processes within system 2. However, it should be emphasized that all the above concept developments do not occur simultaneously. They also do not occur in all students who study mathematics. One should take many mathematics courses and solve a lot of mathematical problems in order to achieve that level. Those who do it should have special interest in mathematics or what can be called mathematical curiosity. It requires, what some people call, a mathematical mind. Is it genetic (Devlin 2000) or acquired? At this point we have reached a huge domain of psychological research which is far beyond the scope of this particular encyclopedic issue.

Cross-References

- ▶ Abstraction in Mathematics Education
- ▶ Concept Development in Mathematics Education
- ▶ Critical Thinking in Mathematics Education
- ▶ Intuition in Mathematics Education
- ▶ Mathematical Proof, Argumentation, and Reasoning
- ▶ Metacognition
- ▶ Problem-Solving in Mathematics Education
- ▶ Theories of Learning Mathematics
- ▶ Values in Mathematics Education
- ▶ Zone of Proximal Development in Mathematics Education

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C

Constructivism in Mathematics Education

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Keywords

Epistemology · Social constructivism · Radical constructivism · Knowledge · Reality · Truth · Objectivity

Background

Constructivism is an epistemological stance regarding the nature of human knowledge, having roots in the writings of Epicurus, Lucretius, Vico, Berkeley, Hume, and Kant. Modern constructivism also contains traces of pragmatism (Peirce, Baldwin, and Dewey). In mathematics education the greatest influences are due to Piaget, Vygotsky, and von Glasersfeld. See Confrey and Kazak (2006) and Steffe and Kieren (1994) for related historical accounts of constructivism in mathematics education.

There are two principle schools of thought within constructivism: radical constructivism (some people say individual or psychological) and social constructivism. Within each there is also a range of positions. While radical and social constructivism will be discussed in a later section, it should be noted that both schools are grounded in a strong skeptical stance regarding reality and truth: *Knowledge cannot be thought of as a copy of an external reality, and claims of truth cannot be grounded in claims about reality.*

The justification of this stance toward knowledge, truth, and reality, first voiced by the skeptics of ancient Greece, is that to verify that one's knowledge is correct, or that what one knows is true, one would need access to reality by means other than one's knowledge of it. The importance of this skeptical stance for mathematics educators is to remind them that students have their own mathematical realities that teachers and researchers can understand only via models of them (Steffe et al. 1983, 1988).

Constructivism did not begin within mathematics education. Its allure to mathematics educators is rooted in their long evolving rejection of Thorndike's associationism (Thorndike 1922; Thorndike et al. 1923) and Skinner's behaviorism (Skinner 1972). Thorndike's stance was that learning happens by forming associations between stimuli and appropriate responses. To design instruction from Thorndike's perspective meant to arrange proper stimuli in a proper order and have students respond appropriately to those stimuli repeatedly. The behaviorist stance that mathematics educators found most objectionable evolved from Skinner's claim that all human behavior is due to environmental forces. From a behaviorist perspective, to say that children participate in their own learning, aside from being the recipient of instructional actions, is nonsense. Skinner stated his position clearly:

> Science . . . has simply discovered and used subtle forces which, acting upon a mechanism, give it the direction and apparent spontaneity which make it seem alive. (Skinner 1972, p. 3)

Behaviorism's influence on psychology, and thereby its indirect influence on mathematics education, was also reflected in two stances that were counter to mathematics educators' growing awareness of learning in classrooms. The first stance was that children's learning could be studied in laboratory settings that have no resemblance to environments in which learning actually happens. The second stance was that researchers could adopt the perspective of a universal knower. This second stance was evident in Simon and Newell's highly influential information processing psychology, in which they separated a problem's "task environment" from the problem solver's "problem space."

> We must distinguish, therefore, between the task environment – the omniscient observer's way of describing the actual problem "out there" – and the problem space – the way a particular subject represents the task in order to work on it. (Simon and Newell 1971, p. 151)

Objections to this distinction were twofold: Psychologists considered themselves to be Simon and Newell's omniscient observers (having access to problems "out there"), and students' understandings of the problem were reduced to a subset of an observer's understanding. This stance among psychologists had the effect, in the eyes of mathematics educators, of blinding them to students' ways of thinking that did not conform to psychologists' preconceptions (Thompson 1982; Cobb 1987). Erlwanger (1973) revealed vividly the negative consequences of behaviorist approaches to mathematics education in his case study of a successful student in a behaviorist individualized program who succeeded by inventing mathematically invalid rules to overcome inconsistencies between his answers and an answer key.

