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5. DIGITAL CURRICULUM RESOURCES IN/FOR MATHEMATICS TEACHER LEARNING

A Documentational Approach Perspective

The relations between teachers' interactions with digital resources and teacher education have been the focus of mathematics education research for many years. This interest has been enhanced by the abundance of digital curriculum resources available for teachers and teacher educators; this is likely to lead to new phenomena in terms of teacher learning. The theoretical frame of the documentational approach to didactics has been developed to study these new phenomena. Drawing on selected research works, we illustrate the use of this approach to study teacher (and teacher educator) learning with digital resources in different settings. We consider settings where the aim of teacher education through the use of digital resources is explicit: the offer (by educational authorities) of educative resources, digital platforms in particular; or teacher education programs using digital resources. We also study the consequences, in terms of teacher learning, of daily interactions with digital resources: in individual teacher's work, and their work in collectives. These interactions are intrinsically linked to teacher design. Digital resources offer new means for teacher design, which we regard as part of teacher documentation work. We claim that teachers require help and support to exploit this potential.

INTRODUCTION

Reform in mathematics teacher education, the rise of digital learning, and the abundance of digital/technology resources represent growing movements in our countries' education policies and practice. The digital "revolution" is transitioning our schools and universities from paper-rich to technology-and-media-rich learning environments. In the midst of these changes, a big issue arises: technology and digital resources in schools and universities can either accelerate the momentum in mathematics (teacher) education, or undermine that momentum. For example, technology can help students visualize and develop deeper understandings in mathematics, whilst teachers gain deeper insights into students' cognition and share their professional growth with a web-connected community. At the same time technology can "water down" mathematics into competitive, drill and practice games for students, whilst relegating teachers to the role of computer "assistants" who are

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increasingly disengaged from their role in helping students to learn and grow in their understandings and appreciation of mathematics.

For clarification, we make a distinction between research on educational technology, and that on digital curriculum resources. Leaning on Pepin, Choppin, Ruthven, and Sinclair (2017), we see the main differences as being the particular attention that research on digital curriculum resources pays to:

> (1) the aims and content of teaching and learning mathematics; (2) the teacher's role in the instructional design process (i.e., how teachers select, revise, and appropriate curriculum materials); (3) students' interactions with the materials in terms of how they navigate learning experiences within a digital environment; (4) the impact of digital curriculum resources in terms of how the scope and sequence of mathematical topics are navigated by teachers and students; (5) the educative potential of digital curriculum materials in terms of how teachers develop capacity to design pedagogic activities. (Pepin et al., 2017, p. 674)

"Curriculum resources" (this term we use as equivalent to "curriculum materials") is an elastic term, ranging from one-off worksheets to a full-blown curriculum scheme/program. However, as curriculum resources digital curriculum resources abide by the "guidelines" for any curriculum resource: it is meant/designed (by the teacher/teacher educator) to be used with a clear educational aim, which in turn is linked to particular curriculum specifications (e.g., age level; curricular topic area). As such, digital curriculum resources are distinct from other types of digital instructional tools or educational software programs, whilst at the same time making use of different types of digital tools and software, and they often incorporate the dynamic features of digital technologies.

The links between teachers' interactions with technology and digital resources and teacher education have been the focus of mathematics education research for some time. For example, Musley, Lamndin, and Koc (2003) have studied a variety of teacher education programs; they distinguish between three types of use of technology. Firstly, the use in teacher education programs of video or other kinds of multimedia resources; second, the use of the Internet or other digital communication means; and third, the use of mathematical software. Naturally the same teacher education program can combine several types of use, for example teacher education programs aiming to support the use of technology by teachers can use themselves videos. Starting from Musley et al.'s (2003) classification, Grugeon, Lagrange, and Jarvis (2010) have proposed to refine it. They claim that the choices for the use of technology in teacher education programs are governed by different views:

1. Views concerning the implementation of technology. Grugeon et al. (2010) consider that these views can be organised along two axes: the contribution of technology considered by the program (ranging from learning improvement to questions about integration); the use of technology by the program itself, ranging from use for communication to preparation of the participants for classroom use;

2. Views about changes in teaching practices, resulting from technology use in class;
3. Views about how to prepare teachers, with again two axes: short term vs long term, and professional proficiency (from classroom teaching skills to professional knowledge).

Teacher education programs can thus be classified according to their position regarding these three different views. According to Grugeon et al. (2010), further research could link this classification to the effectiveness of the program in terms of development of classroom practices with technology. We notice that in their work the focus is mainly on technology use in class, as the aim of the teacher education program. The choice of tasks, the structure and organisation of the teacher education program, prospective or practicing (which can be considered as the orchestration of this teacher education program; Trouche, 2004) seems less important for these authors. The interest of mathematics education research for such orchestrations of teacher education programs has increased over the years. Moreover, the increased access to and availability of digital curriculum resources has raised interest in the consequences of such resources' use in terms of teacher learning. This can develop in a variety of contexts, including teachers' daily practice of lesson preparation, for example.

The documentational approach to mathematics didactics (Gueudet, Pepin, & Trouche, 2012; Trouche, Gueudet, & Pepin, 2018) is a theoretical frame that has been developed in this context: the daily work of mathematics teachers. It proposes a particular interpretation of the interactions between teachers and resources (including educational technology and digital curriculum resources), and of the consequences of such interaction/s in terms of teacher professional learning. The research question we answer in this chapter is the following: How does the documentational approach to didactics inform the processes in teacher professional learning resulting from teacher interaction with digital curriculum resources and educational technology?

