-textbooks can also be analyzed by studying the design process itself and following the choices of content and structure: see Sabra & Trouche (2011), or Gueudet, Sabra, Pepin & Trouche (in press). However, such studies would require specific design and data collection strategies. One way to study the design of an e-textbook would be to use design-based methods, because the e-text will be a document that is constantly changing based on an iterative process. Further, the coherence of the design can be studied by analyzing (a) the book's content, searching for didactic choices common to different sections and different components of the book; and (b) the "e-traces" left by authors/author team: e.g. traces of the design process and traces of the "design contract" (what did the designer team do to ensure the coherence of the e-textbook?) At this macro level, assessing coherence requires to consider a collection of tasks/text, typically a whole textbook for a given grade. Leaning on the works of the 22nd ICMI study on task design (Margolinas, 2013, Theme C), coherence of such published collections is related to "the overall philosophy of the collection . . . beliefs about the nature of mathematics and what it means to do or learn mathematics vary and are manifested in the nature of the texts and tasks" (p. 13).

Product coherence and *coherence-in-use* are closely connected. Whilst an analysis in terms of *product coherence* may show the e-textbook's affordances, for example, to favor a particular learning path (e.g. a hyperlink proposes another notion of the hyperlinked word), the final use of the e-textbook also depends on the teachers' choices (e.g. of tasks in the book). Coherence clearly depends on these two connected factors: the affordances of the book (e.g. the learning paths/trajectories) and the teacher's particular use. In the recent research literature (e.g. Cohen, Raudenbush, & Loewenberg Ball, 2003, Charalambous et al., 2010, Cai, Wang, & Ding, 2014) the term *coherence* is often linked to the *coherence of instruction* (e.g. Hiebert et al., 2003, Stigler & Hiebert, 1999), which was seen more prominently in Japanese lessons, for example, than in U.S. classroom instruction—this links to our notion of coherence-in-use. Moreover, and linking coherence-in-use to product coherence, coherence of curriculum resources/materials and textbooks, as we see it, is related to sequencing of topics (for instructional purposes), for example learning trajectory/ies (e.g. "curriculum coherence," Schmidt & Huang, 2012), to instructional issues. Hence, in our discussion of coherence, we refer more particularly to coherence of e-textbooks with respect to sequencing of topics, learning trajectories, and instructional issues.

Recently, Cai et al. (2014) conducted a study in which they investigated Chinese and U.S. mathematics teachers' views about instructional coherence. They distinguish between "surface" (e.g. designed in advance of the lesson) and "real" coherence (achieved in the lesson) in instruction (as would apply for traditional as well as e-textbooks). Interestingly, studying textbooks and student thinking for lesson preparation were two of the most-mentioned approaches by Chinese teachers in order to achieve coherence at the lesson design stage. Whilst emerging classroom events may disrupt this surface coherence, it is still part of the development of real coherence. The study points to two kinds of instructional coherence: "predesigned"; and "emerging"/"situational," and teachers are said to need both for real coherence. This view goes hand-in-hand with our understandings of coherence in e-textbooks, which is linked to instructional issues, and it suggests to reconceptualize coherence from a "dynamic, co-constructive and situational perspective" (Cai et al., 2014, p. 25).

CONCLUSION

In this chapter, we worked towards a definition of e-textbooks, identified and described three different model types, and analyzed their design and use. Moreover, we have argued that these new technological artifacts have the potential to change traditional relationships between student and textbook, teacher and textbook, and between students and teachers, as well as categories used by educational researchers. The e-textbooks' affordances offer new possibilities for student and teacher agency. This raises questions about differences and similarities of student/teacher agency, and how they play out at different levels/stages of mathematics learning. At the university level, teachers (and perhaps students) have the ability—and in many countries the possibility—to develop their own teaching/learning paths through the topic area, and they are likely to benefit from the collaboration opportunities offered by e-textbooks. The situation may be very different at the primary school level, as primary-school teachers are often not specialized in mathematics and are more likely to rely on the textbook (e.g. to be "assured" that their teaching is in line with governmental guidance, in order to provide an "accepted" coherence in their learning paths/opportunities for pupils). We propose this first limitation of the potential impact of e-textbooks: exploiting their potential requires a certain level of expertise, and the provision of specific professional development programs to support the development of this expertise.

In his article, Friesen (2013) leans on Kuhn (1962) to consider that textbooks, in line with their traditional pedagogic and didactic function, tend to present accumulation of knowledge rather than shifts of paradigms. Friesen (2013) asks: "What is it that textbooks provide pedagogically and epistemologically, besides a reminder of the past?" (p. 498). Textbooks cannot provide access to the process of production of the scientific knowledge/concepts they present. However, students and teachers can—collectively or individually—work and elaborate on e-textbook contents. This certainly changes the relationships between students, teachers, and textbooks. But does it really affect the teaching and learning processes, the educational institution at large? Friesen (2013) sees the textbook as "an evolving pedagogical form," and considers that its components changed "not so much through technological innovation as in synchrony with larger cultural and epistemological developments" (p. 498). With reference to e-textbooks, this begs the question whether the e-textbook changes the user, or the user's intentions/epistemological beliefs change the e-textbook. In line with our research results, we consider that any change happens *in interaction* between user/s and the book (see Yerushalmy & Chazan, 2002, 2008; Gueudet et al., 2012), hence our phrasing of "design-in-use" (Pepin et al., 2013a).

At the same time we concur with Friesen (2013) that textbook features "provide an indispensable animating didactic function," and that textbooks are an "evolving pedagogical form," to point to the synergy between e-textbooks and users. Having said that, we consider that we have contributed to an understanding of e-textbooks by comparing "old" and "new" designs of e-textbooks: for the moment we investigate new designs of e-textbooks as compared to previous designs. That means that we view the developments as stages of evolution rather than revolution. We consider that we do not yet have sufficient examples of research studies that have examined the teaching, learning, and designs of e-textbook communities, and hence the new ideas and concepts, including terminologies, are still vague. However, we have attempted to provide reconceptualizations of categories and concepts that we propose here for researchers, practitioners, and policy makers: from static quality to dynamic quality; from design (and implementation) to design-in-use; from teachers making connections (for their students), to e-textbooks making/providing connections for teachers (also amongst teachers/colleagues/principals) and students (and parents); from static coherence to coherence-in-use; from individual design and use to collective design and use—all due to the affordances of a changing digital environment and the accompanying (potentially) changing practices and mindsets.

If teachers become more engaged in the development of e-textbooks, they will develop, and redevelop, their own categories for the change/development of the e-textbook, and more generally for the design of their teaching. Hence, we consider that a new paradigm in terms of teaching is accompanied by, and necessitates, the development/design of new books, tools, and pedagogically innovative ideas.

However, the existence of new artifacts is not enough. Technology had always created excessive expectations in the field of education, but technology alone has not yet produced sustained changes in education. There are, however, several examples where a material change in object resulted in the development of new paradigms. This is what happened to the picture album when it was transformed into a digital folder, causing pictures to play a completely new visual, social, semiotic role. Music (as text), after it was converted to digital formats, changed from a passive market commodity into a creative, engaged process. Even if at present the only noticeable benefit of digital textbooks is the fact that the object is cheap, efficient, and easy to use anywhere, the examples reviewed in this chapter (see for example the Sésamath experience) suggest that more substantial challenges are just around the corner, opening new ways for teacher collaboration and creativity.

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