The gradual release of mathematics education from the clutches of behaviorism, and infusions of insights from Polya's writings on problem solving (Polya 1945, 1954, 1962), opened mathematics education to new ways of thinking about student learning and the importance of student thinking. Confrey and Kazak (2006) described the influence of research on problem solving, misconceptions, and conceptual development of mathematical ideas as precursors to the emergence of constructivism in mathematics education.

Piaget's writings had a growing influence in mathematics education once English translations became available. In England, Skemp (1961, 1962) championed Piaget's notions of schema, assimilation, accommodation, equilibration, and reflection as ways to conceptualize students' mathematical thinking as having an internal coherence. Piaget's method of clinical interviews also was attractive to researchers of students' learning. However, until 1974 mathematics educators were interested in Piaget's writings largely because they thought of his work as "developmental psychology" or "child psychology," with implications for children's learning. It was in 1974, at a conference at the University of Georgia, that Piaget's work was recognized in mathematics education as a new field, one that leveraged children's cognitive development to study the growth of knowledge. Smock (1974) wrote of *constructivism's* implications for instruction, not *psychology's* implications for instruction. Glasersfeld (1974) wrote of Piaget's genetic epistemology as a theory of knowledge, not as a theory of cognitive development. The 1974 Georgia conference is the first occasion this writer could find where "constructivism" was used to describe the epistemological stance toward mathematical knowing that characterizes constructivism in mathematics education today.

Acceptance of constructivism in mathematics education was not without controversy. Disputes sometimes emerged from competing visions of desired student learning, such as students' performance on accepted measures of competency (Gagné 1977, 1983) versus attendance to the quality of students' mathematics (Steffe and Blake 1983), and others emerged from different conceptions of teaching effectiveness (Brophy 1986; Confrey 1986). Additional objections to constructivism were in reaction to its fundamental aversion to the idea of truth as a correspondence between knowledge and reality (Kilpatrick 1987).

Radical and Social Constructivism in Mathematics Education

Radical constructivism is based on two tenets: "(1) Knowledge is not passively received but actively built up by the cognizing subject; (2) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality" (Glasersfeld 1989, p. 114). Glasersfeld's use of "radical" is in the sense of fundamental – that cognition is "a constitutive activity which, alone, is responsible for every type or kind of structure an organism comes to know" (Glasersfeld 1974, p. 10).

Social constructivism is the stance that history and culture precede and preform individual knowledge. As Vygotsky famously stated, "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first *between* people . . ., then *inside* the child" (Vygotsky 1978, p. 57).

The difference between radical and social constructivism can be seen through contrasting interpretations of the following event. Vygotsky (1978) illustrated his meaning of *internalization* – "the internal reconstruction of an external operation" – by describing the development of pointing:

> The child attempts to grasp an object placed beyond his reach; his hands, stretched toward that object, remain poised in the air. His fingers make grasping movements. At this initial state pointing is represented by the child's movement, which seems to be pointing to an object – that and nothing more. When the mother comes to the child's aid and realizes his movement indicates something, the situation changes fundamentally. Pointing becomes a gesture for others. The child's unsuccessful attempt engenders a reaction not from the object he seeks but *from another person* [sic]. Consequently, *the primary meaning of that unsuccessful grasping movement is established by others* [italics added]. (Vygotsky 1978, p. 56)

Vygotsky clearly meant that meanings originate in society and are transmitted via social interaction to children. Glasersfeld and Piaget would have listened agreeably to Vygotsky's tale – until the last sentence. They instead would have described the child as making a connection between his attempted grasping action and someone fetching what he wanted. Had it been the pet dog bringing the desired item, it would have made little difference to the child in regard to the practical consequences of his action. Rather, the child realized, in a sense, "Look at what I can make others do with this action." This interpretation would fit nicely with the finding that adults mimic infants' speech abundantly (Fernald 1992; Schachner and Hannon 2011). Glasersfeld and Piaget might have thought that adults' imitative speech acts, once children recognize them as imitations, provide occasions for children to have a sense that they can influence actions of others through verbal behavior. This interpretation also would fit well with Bauersfeld's (1980, 1988, 1995) understanding of communication as a reflexive interchange among mutually oriented individuals: "*The [conversation] is constituted at every moment through the interaction of reflective subjects*" (Bauersfeld 1980, p. 30 italics in original).

Paul Ernest (1991, 1994, 1998) introduced the term social constructivism to mathematics education, distinguishing between two forms of it. One form begins with a radical constructivist perspective and then accounts for human interaction in terms of mutual interpretation and adaptation (Bauersfeld 1980, 1988, 1992). Glasersfeld (1995) considered this as just radical constructivism. The other, building from Vygotsky's notion of cultural regeneration, introduced the idea of mathematical objectivity as a social construct.