We draw on the research literature to answer this research question: mathematics education research using the documentational approach to didactics (including our own works) but also other studies concerned with mathematics teacher learning through teacher interaction with resources. In particular, we provide "windows" from projects that illustrate teachers' interactions with particular digital resources.

After this introduction, we first present the documentational approach to didactics. We then consider research works addressing contexts where teachers interact with digital resources designed for or aimed at teacher education. This includes online teacher education programs, but also online and open educational resources offered to teachers to support their work. We next focus on teacher professional learning resulting from their design work in the context of their "usual" professional activity, and/or from their involvement in collective work with colleagues. In the final section we answer the research question, and present and explain our insights developed from the studies reviewed, seen through the lens of the documentational approach to didactics.

THE DOCUMENTATIONAL APPROACH PERSPECTIVE

The documentational approach to didactics (Gueudet et al., 2012; Gueudet & Trouche, 2009) acknowledges the central role of "resources" for teachers' work. It is linked to an understanding of a resource, which was anchored in Adler's (2000) work: this defines a resource as anything likely to "re-source" the teacher's work (e.g., curriculum material/s; a conversation with a colleague). It draws on the "instrumental approach" (Rabardel, 1999; Trouche, 2004) which has been used in mathematics education to study the interactions between students and educational technology. It enlarges the scope of the instrumental approach to encompass different kinds of resources, including digital curriculum resources, and to consider the interactions of teachers with these resources.

The documentational approach to didactics (as the instrumental approach) maintains two main concepts introduced by Rabardel (1995): instrumentation; instrumentalization. For performing a teaching task, a teacher interacts with a set of resources. This interaction combines two interrelated processes: first, the process of instrumentation, where the selected resources support and influence the teacher's or user's activity, that is, they represent an interface between the knowledge, goals, and values of the author (of the resource) and the user. Second, there is the process of instrumentalization, where the teacher or user adapts the resources for his or her needs. Brown (2009) claims that curriculum materials require craft in their use; they are inert objects that come alive only through interpretation and use by a user or practitioner.

This productive interaction between an individual teacher/user, or a group of teachers/users, and a set of resources, guided by a teaching goal, through successive stages of (re-) design and implementation in class, results in a new (hybrid) entity, the "document": defined as a mixed entity integrating a material component (the resources gathered for a given teaching objective), a practice component (the usages of these resources) and a cognitive component (knowledge guiding these usages) (Trouche, Gueudet, & Pepin, 2018). In other words, a document consists of the resources adapted and re-combined; and the ways the teacher/s use/s them ("usage scheme/s" according to Vergnaud, 1998), which include the stable organizations of associated activities and particular usages, and contain the 'knowledge' guiding the usages. The documentational approach to didactics labels this process of developing a document documentational genesis (Figure 5.1).

The different documents developed by a teacher/user are not isolated, but organized in a structured system. This system encompasses the resources and the associated 'usage schemes'; and the resources part constitutes the teacher's resource system. The documentation work can be individual, but it also takes place in groups of teachers. We have evidenced in previous works (see e.g., Gueudet, Pepin, & Trouche, 2013) that the emergence of a teachers' community of practice (Wenger, 1998) is strongly linked with the emergence of a resource system shared by this community.

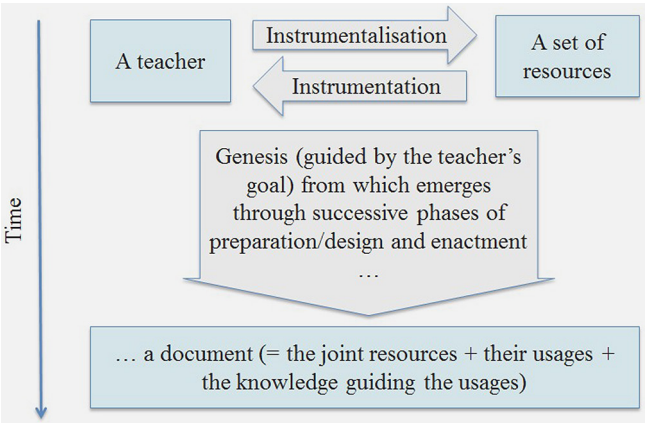

Figure 5.1. A schema of a documentational genesis

USE OF DIGITAL RESOURCES FOR TEACHER EDUCATION (INSTRUMENTATION)

In this section we consider settings where the aim of teacher education through the use of digital resources is present. With a documentational approach to didactics perspective, this means that we will focus mainly on instrumentation or at least intended instrumentation processes. We firstly consider educational resources proposed to teachers; then teacher education programs; and finally programs for teacher educators.

Educational Digital Resources

For quite some time research in mathematics education has investigated the use of (educative) curriculum materials as a mean to produce evolutions in the teachers' practice and knowledge, in particular in the context of reforms (Ball & Cohen, 1996; Davis & Krajcik, 2005). We claim that digital curriculum materials offer new means for this educative aim, and that some institutions already use these means to shape teachers' work. We discuss this claim below, drawing on examples of on-going research works.

The offer of digital curriculum material naturally leads to evolutions in the kinds of resources available for the teachers, which can have consequences in terms of teachers' practices. For example, on the *Digital Educational Resources Bank* [DERB] in France, teachers can find videos about some mathematical methods or notions. This possibility supports the integration in their resources systems of such videos, and can lead to new practices, like flipped classroom, or at least a possible autonomous access of the students to the videos if they need support to understand a given part of the course.