Social constructivism links subjective and objective knowledge in a cycle in which each contributes to the renewal of the other. In this cycle, the path followed by new mathematical knowledge is from subjective knowledge (the personal creation of an individual), via publication to objective knowledge (by intersubjective scrutiny, reformulation, and acceptance). Objective knowledge is internalized and reconstructed by individuals, during the learning of mathematics, to become the individuals' subjective knowledge. Using this knowledge, individuals create and publish new mathematical knowledge, thereby completing the cycle. (Ernest 1991, p. 43).

Ernest focused on objectivity of adult mathematics. He did not address the matter of how children's mathematics comes into being or how it might grow into something like an adult's mathematics.

Radical and social constructivists differ somewhat in the theoretical work they ask of constructivism. Radical constructivists concentrate on understanding learners' mathematical realities and the internal mechanisms by which they change. They conceive, to varying degrees, of learners in social settings, concentrating on the sense that learners make of them. They try to put themselves in the learner's place when analyzing an interaction. Social constructivists focus on social and cultural mathematical and pedagogical practices and attend to individuals' internalization of them. They conceive of learners in social settings, concentrating, to various degrees, on learners' participation in them. They take the stances, however, of an observer of social interactions and that social practices predate individuals' participation.

Conflicts between radical and social constructivism tend to come from two sources: (1) differences in meanings of truth and objectivity and their sources and (2) misunderstandings and miscommunications between people holding contrasting positions. The matter of (1) will be addressed below. Regarding (2), Lerman (1996) claimed that radical constructivism was internally incoherent: How could radical constructivism explain agreement when persons evidently agreeing create their own realities? Steffe and Thompson (2000a) replied that interaction was at the core of Piaget's genetic epistemology and thus the idea of intersubjectivity was entirely coherent with radical constructivism. The core of the misunderstanding was that Lerman on the one hand and Steffe and Thompson on the other had different meanings for "intersubjectivity." Lerman meant "agreement of meanings" – same or similar meanings. Steffe and Thompson meant "nonconflicting mutual interpretations," which might actually entail nonagreement of meanings of which the interacting individuals are unaware. Thus, Lerman's objection was valid relative to the meaning of intersubjectivity he presumed. Lerman on one side and Steffe and Thompson on the other were in a state of intersubjectivity (in the radical constructivist sense) even though they publicly disagreed. They each presumed they understood what the other meant when in fact each understanding of the other's position was faulty.

Other tensions arose because of interlocutors' different objectives. Some mathematics educators focused on understanding individual's mathematical realities. Others focused on the social context of learning. Cobb et al. (1992) diffused these tensions by refocusing discussions on the work that theories in mathematics education must do – they must contribute to our ability to improve the learning and teaching of mathematics. Cobb et al. first reminded the field that, from any perspective, what happens in mathematics classrooms is important for students' mathematical learning. Thus, a theoretical perspective that can capture more, and more salient, aspects for mathematics learning (including participating in practices) is the more powerful theory. With a focus on the need to understand, explain, and design events within classrooms, they recognized that there are indeed social dimensions to mathematics learning and there are psychological aspects to participating in practices and that researchers must be able to view classrooms from either perspective while holding the other as an active background: "[W]e have proposed the metaphor of mathematics as an evolving social practice that is constituted by, *and does not exist apart from*, the constructive activities of individuals" (Cobb et al. 1992, p. 28, italics added).

Cobb et al.'s perspective is entirely consistent with theories of emergence in complex systems (Schelling 1978; Eppstein and Axtell 1996; Resnick 1997; Davis and Simmt 2003) when taken with Maturana's statement that "anything said is said by an observer" (Maturana 1987). Practices, as stable patterns of social interaction, exist in the eyes of an observer who sees them. The theoretician who understands the behavior of a complex system as entailing simultaneously both microprocesses and macrobehavior is better positioned to affect macrobehavior (by influencing microprocesses) than one who sees just one or the other. It is important to note that this notion of emergence is not the same as Ernest's notion of objectivity as described above.

Truth and Objectivity

Radical constructivists take the strong position that children have mathematical realities that do not overlap an adult's mathematics (Steffe et al. 1983; Steffe and Thompson 2000b). Social constructivists (of Ernest's second type) take this as pedagogical solipsism.

The implications of [radical constructivism] are that individual knowers can construct truth that needs no corroboration from outside of the knower, making possible any number of "truths." Consider the pedagogical puzzles this creates. What is the teacher trying to teach students if they are all busy constructing their own private worlds? What are the grounds for getting the world right? Why even care whether these worlds agree? (Howe and Berv 2000, pp. 32–33).