Nevertheless we claim that this is not the main evolution resulting from digital curriculum materials. The platform in France presented in window 1, or similar platforms used in other countries, offer many possibilities for teachers' design and sharing of resources. They can contribute (with other websites offering open educational resources) to enrich the teachers' resources systems (Trouche et al., 2018). But we argue that they are also designed with the intention to influence teachers' documentation work: their choices in terms of organisation of the mathematical content, how they associate resources. For example, in Denmark (Tamborg, 2017), the use of learning platforms is compulsory since the 2016–2017 school year. This decision from the ministry was taken in the context of a new curriculum, objective-driven. Thus one of the aims for imposing the use of these platforms was to lead the teachers to build each of their lessons according to a precise learning objective. One of the platforms used in Denmark, MinUddannelse, even "requires teachers to define a learning objective as the initial step of planning a lesson" (Tamborg, 2017, p. 2431). Here the educational aim appears clearly. It is also present in the case of the French platform DERB, with the possibility to choose resources according to the official competencies.

Window 1: A national resources platform in France

The "Digital Educational Resources Bank" (DERB,[1] Figure 5.2) is a national platform in France. In 2015, the ministry of education launched a call for tenders concerning the production of digital educational resources for different subjects (including mathematics), corresponding to a new curriculum for grades 6 to 9, starting in September 2016. Several private publishers (of paper textbooks) answered and were retained. The DERB is freely accessible for all teachers.

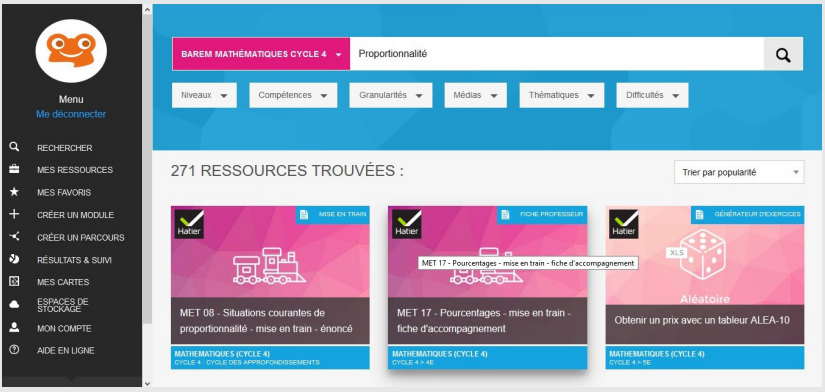

Figure 5.2. A screen of the French Digital Educational Resources Bank. For the keyword "proportionality," 271 resources are available, and can be used to build lessons

Various kinds of contents and media can be found on the DERB: static texts of courses and exercises; interactive exercises, videos, mind maps etc. are the elementary "bricks." The DERB also proposes "modules" associating several bricks. The teacher can also build his/her own modules, associating different bricks found on the DERB but also his/her own resources (created by him/herself or downloaded on another website). When a teacher creates modules for his/her students, he can keep them private or publish them, to share with some chosen colleagues or to publish them at the national level.

The DERB has been offered to teachers by the institution with a clear intention to foster professional development through what we interpret as individual or collective documentation work. The use of the DERB could lead, through instrumentation processes, to evolutions in the teachers' practices corresponding to the new curriculum and more generally to current institutional recommendations, for example:

- The DERB contains indeed resources corresponding to new contents of the curriculum, like algorithmics. More generally many bricks are devoted to the use of various software;
- Some bricks are called "mise en train" (warmers), and propose short activities to start the course (an institutional recommendation);
- Some modules indicate possibilities to propose different exercises to different students, with the aim of managing the heterogeneity of the class (an institutional priority);
- The bricks can also be searched for or selected through official competencies of the curriculum.

Nevertheless, the DERB is only used for the moment by a minority of mathematics teachers. The analysis of answers to an online questionnaire (Gueudet, 2018) on the DERB use indicate that only 19% of the teachers who answered the questionnaire actually used the DERB, the others considering that they miss time to discover the resources (there is no specific teacher education program associated with the DERB). Some interesting emergent uses are described in the answers, in particular uses of the videos to foster students' autonomy, sometimes in a flipped-classroom organisation.

We consider this as a very important evolution, in terms of the educational potential of the resources. Indeed with this new kind of resources, the instrumentation processes do not only concern the mathematical content (e.g., proposing new contents corresponding to a new curriculum), or the pedagogical approaches (e.g., proposing for example the use of dynamic geometry software, or inquiry-based tasks). The platforms propose tools to support teachers' individual or collective documentation work; the features of these tools can shape the documentation work itself.

Depending on the national context (e.g., when teachers are obliged or expected to use these platforms), these new resources can have an important impact on teachers' practices.

Digital Resources and Practicing Teacher Education Programs

Over the past ten years in selected countries many teacher education programs using digital resources have been designed and implemented, side-by-side with a large number of curriculum resources being offered. As evidenced in the ZDM special issue about online mathematics teacher education (Borba & Llinares, 2012), research in mathematics education that investigates the design and the features of such teacher education programs has increased. In those studies the issues of networking and communities are central. More precisely, Borba and Llinares identify two emergent themes in mathematics education research about online teacher education: "(1) the impact of online collaboration on the constitution of communities, and the issue of sustainability and (2) the impact of online collaboration on the learning and development of teachers" (p. 702). The focus in these studies is on how technology (meaning here various digital tools allowing communicating, sharing resources, or working together at a distance) mediates teachers' collective work within online teacher education programs.

In this section our aim is to evidence how the documentational approach to didactics perspective enlightens these issues. Sánchez (2010) has investigated an online practicing teacher education course in Mexico. Introducing the notion of documentational orchestration, he studies the features of this course. He evidences the importance of fostering the collective design of tasks by teachers, leading to the development of a rich shared resource system of the student teachers. The joint emergence of a rich resource system and of a community of student teachers has been observed in several research works that studied blended teacher education programs in France. Gueudet and Trouche (2011) studied a teacher education program concerning inquiry-based teaching with dynamic geometry software for secondary school teachers; Gueudet and Poisard (2018, see window 2 below) analysed a teacher education program for primary school teachers concerning the use of the abacus, material and virtual. Both teacher education programs have a similar organization. A part of the teacher education program takes place in presence, and a part is at distance. The teacher education program uses a platform that offers different resources to scaffold the design of lessons by the student teachers. Tutorials presenting the technical aspects of the software used (dynamic geometry software, or virtual abacus) are offered, and so are examples of lessons using this software. But the most important resources are scenario and observation grids to support the collective design, implementation, observation and improvement of lessons.

Window 2: Online practicing teacher education in France, M@gistère

M@gistère is a national French platform, offering "teacher education paths": structured sets of resources for distant or blending teacher education programs. For primary school teachers, the teacher education program is supposed to last 9 hours, with at least half of it distant. The "Chinese abacus at school" training path (Figure 5.3) is one of the paths offered for primary school teachers and teacher educators, who can use it to design their own teacher education program (Gueudet & Poisard 2018).

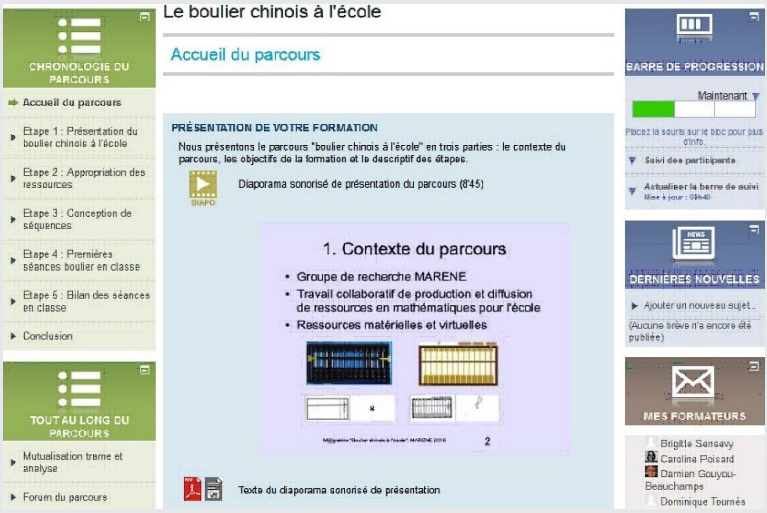

Figure 5.3. The "Chinese abacus at school" training path

The aim of the teacher education program is the integration of the Chinese abacus by teachers, both material and virtual (an online software) in their teaching of numbers and operations, from preschool to grade 5. The teacher education program is built on the documentational approach, more precisely on the link between documentation work and professional development: it proposes to the student teachers, after the presentation and appropriation of the mode of operation of the material and virtual abacii, to design their own teaching with it. The path offers different kinds of resources: videos presenting the modes of operation of the abacus, followed by online quizzes; videos of classroom uses; example of lessons, but also grids to build lesson plans and spaces to share them with other student teachers. It has been used with several teams of student teachers, who used the resources in various ways for the design of their lessons. Sometimes the adaptations depend on the material available in the class. Some

teachers have an interactive white board (IWB) in their classroom; then can project the virtual abacus on it and send a student for manipulations in front of the class. Others prefer to figure an abacus using magnets on a classical white board. But many choices of the student teachers, drawing on the resources of the training path and modifying them, are independent of material conditions. For example, some student teachers used the quizzes (designed to test their own learning of the abacus mode of operation) with their students, because they appreciated these quizzes and found them helpful to learn how the abacus works.

These teacher education programs are inspired by documentational approach to didactics: the evolutions of teaching practices are the outcomes of documentational geneses. The collective involvement of student teachers in a documentational work leads to the joint emergence of communities of student teachers and of a shared resource system. It also leads to the development of documents by the student teachers, which means an evolution of their schemes of use of resources, and of their classroom practices. For example, in the teacher education program about the use of the abacus (window 2), the student teachers used the abacus with their students, and proposed some exercises coming from the training path, like writing in digits a number inscribed on the abacus, or using the abacus to compute the result of a sum. They developed their knowledge about the teaching of integers and operations, in an instrumentation movement. They also designed their own teaching, in an instrumentalization movement, for example when they use magnets to figure an abacus on a white board. The discussions in the student teachers' teams foster the design of lessons and the didactical reflection (for example about the interest of the abacus to evidence the exchanges between tenth and units when computing a sum).

These blended teacher education programs have been used, or are still used (for the teacher education programs around the abacus) each year in different regions of France. Nevertheless, the number of student teachers remains limited, each trainer working with a maximum of 15 student teachers. The up-scaling of teacher education programs allowing collective documentational works leads to forsaking teacher education "in presence" for distance teacher education. Different Massive Open Online Courses [MOOCs] (Taranto, Arzarello, & Robutti, 2018; Panero, Aldon, & Trouche, 2017) afford such collective documentation work for teachers, and evidence the possibility of up-scaling. The MOOC "Teaching and Training with Technology in Mathematics" (Panero et al., 2017) proposes to teams of distant student teachers to develop lessons using technology, and at the same time to evaluate the lessons developed by other teams. The evaluation grid, designed by researchers, is an important resource for the student teachers. Firstly, it is a resource to evaluate the project of another team. Secondly, it becomes a resource to design their own project, because it is said to raise their awareness concerning important aspects of lessons using technology to enhance the learning of mathematics.