Howe and Berv made explicit the social constructivist stance that there is a "right" world to be got – the world of socially constructed meanings. They also revealed their unawareness that, from its very beginning, radical constructivism addressed what "negotiation" could mean in its framework and how stable patterns of meaning could emerge socially (Glasersfeld 1972, 1975, 1977). Howe and Berv were also unaware of the notion of *epistemic subject* in radical constructivism – the mental construction of a nonspecific person who has particular ways of thinking (Beth and Piaget 1966; Glasersfeld 1995). A teacher need not attend to 30 mathematical realities with regard to teaching the meaning of fractions in a class of 30 children. Rather, she need only attend to perhaps 5 or 6 epistemic children and listen for which fits the ways particular children express themselves (Thompson 2000).

A Short List: Impact of Constructivism in Mathematics Education

- Mathematics education has a new stance toward learners at all ages. One must attend to learner's mathematical realities, not just their performance.
- Current research on students' and teachers' thinking and learning is largely consistent with constructivism – to the point that articles rarely declare their basis in constructivism. Constructivism is now taken for granted.
- Teaching experiments (Cobb and Steffe 1983; Cobb 2000; Steffe and Thompson 2000b) and design experiments (Cobb et al. 2003) are vital and vibrant methodologies in mathematics education theory development.
- Conceptual analysis of mathematical thinking and mathematical ideas is a prominent and

widely used analytic tool (Smith et al. 1993; Glasersfeld 1995; Behr et al. 1997; Thompson 2000; Lobato et al. 2012).
- What used to be thought of as *practice* is now conceived as *repeated experience*. Practice focuses on repeated behavior. Repeated experience focuses on repeated reasoning, which can vary in principled ways from setting to setting (Cooper 1991; Harel 2008a, b).
- Constructivism has clear and operationalized implications for the design of instruction (Confrey 1990; Simon 1995; Steffe and D'Ambrosio 1995; Forman 1996; Thompson 2002) and assessment (Carlson et al. 2010; Kersting et al. 2012).

Cross-References

► Constructivist Teaching Experiment
► Misconceptions and Alternative Conceptions in Mathematics Education
► Sociomathematical Norms in Mathematics Education

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Constructivist Teaching Experiment

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Constructivism · Methodology · Teaching experiment · Instrumental understanding

Introduction

The constructivist is fully aware of the fact that an organism's conceptual constructions are not fancy-free. On the contrary, the process of constructing is constantly curbed and held in check by the constraints it runs into. (Ernst von Glasersfeld 1990, p. 33).

The constructivist teaching experiment emerged in the United States circa 1975 (Steffe et al. 1976) in an attempt to understand children's numerical thinking and how that thinking might change rather than to rely on models that were developed outside of mathematics education for purposes other than educating children (e.g., Piaget and Szeminska 1952; McLellan and Dewey 1895; Brownell 1928). The use of the constructivist teaching experiment in the United State was buttressed by versions of the teaching experiment methodology that were being used already by researchers in the Academy of Pedagogical Sciences in the then Union of Soviet Socialist Republics (Wirszup and Kilpatrick 1975–1978). The work at the Academy of Pedagogical Sciences provided academic respectability for what was then a major departure in the practice of research in mathematics education in the United States, not only in terms of research methods but more crucially in terms of the research orientation of the methodology. In El'konin's (1967) assessment of Vygotsky's (1978) research, the essential function of a teaching experiment is the production of models of student thinking and changes in it:

Unfortunately, it is still rare to meet with the interpretation of Vygotsky's research as modeling, rather than empirically studying, developmental processes. (El'konin 1967, p. 36).

Similarly, the primary purpose of constructivist teaching experiments is to construct explanations of students' mathematical concepts and operations and changes in them. Without experiences of students' mathematics afforded by teaching, there would be no basis for coming to understand the mathematical concepts and operations students construct or even for suspecting that these concepts and operations may be distinctly different from those of teacher/researchers. The necessity to attribute mathematical concepts and operations to students that are independent of those of teacher/researchers has been captured by Ackermann (1995) in speaking of human relations:

In human relations, it is vital to attribute autonomy to others and to things—to celebrate their existence independently from our current interaction with them. This is true even if an attribution (of existence) is a mental construct. We can literally rob others of their identity if we deny them an existence beyond our current interests (p. 343).

Students' mathematical concepts and operations constitute first-order models, which are models that students construct to organize, comprehend, and control their own experience (Steffe et al. 1983, p. xvi). Through a process of *conceptual analysis* (von Glasersfeld 1995), teacher/researchers construct models of students'