Professional Development of Teacher Educators

Studying mathematics teacher education also leads to consider the education of teacher educators: their role, their skills and their professional development. For many years, technology mediation has also been the focus of research on innovative professional development programs (for teacher educators) and their impact (see e.g., Kynigos, 2007). The documentational approach to didactics can be used to study the professional development of teacher educators or future teacher educators. It is indeed possible to consider that teacher educators, when designing a teacher education course (prospective or practicing), search for resources, associate them, modify them etc. Along this documentation work they develop documents. Moreover, the generalized availability of online resources also concerns trainers –some of these resources being explicitly designed for this purpose.

For example, Gueudet, Sacristan, Soury-Lavergne, and Trouche (2012) have studied the issue of the specific skills required from teachers educators to set up a blended teacher education program, and in particular its distance part. Their work is situated in the frame of a French national project, Pairform@nce, offering "training paths" on a national platform, that the teacher educator can use to set up blended teacher education programs in their different regions. Using documentational approach to didactics, the authors consider these training paths as resources for the trainers and observe that the interactions between the trainers and these resources lead to professional development for teacher educators. Some of the teacher educators involved, novice in blended teacher education, developed through these interactions new skills concerning the distant work: use of an agenda sent before the beginning of the teacher education program, writing and sending reports for all the sessions in presence, to foster the distant activity of the student teachers. In this example, the teacher educators interact with resources in the design of their teacher education program, and these interactions lead to professional development through documentational geneses. In other cases, teacher educators or future teacher educators attend specific professional development programs (Window 3).

Window 3: Becoming a teacher educator for technology-enhanced mathematics

Psycharis and Kalogeria (in press) study a teacher education program in Greece, whose aim is to educate future teacher educators, in particular to train mathematics teachers to integrate technology in their teaching.

In this program the future teacher educators are invited to design their own material and to use it in teacher education contexts, then to modify this material in an observation–reflection–design–implementation cycle.

The researchers analyse the documentational work and documentational geneses of the future teacher educators during the program.

They observe for example that some teacher educators have designed tools, like a scenario grid, to support the design of lessons by the teachers. The discussions and observations of teacher education classes have indeed lead the future teacher educators to identify the interest of teacher education programs where the teachers themselves have the opportunity to design their own teaching. Thus, instead of writing very precise lesson plans (as they do for themselves as teachers), they started writing incomplete scenarios, and providing tools like e.g. scenario grids to the teachers. These scenarios and grids are resources that the teachers can appropriate to build their own lessons.

The future teacher educators also became aware during the program of the complexity of the double instrumental geneses (Haspekian, 2014) for teachers. The teachers attending a teacher education program are not only acting as students learning how the technology works; they must also reflect in terms of didactics and pedagogy about how this technology can be used for their teaching objectives. So they designed on the one hand resources to support the appropriation by the teachers of the technological tools involved; and on the other hand to support the pedagogical use of the same tools.

The research works concerning the education of teacher educators and using the documentational approach to didactics (window 3 above) suggest for the education of teacher educators features similar to those suggested for teacher education programs: organising the teacher education program along reflection-design-implementation cycles for teams of teacher educators. For such teacher education programs, as we observed for teachers, digital means open new possibilities for design by the student teachers and for distant collaboration. Along their activity in these teacher education programs teacher educators or future teacher educators develop documents for educating teachers. These documents comprise in particular professional knowledge.

DIGITAL RESOURCES ENRICHING TEACHERS' DOCUMENTATION WORK: INSTRUMENTALIZATION AND TEACHERS' DESIGN

In this section we consider the professional learning of teachers resulting from their interactions with digital resources during their documentation work. This work can be conducted as part of teachers' individual daily work, when they prepare for instruction (at home or in school), or when they work collectively in groups or associations designing shared resources.

Teacher Documentation Work as Daily Practice

Teachers interact in their daily practice with a variety of resources, including digital curriculum resources and in particular open educational resources. In their daily documentation work, they choose, modify and implement such resources.

Window 4: documentation work as a daily practice, the case of Valeria

Valeria (see Trouche et al., 2018) was an experienced mathematics teacher, working for 29 years at upper secondary high school. In 2005-2006, she had in particular a grade 10 class; a part of the teaching is devoted to functions: examples of particular functions, variation of functions.

Valeria considered that pupils entering grade 10, coming from different lower secondary schools have very different background knowledge about functions. This conviction led her to use LaboMep, which offered in particular the online exercises of Mathenpoche. LaboMep provided her with opportunities to program/provide different Mathenpoche exercises for different students, and she chose particular exercises selected on the basis of particular mathematical objectives.

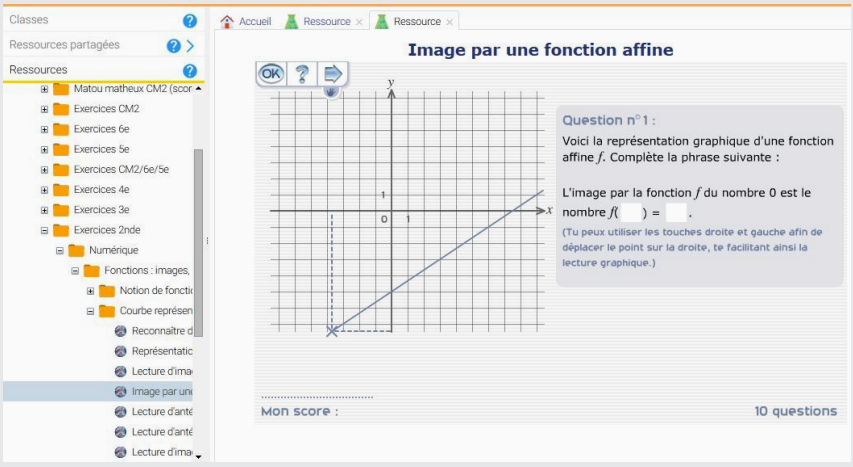

Figure 5.4. LaboMEP, choosing interactive exercises about functions

Moreover Valeria was convinced of the importance of providing rich introductory problems to start her introduction of a topic area, in this case the variations of functions. To search for such a problem, she first typed her aim into an Internet browser, "introducing variations of functions," which provided her with a list of links. At this stage she made a first choice following only the links corresponding to institutional repositories she trusted. Subsequently, she reached an institutional repository (http://eduscol.education.fr), and used its browser in a second step. With a list of 22 offers, Valeria had to choose again: she used didactical criteria associated with her objective to dismiss inadequate activities. In fact, most of the activities addressed "optimization," and not

"variations." Moreover, she also contended that the activity should allow for discovering variations, at the very beginning of the chapter, and that the activity should start from a "concrete" and authentic situation – this could be regarded as an instrumentalization process linked with her conviction that "a concrete situation fosters students' interest and motivation."

Only three activities in the list corresponded to her objectives. For each of these three she followed the link giving access to details. Then she compared these three activities with three others, found in different textbooks (on paper).

She finally retained an activity found in the Internet and entitled "Graphical approach of functions variations," because it used Geogebra.

She printed the original text and re-typed it completely. Simultaneously she adapted it for her students: she added some questions, rephrased others, modified a graphic etc. She used it in class, and noticed some possible improvements, for a future use: shorten the initial modelling activity, which was time-consuming and not directly linked with the central objective in particular.

In window 4 we observe that Valeria designs resources for her own teaching, with precise mathematical aims. She works with many different open educational resources: lesson scenarios proposed on websites, a dynamic geometry software (GeoGebra), an environment (LaboMEP) which offers in particular interactive exercises. These open educational resources enrich her documentation work; some of them are involved in documents she developed. For example, we claim that she developed a document for the aim "managing the heterogeneity of students" with LaboMEP. The features of this resource, allowing the choice of different online exercises for different students, lead to an evolution in her practice. She now programs online exercises for her grade 10 students before each new chapter. Some studies evidence such processes of teacher professional development resulting from the interactions between teachers and open educational resources or other digital curriculum resources. Some of these use the documentational approach (e.g., Gueudet & Trouche, 2012) while others do not use it, but could be also interpreted in terms of documentational geneses (e.g., Choppin, 2018). Similarly, some works evidence documentational geneses for teacher educators: for example, Gueudet and Poisard (2008) analyze documentational geneses of a teacher educator working with the "Chinese abacus training path" (window 2).

With regards to teaching (and teachers' lesson preparation), open educational resources do not only offer possibilities for a rich documentation work. They can also pose a threat to teachers' work in terms of providing curricular coherence for their students. In a time when open educational resources are increasingly available, it is imperative that teachers are provided with curricular materials, or that they develop materials, that clearly lay out well-reasoned organisations of student learning trajectories/progressions (e.g., with regard to mathematical content). A coherent,

well-articulated curriculum is an essential tool for guiding teacher documentation work, goal setting, analysis of student thinking, and the enactment of the goals/prepared curricular materials. Coherence means here that connections are made: for example, from one year to another, from one mathematical idea to the next, from one representation to another. Coherence can be applied pedagogically, logically, conceptually, and with links to relevant contexts, for example.

Although teachers and schools have now access to an immensity of digital tools and resources (e.g., on the web) for developing their instructional materials, the knowledge and skills required to develop high-quality curriculum materials is complex, and often not understood or appreciated. The risks of open educational resources can include the following:

4. Teachers choose open educational resources because there are insufficient financial resources for quality resources.
5. Teachers are provided with little or no support for choosing and organizing quality open educational resources into a coherent learning program.

At the same time open educational resources can offer opportunities for vibrant discussions about mathematics teaching and learning when teachers work collectively. We consider such cases in the next subsection.

Teacher Interaction with Digital Resources in Collectives

We have discussed in a previous section of this chapter the possibilities provided by digital resources for teacher collective documentation work as a mean for practicing teacher education. Here we consider collective work with digital resources in informal settings. Several studies evidence that professional development also takes place in these settings (see e.g., van Bommel & Liljekvist, 2016, about the professional development of teachers using a social media to discuss professional questions and to share resources). This issue can also be studied in terms of the documentational approach to didactics; we discuss this in what follows. In our window 5 below, two French mathematics teachers (at lower secondary level) work together on a newly introduced topic area, "algorithmics."

Window 5: Collective documentation work and professional development in a school

Anna and Cindy are two teachers at lower secondary school in France. They are both experienced; they work for more than ten years in the same school, and are used to work together. They are also both involved in professional groups, in particular in a group (Sésames) working with researchers in mathematics education to design resources for the teaching of algebra. Moreover, Cindy works as part-time primary school teacher educator in the teacher education

school. In September 2016, a new curriculum was introduced, incorporating in particular the teaching of algorithmics (using Scratch in particular) in the mathematics course. This content is completely new for both teachers, and they decided to prepare together their course on this topic. Their common preparation work was video recorded (Rocha, 2018), and its analysis evidences in particular how digital resources enrich their documentation work.

Anna and Cindy use repertoires of digital resources: a shared folder of the mathematics teachers of their school, a national platform (Viaeduc) mostly used by teacher educators. They also use the website of the Sésames group, and in particular a resource designed in this group called "mise en TRAIN." This resource is in fact a structured model, whose aim is to support the design by teachers of classroom activities where the students are quickly involved in an inquiry.

Along their work in Sésames team, Anna and Cindy developed some schemes; they share this way some common convictions which guided this new step in their common documentational work, like: "a new content must be encountered through problem-solving"; "learning processes are fostered by a balanced use of the symbolic and usual language." They also have some more personal convictions, e.g. "the need to be clear on the meaning of operations" for Cindy (coming from her experience as a primary school teacher educator).

Along their common documentation work for algorithmics, they discuss several aspects of the didactic approach to algorithmics, drawing on the resources they found and on their own experience for other topics (algebra in particular). They have in particular a debate on the meaning of the "variable" concept, which is different in algebra and in algorithmics (where writing for example $x = x + 1$ makes sense).

Their common documentation work draws on their previous experiences and resources. It is enriched by digital resources that they can find on different platforms. The use of a familiar digital model for the design of teaching resources ("mise en TRAIN") also supports the integration of new resources. The outcome of their work will be shared with other colleagues of their school through the shared folder.

Along their collective work, using and producing resources, Anna and Cindy developed new practices and new knowledge. Their aim was to prepare a new teaching together, concerning a new topic: algorithmics, and involving a new software (Scratch). Many online resources concerning this topic were available on different websites. In their preparation work Anna and Cindy also designed shared digital resources, drawing on their familiar digital model. These resources will be shared further, with other colleagues, using a shared folder. Along this process Anna and Cindy drew on their previous experience, but also developed new knowledge.

Many research works confirm this result. For example, Trgalova and Rousson (2017) have investigated the process of "appropriation" of a (digital) resource by a teacher, and they drew on the model of instrumentalization (i.e., adaptation and reshaping) and instrumentation (i.e., evolution of teachers' professional knowledge), which are the underpinning processes of the documentational approach to didactics. Results showed that in their case study

> the nature of the resource (a game with activities of increasing difficulty, requiring an enactment over several sessions) led the teacher to think of continuous formative assessment in order to monitor pupil progress, [and] look for the most appropriate instrument orchestrations. (p. 782)

Another two examples (for collaborative documentation work) come from the European Union funded project "MC Squared." In a study by Essonier, Kynigos, Trgalova, and Daskolia (2016), the team investigated the role of context in "social creativity" for the collaborative design of digital educational resources (e.g., c-books) with a new technology enabling the meshing of text with dynamic digital widgets. It appeared that supported by the appropriate technology, alternative, rich and promising designs, solutions and implementations were produced, due to the different backgrounds and set of personal and professional concerns of the team (as designers). Results from the study by Kynigos and Kolovou (2016) suggested that (during the design process of a c-book unit) the socio-technological environment allowed the communication and coordination of diverse perspectives. In a study by the Israeli team under Michal Yerushalmy, Naftaliev (2016) has investigated the professional learning of prospective teachers, when developing lesson plans. She focused on "interactive diagrams" (i.e. interactive text as a key component of an e-textbook). The findings showed that when analysing scenarios of classroom situations, the prospective teachers got involved with student thinking with the interactive diagrams, and they could identify and understand students' learning paths for the construction of mathematical meaning with the diagrams.

In our own work with the French e-textbook Sésamath, we have reported on the collective design (by a selected Sésamath teacher team) of a grade 10 function chapter (Gueudet, Pepin, Sabra, & Trouche, 2016). The results have shown different design processes, in particular the factors shaping the choices of content and structure for this chapter, and the implications of this design for the community.

In all these different contexts similar processes were observed. The digital means have fostered collective work, sometimes only by exchanging files via e-mail, sometimes using very elaborated platforms. Along this collective work, teachers shared resources, and discussed important didactical choices about the mathematical content, the relevant tasks for the students, the structure and organization of the content. This had important consequences in terms of teacher professional development.

Window 6: Collective and individual documentation work in the PRIMAS project

In the study by Pepin, Gueudet, and Trouche (2017) we have shown both collaborative and individual documentation work under the umbrella of the European Union supported project PRIMAS. The collective work of preparing the "designer-made" digital materials was mainly done by a team of experienced academic curriculum designers.

These teaching resources (see Figure 5.5: e.g. mathematics and science tasks, professional development modules) were made freely available on the web (in order to help to effect change in practices in terms of inquiry-based learning and teaching).

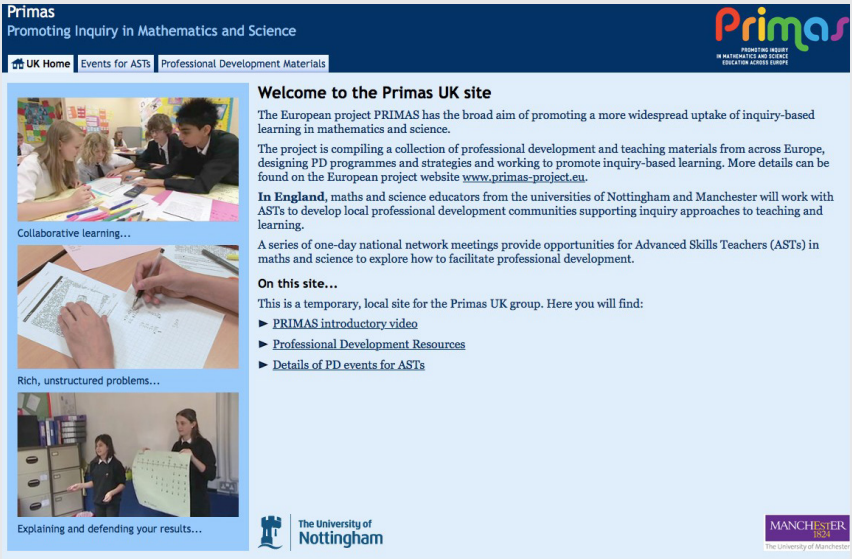

Figure 5.5. A screenshot from the PRIMAS platform

Our case teacher Cora worked both individually with those resources, as well as collectively with her colleagues. She adapted the PRIMAS materials to suit her own instruction, and she amended the professional development modules to work with her teacher colleagues. The goal of her adaptations was to enhance/develop her teaching, so that students could gain first-hand experience of scientific inquiry. From interviews we collected evidence about Cora's developing professional knowledge, in terms of: (a) clearer and more goal-directed teaching preparations; (b) enhancement of principled lesson

preparations in terms of "latching onto pupil thinking" and formative assessment; (c) enhancement of didactic flexibility with respect to differentiation and preparation of differentiated tasks; (d) in the moment "enactment" of principled beliefs (e.g. question posing).

CONCLUSIONS

In this chapter we studied the following research question: How does the documentational approach to didactics inform the processes in teacher professional learning resulting from teacher interaction with digital curriculum resources and educational technology?

The documentational approach considers teachers' documentation work: searching for and choosing resources, modifying them etc., as central in teacher professional activity. This documentation work takes place in a teacher's daily activity, when she or he prepares and sets up his or her teaching in class. It also takes place in different groups, or particular settings: when the teacher works with a colleague in his or her school, when she or he attends a teacher education program. The documentational approach also invites to consider teachers' life-long learning trajectories, linking all successive or simultaneous events: Rocha (2018) has called these teachers' documentational trajectories. Digital resources play important roles in these trajectories.

As evidenced in the second section, digital curriculum resources and digital means offer specific possibilities for the education of teachers (and also of teacher educators). The collective documentation work, drawing on digital resources, using shared folders or other distant communication means, and producing shared digital resources, fosters professional development. Outside of structured teacher education programs, educative (digital) resources (Davis & Krajcik, 2005; Pepin, 2018) can also lead to professional evolutions, through instrumentation processes. In several countries "educative platforms" are offered by the institution. The documentational approach to didactics enlightens this as a significant recent evolution, going beyond the mere offer of educative curriculum resources. The platforms propose digital tools for designing lessons that shape the documentation work itself and foster collective processes to share the resources designed.

The documentational approach to didactics evidences the strong links between instrumentation and instrumentalization processes. Even in the settings where resources are offered with a clear educative aim (e.g., practicing teacher education programs), teachers are engaged in design work developing instrumentalization processes. We have observed in a previous section of the chapter that these processes can take place during their individual daily work; and that they are especially developed within collective work (e.g., in design teams). Digital curriculum resources open new possibilities in particular for teacher design. The recently introduced field called "curriculum ergonomics" (Choppin, Roth Mc Duffy, Drake, & Davis, 2018)

incorporates research works with different theoretical perspectives (including documentational approach to didactics), investigating issues closely related with those listed here, with a focus on resource features. For example, "which features of curriculum resources can be designed to achieve a given educative (for teachers) purpose?"

In an ideal situation, teachers have access to a high-quality curriculum that supports them to make informed choices about choosing, adapting, designing and implementing tasks, and coherent learning trajectories. Moreover, teachers need time and well-facilitated work with colleagues. From the above examples it appears that the knowledge and skills required to develop high-quality and coherent instructional materials and learning trajectories is expected of mathematics teachers. In other words, they are expected to become "(co-) designers" of their own lessons and mathematical tasks. In reality teachers are often left on their own to develop, or adapt high quality materials – this is where digitalization becomes important: teachers can join design groups (and platforms) that provide support and inspiration for lesson (and progression/learning trajectory) design. Moreover, a large number of quality tasks can be found freely on the web, albeit teachers need to develop knowledge to assess their quality for their particular instructional purposes. The task/lesson design work is in principle documentation work, which is likely to contribute to their professional learning. The documentation work of teachers, in particular their design of (digital) curriculum resources, can enhance teacher learning. The collective work by teachers can also support their documentation work, and hence be an important means for teacher education.

What we have learnt from the research literature (e.g., Trouche et al., 2018) is the following: whether searching for tasks to supplement a given learning sequence, or planning learning paths through a flexible e-textbook, or adapt a given learning sequence to specific contingencies of their classroom (e.g., Visnovska & Cortina, 2018), teachers will require help and support for this documentation work (often provided in teacher collectives). This is particularly relevant at times of curriculum change, as Ball and Cohen (1996) have argued – they regarded curriculum materials as a lever for effecting change in classrooms.

Arguing that documentation work can be regarded as design (e.g., Pepin et al., 2017) is in line with a range of cognitive theories that

> emphasize the vital partnership that exists between individuals and the tools they use to accomplish their goals. … And it is not just the capacities of individuals that dictate human accomplishment, but also the affordances of the artefacts they use. (Brown, 2009, p. 19)

These theories see this relationship in the same way as we do, as an interrelationship: that is, the activity of "designing" is not only dependent on the teacher's competence, but it is an interrelationship between the teacher(s) and the (curriculum) material(s), the teacher-tool relationship, that is at play here, and hence the affordances of the curriculum materials influence this relationship. It can also be argued that the

knowledge (and hence teacher learning) does not reside in the teacher alone, or in the curriculum resource, but is developed in the interaction and the use of particular resources. This is to say that different professional expertise (e.g., Pepin, Xu, Trouche, & Wang, 2016) is, and will be, developed when working with interactive digital rather than traditional text resources. What exactly this professional expertise entails is to be investigated in further research.

NOTE

[1] http://www.barem-hatier.fr/

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